Aging and Retirement

A Comprehensive Life-cycle Fund Design for the U.S. Pension System

JANUARY | 2025

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A Comprehensive Life-cycle Fund Design for the U.S. Pension System

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Contents

Executive Summary

Life-cycle funds have become the most popular default investment alternative for the U.S. retirement system, with 90% of retirement plans using a life-cycle fund as their default strategy in 2022 (Vanguard, 2023). However, these funds should be designed under certain demographic, economic, and financial uncertainties and additional parameters over the life cycle. While previous studies generally focused on these risks and parameters separately, little research concentrates on a step-by-step sensitivity analysis of asset allocation in life-cycle funds. Moreover, most academic studies use complex methodologies to optimize the long-term portfolio choice for life-cycle funds, which may be less appealing for practitioners.

This study aims to design a comprehensive life-cycle fund model in an intuitive way by including human capital risk, social security benefits, longevity risk, bequest motive, parameter uncertainty, and different types of annuities. We also conduct a detailed sensitivity analysis for the optimal portfolio allocation in life-cycle funds in terms of risk aversion coefficients, discount rates, contribution rates, permanent shocks to labor income, capital market assumptions, correlation between human capital and stock returns, social security benefits, bequest motive, and parameter uncertainty. Finally, we compare the performance of our optimized life-cycle funds with different investment strategies for the pre- and post-retirement periods.

The main findings of the study can be summarized as follows:

- The level of risk aversion, permanent shocks, and positive correlation between human capital and stock returns have a significant effect on the portfolio allocation of life-cycle funds. On the other hand, discount rates, contribution rates, and capital market assumptions do not have a substantial impact on the asset allocation between equities and bonds.

- Social security acts as a relatively safe asset in the life-cycle portfolio allocation, so that it increases the optimal equity ratio in the post-retirement stage.

- While longevity risk becomes particularly important after the age of about 90, the partial bequest motive does not have a major impact on portfolio allocation over the life cycle.

- Parameter uncertainty affects optimal equity allocation in every age group. While the share of equities relative to bonds decreases at the pre-retirement stage due to parameter uncertainty, the share of fixed annuities relative to variable annuities increases at the post-retirement phase.

- The proposed life-cycle funds outperform traditional, 100-minus age, and contrarian investment strategies, in most cases, in terms of expected utility and portfolio success rates. Moreover, they achieve the highest retirement wealth accumulation for adverse outcomes.

The study contributes to life-cycle investing and human capital, as well as the mean reversion and parameter uncertainty literature by modeling the optimal life-cycle fund portfolio structure for the U.S. pension system. To our knowledge, this study is the first to design a U.S. life-cycle fund in such a comprehensive manner with a detailed sensitivity analysis for the pre- and post-retirement stages. The findings of the study can contribute to the understanding of long-term portfolio optimization and be applied by retirement funds and asset management companies.

Section 1: Introduction

Over the past few decades, defined contribution (DC) retirement plans have experienced significant growth at the expense of defined benefit (DB) schemes in the U.S., mainly driven by the population aging phenomenon. However, as empirical evidence shows, DC plan members are largely incapable (e.g., financial illiteracy) or unwilling (e.g., behavioral biases) to make their own investment decisions, which implies the risk of having inadequate income at retirement (Benartzi and Thaler, 2007; Lusardi and Mitchell, 2011). To address this problem, retirement plans offer default funds that are automatically selected when the plan member does not actively choose an investment strategy. In the U.S., life-cycle funds (also called target-date funds) have become the dominant investment choice in default options, with 90% of retirement plans using a life-cycle fund as their default strategy in 2022 (Vanguard, 2023).

Life-cycle funds are long-term investment vehicles that automatically reduce investors' exposure to risky assets as they approach retirement. For example, in typical U.S. life-cycle funds, young investors generally have over 90% equity exposure for several decades, and this ratio decreases linearly with age until the retirement date. Life-cycle funds make possible: (i) age-based portfolio allocation and (ii) automated diversified portfolios at low cost.

According to the recent literature, there are various risks and parameters that need to be integrated into the design of life-cycle funds. The first important risk is human capital risk. Ibbotson (2007) defines human capital as the present value of future labor income. Relative to this definition, we take a step further by modelling human capital stochastically. In the context of this study, human capital risk refers to the temporary (e.g., maternity leave and short-term unemployment) and permanent shocks (e.g., permanent disability and promotion) to labor income to which individuals are subject during their working life. Permanent shocks lead to a change in the subjective expectation of labor income (Viceira, 2001; Cocco et al., 2005). The second important risk is longevity risk, which can be defined as the risk of a retiree outliving his or her retirement benefits due to improvements in life expectancy (OECD, 2008). Finally, the third important risk is parameter uncertainty, which refers to the uncertainty in predicting expected return and variance parameters from the perspective of long-term investors (Barberis, 2000; Pastor and Stambaugh, 2012; Carvalho et al., 2018).

In terms of additional parameters in our comprehensive life-cycle fund design, we include the following: (i) social security benefits; (ii) bequest motive; and (iii) fixed annuities and variable annuities. According to the U.S. Social Security Administration (SSA) (2023), nearly 9 out of 10 people aged 65 and older are receiving a social security benefit and, on average, these benefits represent about 30% of the income of people over 65. In this respect, social security is the largest source of income for most retirees and should be included in the life-cycle fund design for the U.S. pension system. Secondly, we include the bequest motive – an economic incentive to accumulate financial wealth presently for inheritance by heirs in the future – as an additional parameter in our life-cycle fund analysis. Indeed, as previous literature shows, a significant fraction of total financial wealth is motivated by the desire to leave bequests in the U.S. (Nardi and Yang, 2014; Lee and Tan, 2023). Finally, to examine the life-cycle funds' portfolio structure with different asset classes, we include fixed annuities and variable annuities in our analysis, which play a particularly important role in protecting retirees from outliving their savings.[1]

[1] It is noteworthy to mention that when we discuss the role of fixed and variable annuities in protecting retirees from outliving their savings, we assume that they have adequate accumulated assets for retirement.

While there are some studies that separately consider the risks and parameters above in designing life-cycle funds, in this paper, we aim to design a comprehensive life-cycle fund model with a detailed sensitivity analysis. To our knowledge, this study is the first to optimize a life-cycle fund in such a comprehensive way for the pre- and post-retirement stages, including human capital risk, social security benefits, longevity risk, bequest motive, parameter uncertainty, and annuities.

In this respect, firstly, stochastic human capital is modeled for the U.S. according to the methodology of Campbell and Viceira (2002) and Cocco et al. (2005). Secondly, portfolio allocations of life-cycle funds are estimated for the pre-retirement stage based on the human capital structure, which is both stochastic and correlated with stock returns. These portfolio allocations are checked with a sensitivity analysis for different risk aversion coefficients, contribution rates, discount rates, permanent shocks, capital market assumptions, and correlation coefficients between human capital and stock returns. Next, we optimize portfolio allocations of life-cycle funds with respect to longevity risk, social security benefits, bequest motive, parameter uncertainty, and annuities for the pre- and post-retirement stages. Finally, we compare the performance of our life-cycle funds with other investment approaches over the life cycle.

In line with the studies of Cocco et al. (2005), Gomes et al. (2008), and Bagliano et al. (2019), we find the following results: (i) the level of risk aversion, permanent shocks, and positive correlation between human capital and stock returns has a significant effect on the portfolio allocation of life-cycle funds. On the other hand, discount rates, contribution rates, and capital market assumptions do not have a substantial impact on the asset allocation between equities and bonds; (ii) social security acts as a relatively safe asset in the life-cycle portfolio allocation, so that it increases the optimal equity ratio at the post-retirement phase; (iii) longevity risk becomes particularly important after the age of about 90, whereas partial bequest motive does not have a major impact on the portfolio allocation over the life cycle; (iv) parameter uncertainty affects optimal equity allocation in every age group. While the share of equities relative to bonds decreases until the retirement age due to parameter uncertainty, the share of fixed annuities relative to the variable annuities increases at post-retirement phase; and (v) the proposed life-cycle funds outperform traditional, 100-minus age, and contrarian investment strategies, in most cases, in terms of the expected utility and portfolio success rates. Moreover, they achieve the highest retirement wealth accumulation for adverse outcomes.

The findings of the study can contribute to the understanding of long-term portfolio optimization and be applied by retirement funds and asset management companies. For future studies, it is recommended that the correlation of stochastic human capital with stocks be examined in detail for different business sectors. Moreover, real estate, which plays an important role in household investments, may be included as an alternative asset in the analysis of life-cycle funds.

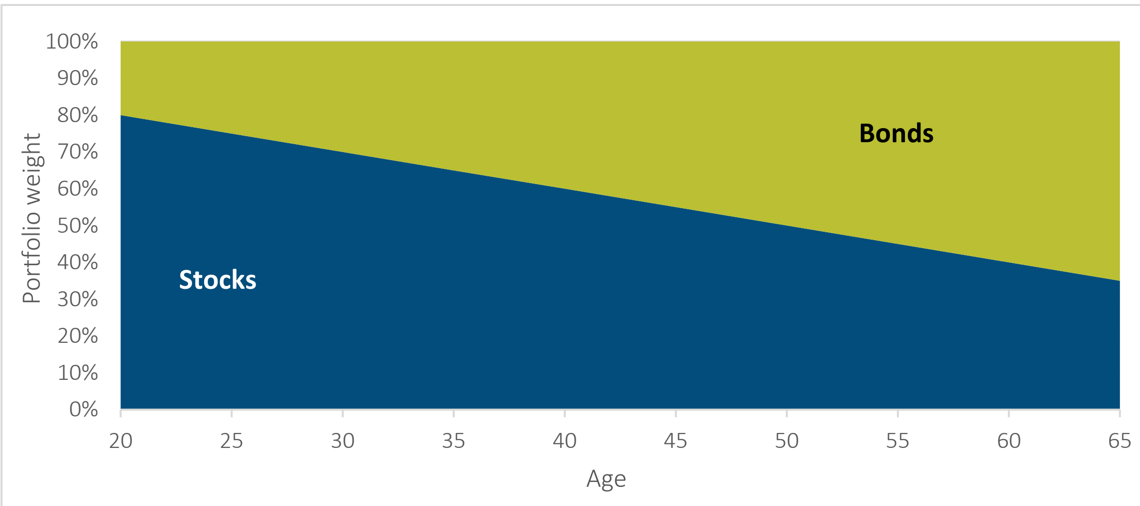
The remainder of this study is organized as follows: Section 2 reviews the related literature, Section 3 introduces the data, Section 4 illustrates the methodology for modeling human capital, parameter uncertainty, and portfolio optimization problem, Section 5 presents the results, and Section 6 discusses the implications of the findings.

Section 2: Literature Review

While in typical life-cycle funds the share of equities in total portfolio investments decreases as investors age, there are different methodologies in calculating the optimal equity ratio in the long run. According to our literature review, there are at least four approaches in academic and industry studies considered in the design of U.S. life-cycle funds, which can be described as below:

- **100-minus age rule:** The simplest alternative for formulating optimal equity ratio in life-cycle funds is the "100-minus age rule," where the proportion invested in stocks equals 100 minus one's age. For example, based on this strategy, at age 45, 55% of investments should be allocated in stocks and 45% in bonds (see Figure 1).

Figure 1
100-MINUS AGE RULE



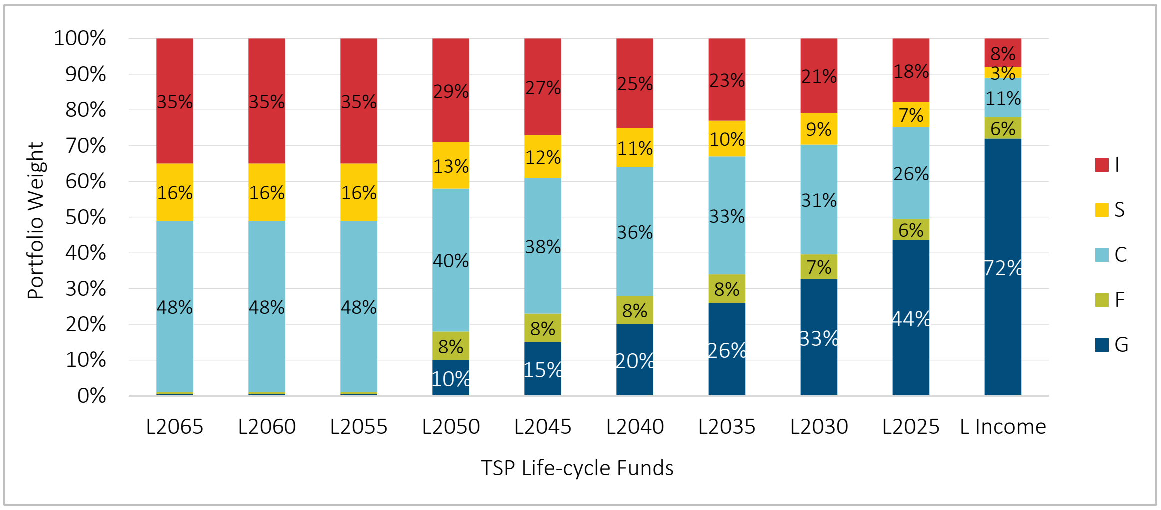
- **Thrift Savings Plan (TSP) life-cycle funds:** The Thrift Savings Plan (TSP) is a retirement savings plan for federal employees and military service members. Each of the ten TSP's life-cycle funds (L funds) consists of different combinations of five individual funds (G, F, C, S, and I) based on different amounts of investment risk (see Figure 2).[2] The portfolio of TSP funds gradually shifts to more conservative assets as investors approach retirement. The objective of the L funds is to get the best expected return for the amount of risk expected that is appropriate for retirement plan participants.

  Every quarter, the optimal asset allocation of each L fund is rebalanced daily to maintain that fund's target allocation. When a fund reaches its horizon, it will roll automatically into the L Income Fund, which generally maintains the same target asset allocation. For example, in 2030, the L2030 Fund will be turned into the L Income Fund.

[2] According to the TSP (2023), "Each of the ten TSP's life-cycle funds are professionally designed to let investors invest their entire portfolio in a single L Fund and get the best expected return for the amount of expected risk that is appropriate for them, based on when they will need their money. Investors can choose their own mix of investments from individual TSP investment funds (G, F, C, S, and I Funds). These funds include a short-term U.S. Treasury security and index funds made of stocks and bonds."

Figure 2

TSP LIFE-CYCLE FUNDS (As of July 1, 2020)

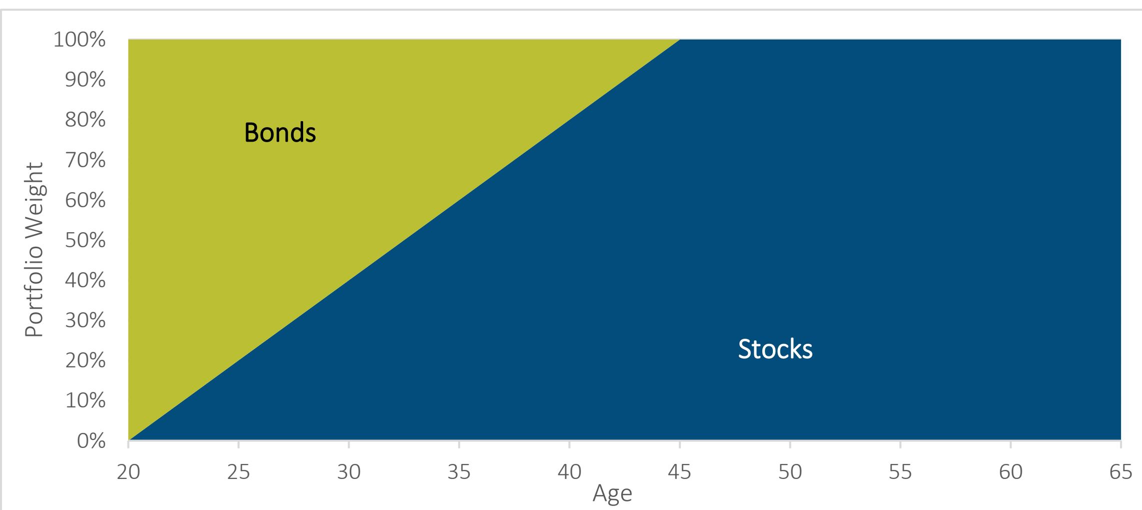


Source: TSP (2023) 'Investment Funds, Fund Options, Lifecycle Funds'

- **Contrarian life-cycle funds:** Basu and Drew (2009) show that the performance of contrarian investment strategies, where the share of equities increases as investors age, is superior compared with typical life-cycle funds. According to Basu and Drew (2009), the main reason for this fact is the 'portfolio size effect.' The size of investments is relatively small in the early years of life-cycle planning, leading to a smaller portfolio size. As retirement plan participants approach retirement, the portfolio size grows, and small changes in portfolio returns result in large differences in accumulated savings. Figure 3 shows an example of a contrarian investment strategy (20, 20), which means that, in the first 20 years, linear step increases for stocks and then holds 100% stocks in investors' portfolios.

Figure 3

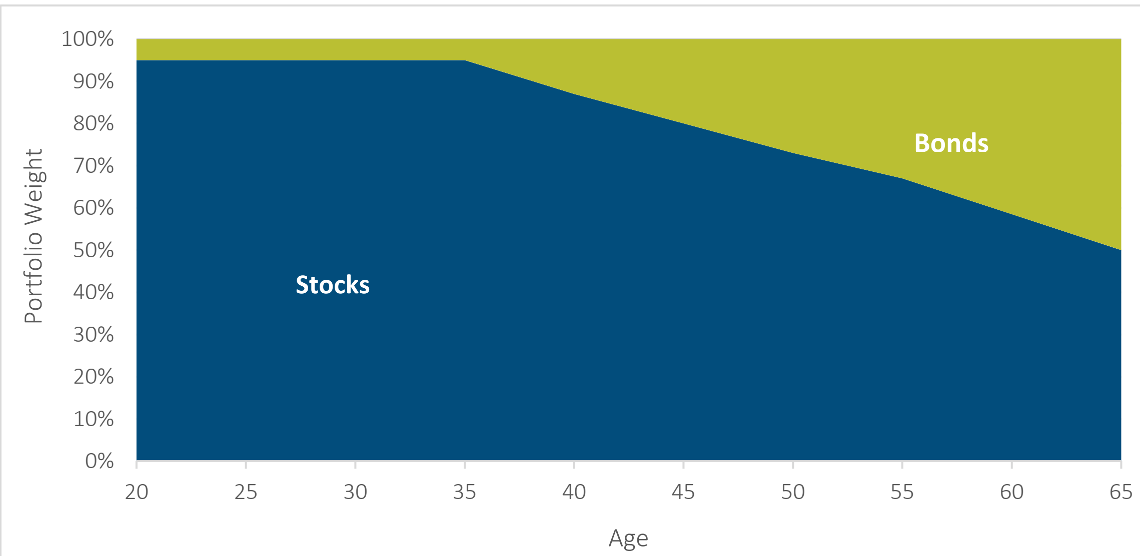
CONTRARIAN LIFE-CYCLE FUNDS (20,20)



Source: Basu and Drew (2009) 'Portfolio Size Effect in Retirement Accounts: What Does It Imply for Lifecycle Asset Allocation Funds?'

- **Human capital/financial capital framework:** This framework is related to the academic literature on life-cycle investment theory, where human capital, in addition to financial capital, is considered one of the key components of an individual's total wealth. From the perspective of this theory, non-risky human capital (the discounted value of future labor income) implies a risk-free asset since it plays a buffer role in preserving an individual's savings from market fluctuations over the long term (Bodie, Merton, and Samuelson, 1992). Given that, while young investors with riskless human capital should invest in riskier assets, older investors with less human capital should prefer a more conservative asset allocation. The practical application of this theory can be found in life-cycle funds (target-date funds) offered by U.S. retirement plans (e.g., Vanguard, Fidelity, and T. Rowe Price). These funds support a substantial allocation to equities (about 90%) for young individuals (see Figure 4).

Figure 4
TYPICAL U.S. LIFE-CYCLE FUNDS



Source: T. Rowe Price (2023)

As mentioned above, while some U.S. life-cycle funds consider the effect of human capital in long-run portfolio optimization, recent studies show that human capital should be viewed as a risky asset due to the temporary and permanent fluctuations that individuals are subject to during their working lives. Temporary shocks (e.g., maternity leave and short unemployment) have transitory effects on the future earnings of workers, whereas permanent shocks (e.g., disability and promotion) lead to a change in the subjective expectation of labor income. Depending on the degree of these random shocks, life-cycle funds may have a totally different portfolio structure from the typical U.S. type of target-date funds (Coco, Gomes, and Maenhaut, 2005; Blanchett and Straehl, 2015; Bagliano et al., 2019; Izdorek and Kaplan, 2024).

Longevity risk is another important factor in life-cycle investment choices (Lussier, 2019). There are a number of research studies in the literature that emphasize the importance of longevity risk in long-run portfolio optimization. For example, Maurer et al. (2013) examined the impact of systematic longevity risk on household portfolios and assess the effect of variable investment-linked deferred annuities on life-cycle investment strategies. Recently, Simsek et al. (2018) investigated the impact of uncertainty in life expectancy on long-term portfolio allocations. Using a multi-stage stochastic program, the study solves the optimal asset allocation problem of a retired couple with an uncertain life expectancy in the presence of a term life insurance policy.

Parameter uncertainty can also affect long-term portfolio allocation over the life cycle. Indeed, previous studies arguing that stock investments are less volatile in the long run attempted to explain this phenomenon with mean reversion (Siegel, 2021). Recent studies, on the other hand, emphasized that parameter uncertainty should be taken into account in addition to the effect of mean reversion in long-term portfolio optimization (Barberis, 2000; Pastor and Stambaugh, 2012; Hoovenaars et al., 2014; Hoang, 2023). According to the main argument in this literature, the expected return and variance parameters cannot be predicted precisely from the perspective of investors, and this uncertainty significantly affects portfolio allocation in the long run.

One of the most comprehensive studies of parameter uncertainty (Pastor and Stambaugh, 2012) showed that stocks are more volatile in the long run than in the short run. By applying the MCMC methodology, the authors considered four different uncertainty effects (i.i.d. uncertainty, uncertainty about future expected returns, uncertainty about the current expected return, and estimation risk) along with the mean reversion of stock prices. Finally, a recent study conducted by Carvalho et al. (2018) showed that, unless investors possess extreme beliefs (priors) about return and variance parameters, stocks would be less volatile in the long run, contrary to the results of Pastor and Stambaugh (2012).

In addition to the risks mentioned above, some additional parameters should also be included in the long-term portfolio optimization of life-cycle funds. In this respect, some studies considered the effect of social security benefits on the asset allocation of life-cycle funds and found that social security can act as a safe asset and provide a capacity for retirees to take an additional risk in their portfolio allocation (Gomes et al., 2008). Secondly, the bequest motive can also affect the portfolio allocation of life-cycle funds. Indeed, a significant fraction of total financial wealth is motivated by the desire to leave bequests in the U.S. (Nardi and Yang, 2014; Lee, 2023). On the other hand, few studies concentrate on the impact of bequest motive in the portfolio allocation of life-cycle funds and report that a higher level of bequest motive can reduce the risky asset allocation for the post-retirement phase (Ibbotson, 2007). Finally, in order to protect retirees from outliving their savings, annuities can play a particularly important role in long-run portfolio optimization. While most studies adopt a two-asset approach (equities and bonds) in the optimization of life-cycle funds, recent studies also integrate fixed annuities and variable annuities for portfolio choice over the life cycle (Maurer, 2013; Shoven and Walton, 2023).

To our knowledge, this study is the first to design a life-cycle fund in such a comprehensive way for the U.S. pension system, including human capital risk, social security benefits, longevity risk, bequest motive, parameter uncertainty, and annuities, and provide an extensive and detailed sensitivity analysis.

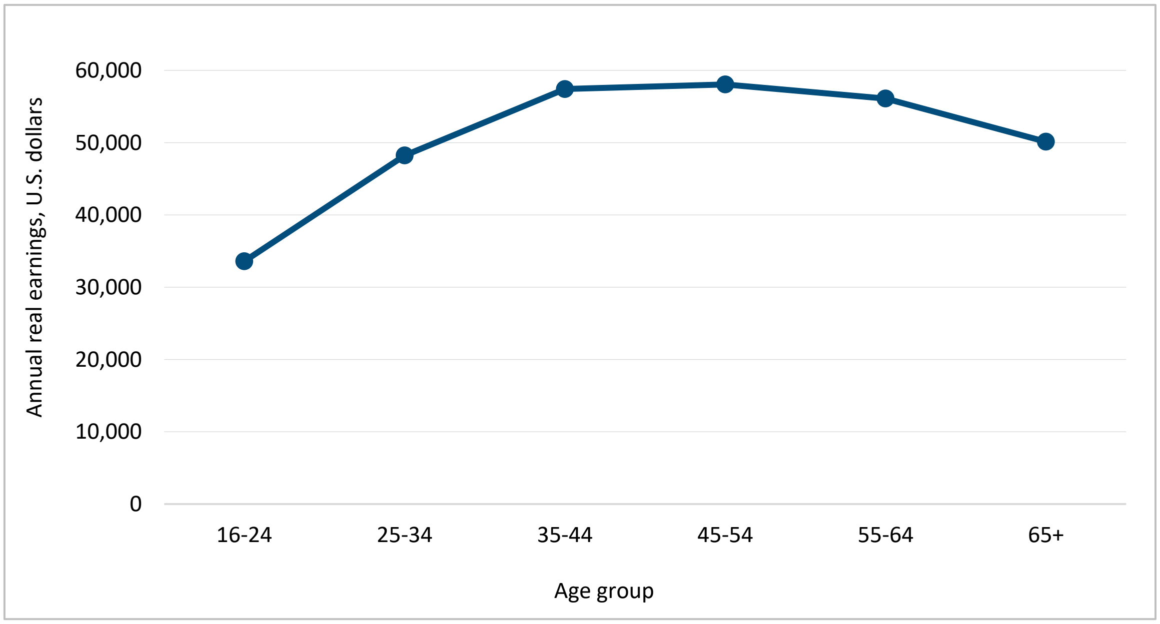
Section 3: Data Description

The present study is based on three different datasets. First, earnings-by-age data from the U.S. Bureau of Labor Statistics (2022) are used to calculate the average annual labor income based on different age groups. This data covers the median earnings of full-time wage and salary workers aged 16 and above. Annual nominal labor income is adjusted for inflation to calculate real labor earnings by age cohorts.[3]

The annual average real labor income for different age groups is shown in Figure 5. For the youngest 16-24 age cohort, labor income is $33,596 USD; it reaches its peak of approximately $57,420 USD for the 45-49 age cohort and decreases to $50,127 USD for the 65+ age group. The humpback structure of labor income in the U.S. is similar to some countries, such as Germany and Turkey (Aktug et al., 2021).

Figure 5

LABOR INCOME PROFILE IN THE U.S.



Source: U.S. Bureau of Labor Statistics

The second dataset employed in the study contains real return capital market assumptions for equities and bonds. Table 1 illustrates the annual expected return, standard deviation, and correlation coefficient assumptions for our baseline scenario.[4] These assumptions are in line with the long-term forecasts of asset management companies, such as Blackrock, Fidelity Investments, and Credit Suisse Group, in terms of equity premium. To test the sensitivity of different capital market assumptions to our results, we also consider two alternative scenarios. In the second scenario, we use assumptions from a survey conducted by Horizon Actuarial Services, which includes the average opinion of 42 different investment firms (see Table 2). In the third scenario, equity and bond returns are based on historical real return averages calculated from Damodaran's database covering the period from 1928 to 2022 (see Table 3). In each scenario, we conduct 100,000 simulations with a multivariate lognormal distribution based on these parameters.

[3] We used the consumer price index (base year = 2010), published by the World Bank, to calculate real labor earnings by age cohorts.

[4] The expected returns for equities and bonds are equal to their arithmetic means.

Table 1
CAPITAL MARKET ASSUMPTIONS (BASELINE)

Assets	Expected Return	Standard Deviation	Correlation Coefficient
Stock	5.13%	15.00%	0.00%
Bond	1.13%	5.00%	

Table 2
CAPITAL MARKET ASSUMPTIONS (HORIZON SURVEY)

Assets	Expected Return	Standard Deviation	Correlation Coefficient
Stock	5.49%	16.64%	6.00%
Bond	0.80%	1.09%	

Source: Horizon Actuarial: Survey of Capital Market Assumptions

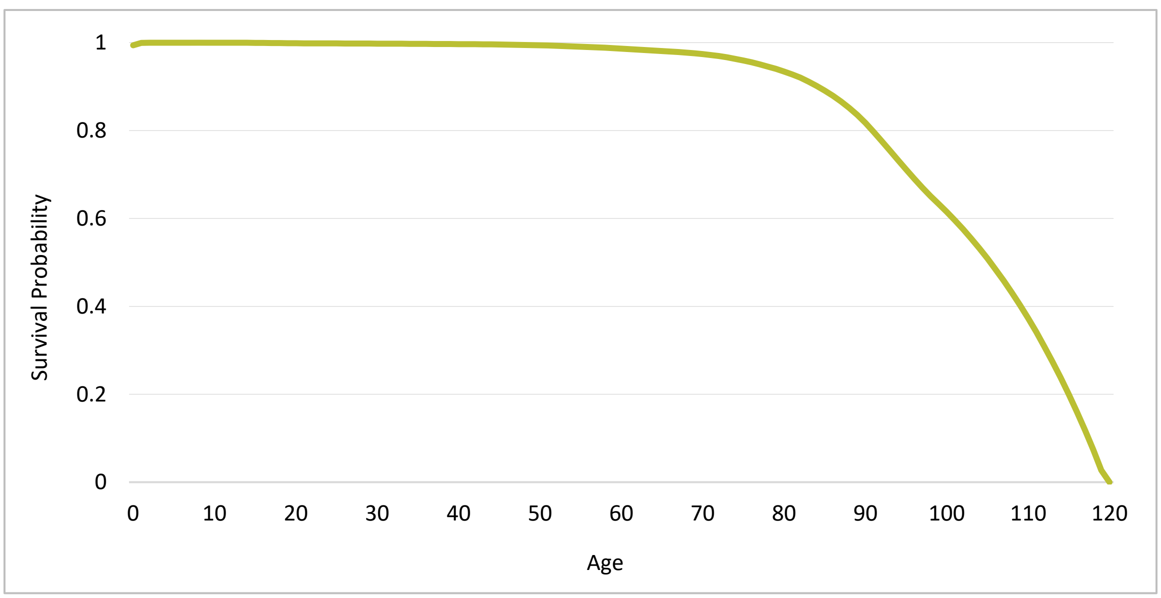
Table 3
CAPITAL MARKET ASSUMPTIONS (HISTORICAL DATA)

Assets	Expected Return	Standard Deviation	Correlation Coefficient
Stock	8.40%	19.32%	9.24%
Bond	1.86%	8.91%	

Source: Damodaran's Database

The third dataset employed in the study is survival probabilities calculated from the Actuarial Life Table of the U.S. Social Security Administration (SSA). *"This is a period life-table based on the mortality experience of a population during a relatively short period of time. It presents the life table for the Social Security area population. For this table, the period life expectancy at a given age is the average remaining number of years expected prior to death for a person at that exact age, born on January 1, using the mortality rates for 2020 over the course of his or her remaining life."* Figure 6 illustrates survival probabilities by age for the U.S. population based on this table. These probabilities are used for modeling longevity risk and are integrated into the calculation of the total wealth of households.

Figure 6
SURVIVAL PROBABILITIES BY AGE



Source: U.S. Social Security Administration (SSA)

We also use survival probabilities (p) to calculate capital market assumptions for immediate fixed annuities and immediate variable annuities in our life-cycle funds' portfolio analysis. In the context of this study, we use the definition and calculation methodology of Ibbotson (2007) for annuities and interchangeably use the terms "life-time annuity," "payout annuity," and "immediate annuity." Based on Ibbotson's (2007) definition, "a fixed-payout annuity pays a fixed nominal dollar amount each period" and "a variable annuity's payments fluctuate in accordance with the performance of the fortunes of the underlying investments chosen by the buyer of the annuity." Payments from both types of annuities are contingent on the investor's life expectancy.

Table 4 illustrates the annual real expected return, standard deviation, and correlation coefficient assumptions for fixed and variable annuities. These assumptions are applicable for the alive state since annuities do not payout when individuals die. In the death state, the expected return on both annuities is zero. Since fixed annuities provide a bond-like return, their performance is linked to the expected return on bonds. On the other hand, the investment performance of variable annuities is linked to the expected return on stocks due to their sensitivity to market risk.

Table 4

CAPITAL MARKET ASSUMPTIONS FOR FIXED AND VARIABLE ANNUITIES

Assets	Expected Return	Standard Deviation	Correlation Coefficient
Fixed Annuities	$(1+r_b)$ / p -1	σ_b	0
Variable Annuities	$(1+r_s)$ / p -1	σ_s	

Note: r_b = expected return on bonds, r_s = expected return on stocks, σ_b = standard deviation on bonds, σ_s = standard deviation on stocks, and p = survival probability. The assumptions for r_b , r_s , σ_b , and σ_s are based on Table 1.

Section 4: Methodology

4.1 HUMAN CAPITAL MODEL

In line with Cocco et al. (2005) and Bagliano et al. (2019), the logarithmic labor income of an investor is modeled as follows:

$$ln(w_x) = f(x, Z_x) + v_x + \epsilon_x$$

where $f(x, Z_x)$ is a deterministic function of the age x and income level Z_x ; $\epsilon_x \sim N(0, \sigma_\epsilon^2)$ assumes that the temporary shock follows a normal distribution; $v_x = v_{x-1} + u_x$, where $u_x \sim N(0, \sigma_u^2)$ describes the permanent shock.

The average real labor income at age x, $f(x, Z_x)$, is obtained from the U.S. Bureau of Labor Statistics dataset. The two random components in the model, representing permanent and temporary shocks, are assumed to be uncorrelated. The magnitude of the shocks as a fraction of labor income was obtained from Cocco et al. (2005) and Viceira (2009). Based on their estimation, the annual standard deviation of permanent and temporary shocks is 10.95% and 13.89%, respectively.

Following Campbell and Viceira (2002), human capital is estimated as follows:

$$HC_x = \sum_{j=0}^{J} \{w_{x+j} exp[-j(r_f + \xi)]\}$$

where HC_x is a random variable and refers to the total human capital and w_x is the labor income at age x; r_f denotes the risk-free rate; and ξ is the risk premium of human capital.[5]

4.2 PARAMETER UNCERTAINTY MODEL

The parameter uncertainty model is based on the i.i.d. stock returns at time t:

$$r_t = \mu + \epsilon_t$$

where $\epsilon_t \sim N(0, \sigma^2)$. Since the expected return μ and variance σ^2 cannot be known precisely from the perspective of investors, their distribution should also be modeled. This can be achieved by specifying the prior distributions and estimating the posterior distributions using the Bayesian approach.

In line with Barberis (2000) and Hoevenaars (2014), an investor facing parameter uncertainty is assumed to have a non-informative prior:

$$p(\mu, \sigma^2) \propto \frac{1}{\sigma^2}$$

[5] Based on Campbell and Viceira (2002) specifications, we define the total discount rate as $(r_f + \xi)$ for future labor earnings. On the other hand, in our calculations and sensitivity analysis, we did not decompose the total discount rate and used some benchmark values depending on the literature. The main reason for this approach is that even high discount rates do not have a major impact on the asset allocation of life-cycle funds.

For the non-informative prior, the posterior distribution of σ^2 is the inverse gamma distribution (Zellner, 1971):

$$\sigma^2|r \sim IG\left(\frac{T-1}{2}, \frac{1}{2}\sum_{t=1}^{T}(r_t - \overline{r})^2\right)$$

where $\overline{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$ and $r = (r_1, r_2, \ldots, r_T)$. After the variance has been sampled from the posterior inverse gamma distribution, it can be used to derive the posterior distribution of the expected return:

$$\mu|\sigma^2, r \sim N\left(\overline{r}, \frac{\sigma^2}{T}\right)$$

We combined prior information for μ with data by using the assumptions indicated in Table 1. The posterior distributions of expected return and variance are obtained after 100,000 Monte Carlo simulations.

An investor with no parameter uncertainty can choose portfolios with a distribution based on the average values of the expected return and variance parameters. In this case, the investor will use the distribution of stock returns $p(R_{T+\hat{T}}|r, \mu, \sigma^2)$ conditioned on the fixed parameters. Here, $R_{T+\hat{T}} = r_{T+1} + r_{T+2} + \cdots + r_{T+\hat{T}}$ represents the cumulative stock returns over $\hat{T}$ periods, where $\hat{T}$ denotes the investor's investment horizon. However, since the investor is exposed to parameter uncertainty, he also considers the uncertainty of the expected return and variance parameters and will use the estimated distribution $p(R_{T+\hat{T}}|r)$ based on $r = (r_1, r_2, \ldots, r_T)$ when modeling stock returns.

In this study, we consider two degrees of parameter uncertainty: medium and high. In medium parameter uncertainty, we assume that investors are confident in the parameters of stock returns (expected return and variance) as if they had observed them for 50 years. For the high parameter uncertainty, the parameters of stock returns are assumed to be observed for 20 years.

4.3 PORTFOLIO OPTIMIZATION MODEL

The optimal portfolio allocations for different age groups are analyzed for various degrees of risk aversion, contribution rates, permanent shocks, discount rates, capital market assumptions, and correlation parameters. The wealth maximization problem of an investor is defined for a constant relative risk aversion (CRRA) utility function:[6]

$$max_{w_x} E[U_{x+1}]$$

where $U_{x+1} = \frac{(FC_{x+1}+HC_{x+1})^{1-\gamma}}{1-\gamma}$ and $\gamma > 0$, and the financial capital – for stocks and bonds – follows the stochastic process that can be described as:

$$\frac{1}{I}\sum_{i=1}^{I}\{(1-w\}\exp\left(r_b\hat{T}\right) + w\exp(r_s\hat{T} + R_{T^{(i)}+\hat{T}})\}$$

FC – financial capital

[6] In order to simplify our methodology and assumptions, we used one-period portfolio optimization framework with annual rebalancing. There are also some recent studies that adopt a (multi-period) approach in their portfolio optimization framework (Moallemi and Saglam, 2017; Warren, 2019). On the other hand, these studies only consider financial capital – and not human capital – in their models.

HC – human capital
γ – risk aversion coefficient
w – investment ratio for stocks ($0 \leq w \leq 1$)
1- w – investment ratio for bonds
r_b – expected return on bonds
r_s – expected return on stocks
$\hat{T}$ – investment horizon
$R_{T^{(i)}+\hat{T}}$ – one of the two possible distributions, $p(R_{T+\hat{T}}|r)$ or $p(R_{T+\hat{T}}|r, \mu, \sigma^2)$
I – the number of simulations (100,000)
x – age

Section 5: Results

5.1 STOCHASTIC HUMAN CAPITAL ANALYSIS

The findings for stochastic human capital are reported for the representative investor who starts working at age 20 and is expected to retire after reaching age 65 (with zero human capital). The simulation process is based on 100,000 Monte Carlo replications of the labor income process with age-income-profile.

The expected human capital distributions for ages 20, 30, 45, and 60 are presented in Figure 7, along with their medians and standard deviations. The median human capital of the representative investor at age 20 is $901,510 USD, which is lower than at age 30 ($931,599 USD) and higher than at age 45 ($734,446 USD); it reaches its peak in the 30s and drops to $245,945 USD at age 60.

Human capital for all ages in Figure 7 is affected by temporary and permanent shocks to labor income. For example, the standard deviation of human capital increases from age 20 ($395,415 USD) to age 30 ($535,537 USD) and age 45 ($599,564 USD) due to the fact that investors are exposed to significant temporary and permanent shocks during their working lives. More importantly, the volatility of human capital always increases with age in relation to its median, e.g., the ratio of the standard deviation to median human capital at age 20 is less than 0.5, but it exceeds 1 at age 60 (Panel A and Panel D). The main reason for these differences is the uncertainty that is caused by permanent shocks in labor income (Viceira, 2009). While temporary shocks affect labor income equally at all ages, permanent shocks modeled by the AR(1) process grow cumulatively with age. For this reason, the standard deviation for older ages approaches and can even exceed the median, e.g., the standard deviation of human capital for a 60-year-old individual ($264,190 USD) is higher than its median value ($245,945 USD) (Panel D).

Figure 7
STOCHASTIC HUMAN CAPITAL FOR DIFFERENT AGES

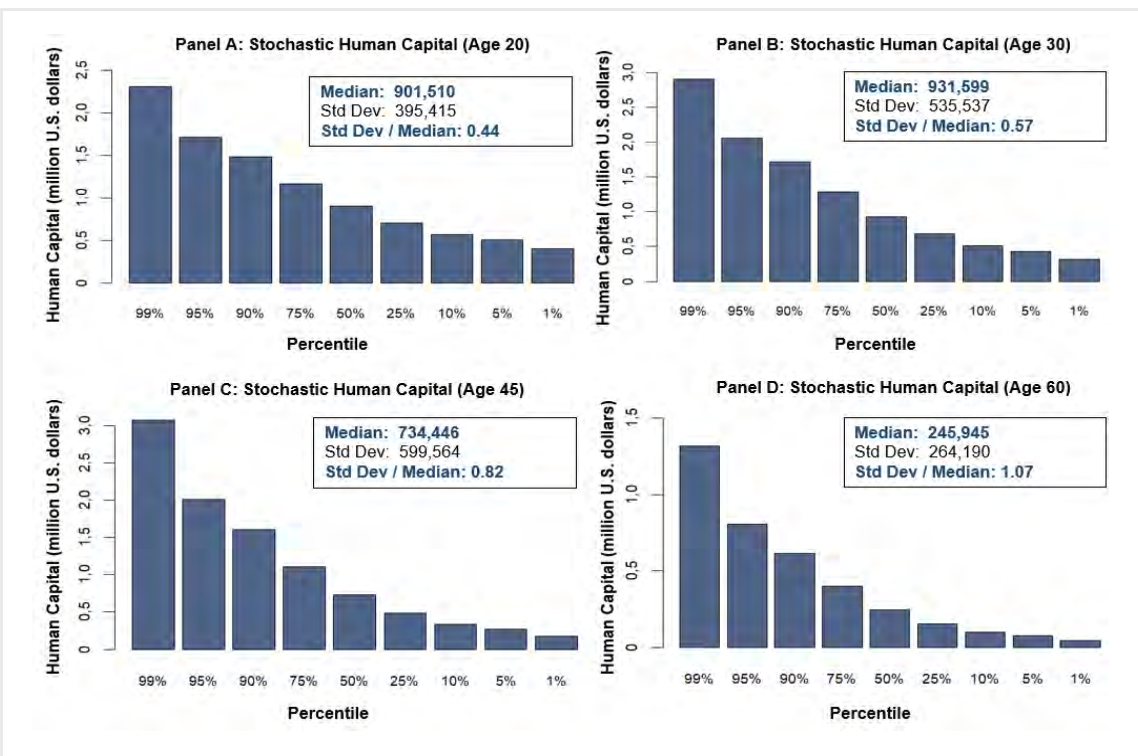
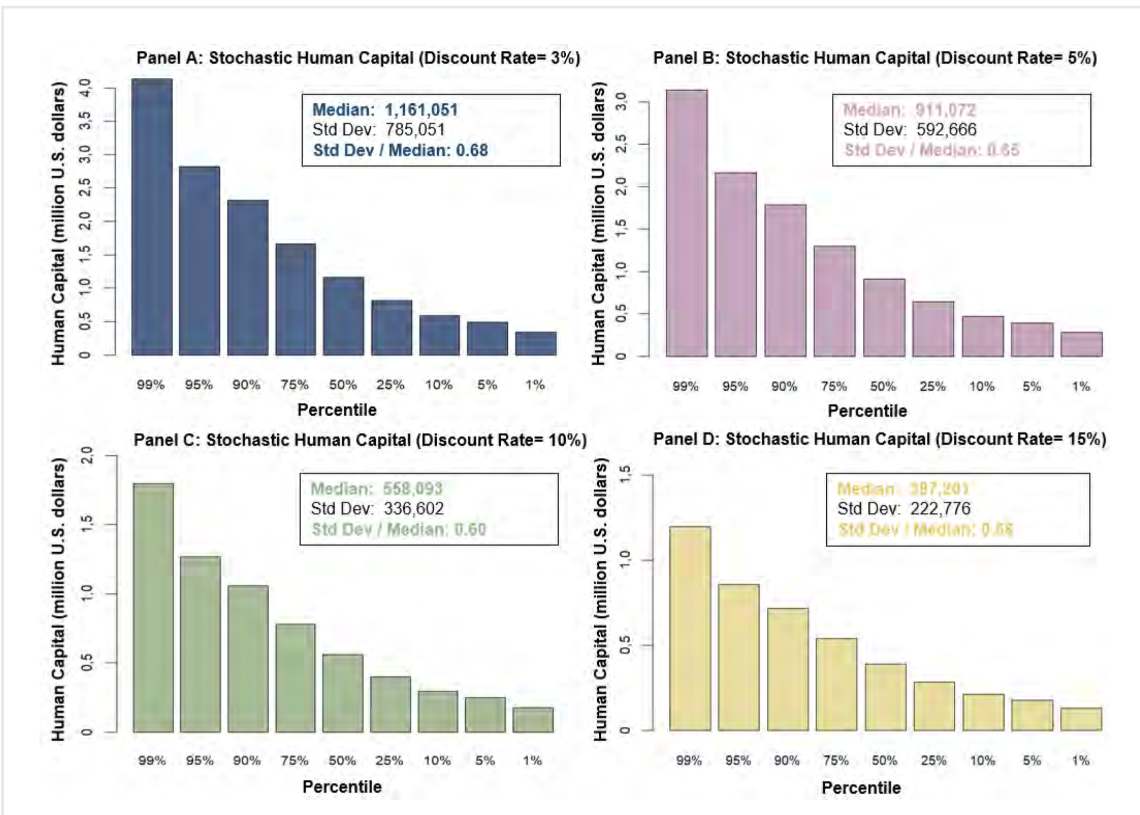


Figure 8 shows how different discount rates affect human capital distributions for the same age by comparing the distributions for the 35-year-old representative investor exposed to the 3%, 5%, 10%, and 15% discount rates.[7] As the discount rate increases, the present value of labor income decreases, leading to a lower median of human capital. For example, the median human capital of the representative investor at a 3% discount rate is $1,161,051 USD, which is higher than at a 5% discount rate ($911,072 USD), a 10% discount rate ($558,093 USD), and a 15% discount rate ($387,201 USD).

As the discount rate increases, the uncertainty resulting from permanent shocks has less effect on the present value of human capital. However, the contribution of permanent shocks remains the same irrespective of the discount rate, since the age is the same for all the panels in Figure 8. For this reason, while the standard deviation to the median human capital ratio decreases as the discount rate increases, this effect is relatively small compared to Figure 7, e.g., the standard deviation to median human capital ratio is 0.68 for the 3% discount rate, which is slightly higher than 0.58 for the 15% discount rate.

Figure 8

STOCHASTIC HUMAN CAPITAL FOR DIFFERENT DISCOUNT RATES AT AGE 35



[7] We present the stochastic human capital analysis for different discount rates at age 35, since the standard deviation to median human capital ratios have similar trends for varying discount rates at different ages.

5.2 THE OPTIMAL PORTFOLIO ALLOCATIONS (PRE-RETIREMENT PERIOD)

This section presents life-cycle funds' portfolio allocation to stocks in a detailed sensitivity analysis. First, the optimal equity ratios of life-cycle funds under stochastic human capital are analyzed in relation to different risk aversion coefficients, discount rates, contribution rates, permanent shocks, and capital market assumptions. Then, the optimal portfolios under stochastic and correlated human capital (with positive and negative correlation coefficients) are presented. Finally, the portfolio allocations are compared, assuming the human capital to be deterministic, stochastic, or correlated.

The baseline assumptions used in modeling life-cycle funds are as follows:

• The age at which the representative investor starts investment: 20;
• The retirement age of the representative investor: 65;
• Risk aversion coefficient: 5;
• Discount rate: 5%;
• Contribution rate: 10%;
• Standard deviation for temporary and permanent shocks: (13.89%,10.95%); and
• Utility Function: $U = \frac{(FC_{t+1}+HC_{t+1})^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$

$U = \ln{(FC_{t+1} + HC_{t+1})}$ for $\gamma = 1$

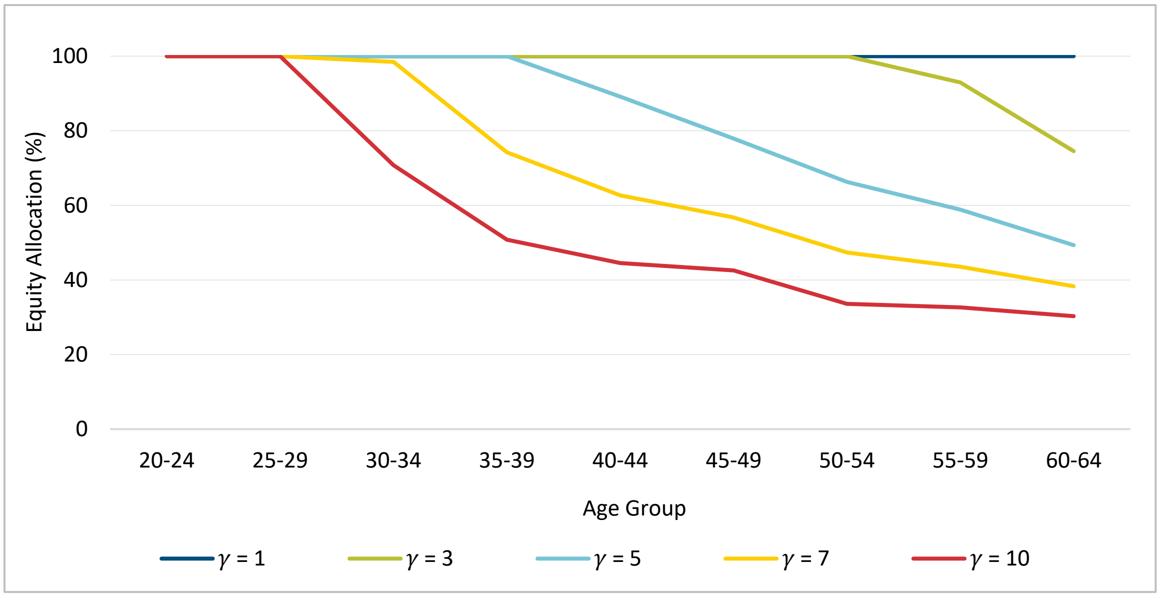
First, the optimal stock allocations of stochastic life-cycle funds for different risk aversion coefficients (γ) (1,3,5 (baseline), 7, and 10) are presented in Figure 9. The coefficient 10 is used to indicate the highest degree of risk aversion, whereas the lowest risk aversion coefficients are either 1 or 2 (Cocco et al., 2005; Azar, 2006). [8]

As it is evident from Figure 9, risk aversion greatly affects the share of the portfolio allocated to stocks. Although the representative investor with a risk aversion coefficient of 1 has risky human capital, he selects a portfolio distribution with a 100% stock ratio for all age groups, and the individual who has a risk aversion coefficient of 3 retires with an approximately 75% equity allocation while investing 100% in stocks up to age 50. This result is also similar to the findings of Pfau (2009), where 100% stock allocation is reported as optimal for individuals with risk aversion coefficients of 1 and 2.

Unlike Pfau (2010), in our study, the optimal equity ratio decreases more rapidly for individuals with a risk aversion coefficient of 3 or higher because human capital is modeled as stochastic rather than deterministic. Investors with risk aversion coefficients of 5, 7, and 10 have a much lower optimal stock allocation for the same age groups. For example, according to the portfolio optimization model, it is predicted that an individual who has a risk aversion coefficient of 7 starts reducing the 100% equity ratio after the age of 30 and retires with an equity allocation of approximately 40%.

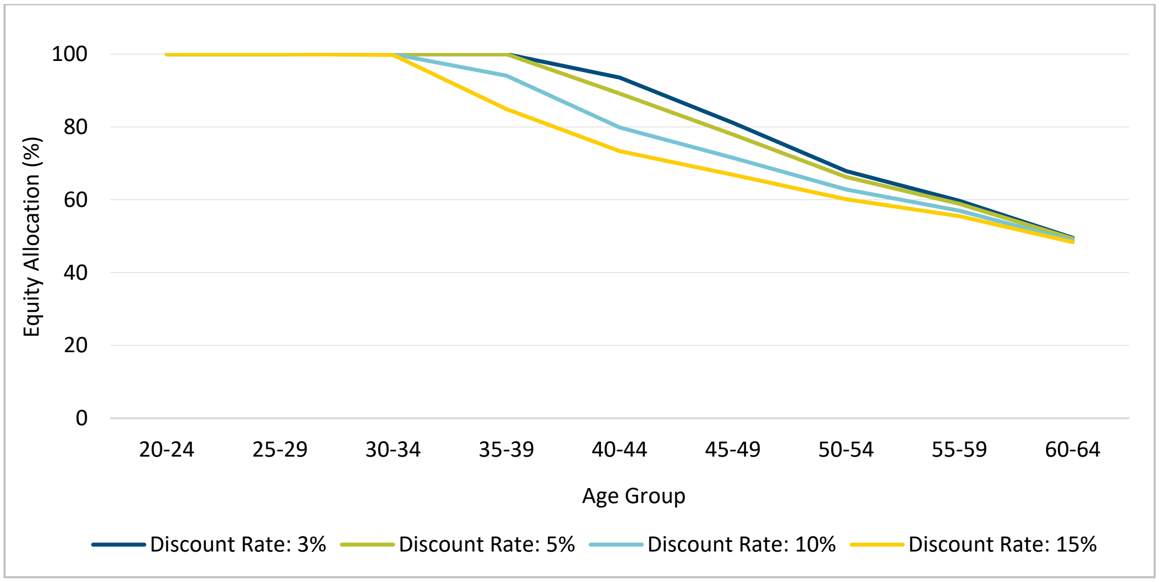
[8] Some studies report that positive estimates of the risk-aversion coefficients are around 7 to 9 (Cederburg and O'Doherty, 2017).

Figure 9
OPTIMAL EQUITY ALLOCATIONS (DIFFERENT RISK AVERSION COEFFICIENTS)



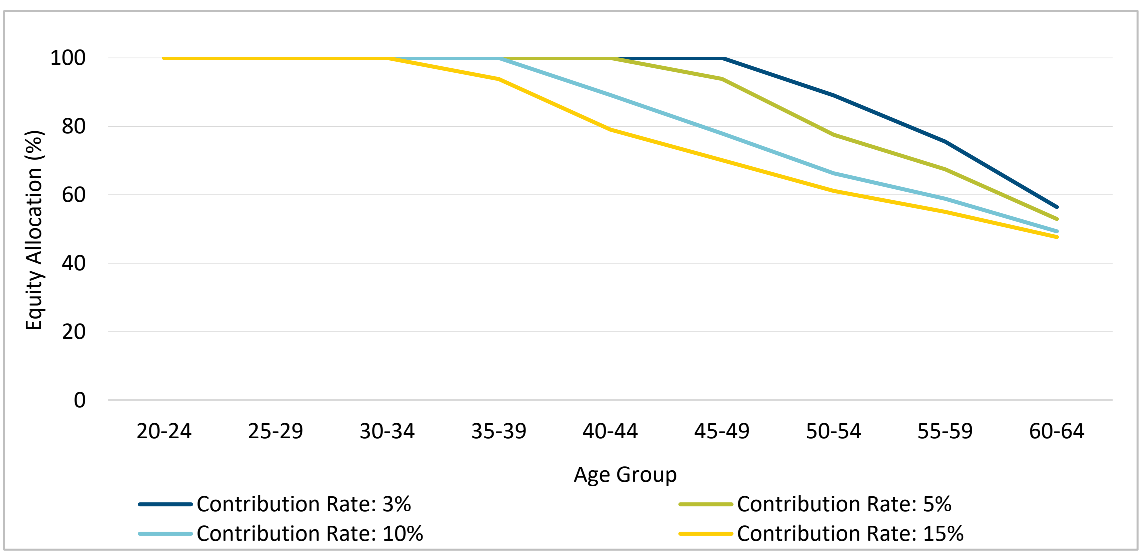
Second, the distribution of life-cycle fund portfolios is analyzed according to different discount rates (3%, 5% (baseline), 10%, and 15%). As is evident from Figure 10, no substantial differences are detected in the optimal portfolio distributions for varying discount rates. Although higher discount rates reduce the amount of human capital, the change in optimal stock allocations is small since human capital is more dominant than financial capital in the wealth maximization problem until the age of 50. Even for the 35-39 age group with the highest dispersion in portfolio allocations due to varying discount rates, the optimal equity stakes are 100%, 100%, 94%, and 85% for 3%, 5%, 10%, and 15% discount rates, respectively. However, for the 60-64 age group, the equity stake is approximately 48% for all three discount rates. The reason for this unanimity is that the stock ratio is optimized primarily for financial capital in older age groups.

Figure 10
OPTIMAL EQUITY ALLOCATIONS (DIFFERENT DISCOUNT RATES)



Third, in Figure 11, we present the optimal stock allocations of life-cycle funds for different contribution rates (3%, 5%, 10% (baseline), and 15%). When the contribution rate increases, retirement plan participants allocate a larger share of their labor income to pension savings. In this respect, higher contribution rates allow individuals to accumulate larger pension savings during the pre-retirement period. As a result of this, in higher contribution rates, participants start reducing the share of equities in the portfolio at a younger age and, in lower contribution rates, the situation is vice versa, e.g., the optimal allocation to equities starts decreasing after the 45-49 age group for 3% contribution rates, whereas it starts decreasing after the 30-34 age group for 15% contribution rates.

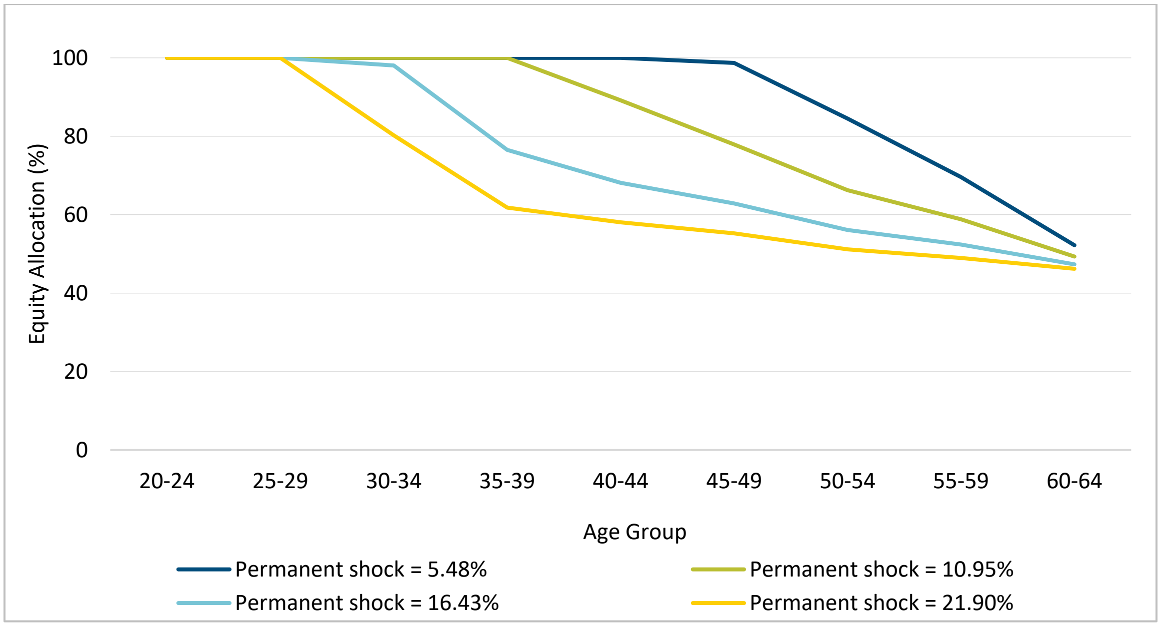
Figure 11
OPTIMAL EQUITY ALLOCATIONS (DIFFERENT CONTRIBUTION RATES)



Fourth, the optimal equity allocations for life-cycle funds at different levels of permanent shocks (5.48%, 10.95% (baseline), 16.43%, and 21.90%) are presented in Figure 12. It is clear that larger standard deviations in permanent shocks decrease the optimal share of equities in investors' portfolios. Since permanent shocks are modeled with the AR (1) process and grow cumulatively with age, they significantly affect portfolio allocation, e.g., the optimal allocation to equities starts decreasing after the 40-44 age group for 5.48% permanent shock, whereas it starts decreasing after the 20-24 age group for 21.90% permanent shock.[9] On the other hand, the optimal equity allocation for the 60-64 age group looks similar under different degrees of permanent shock. This similarity is due to the fact that the stock ratio is optimized primarily for financial capital in older age groups.

Figure 12

OPTIMAL EQUITY ALLOCATIONS (DIFFERENT DEGREES OF PERMANENT SHOCK)



Fifth, in order to test the sensitivity of our results to different capital market assumptions, we consider two alternative scenarios in addition to our baseline expectations in Table 1. The first one was obtained from the Horizon Survey, where the real annualized capital market assumptions are indicated in Table 2. The second alternative scenario is based on historical returns obtained from Damodaran's dataset and presented in Table 3. Simulations are conducted in each case with a multivariate lognormal distribution defined by these parameters.

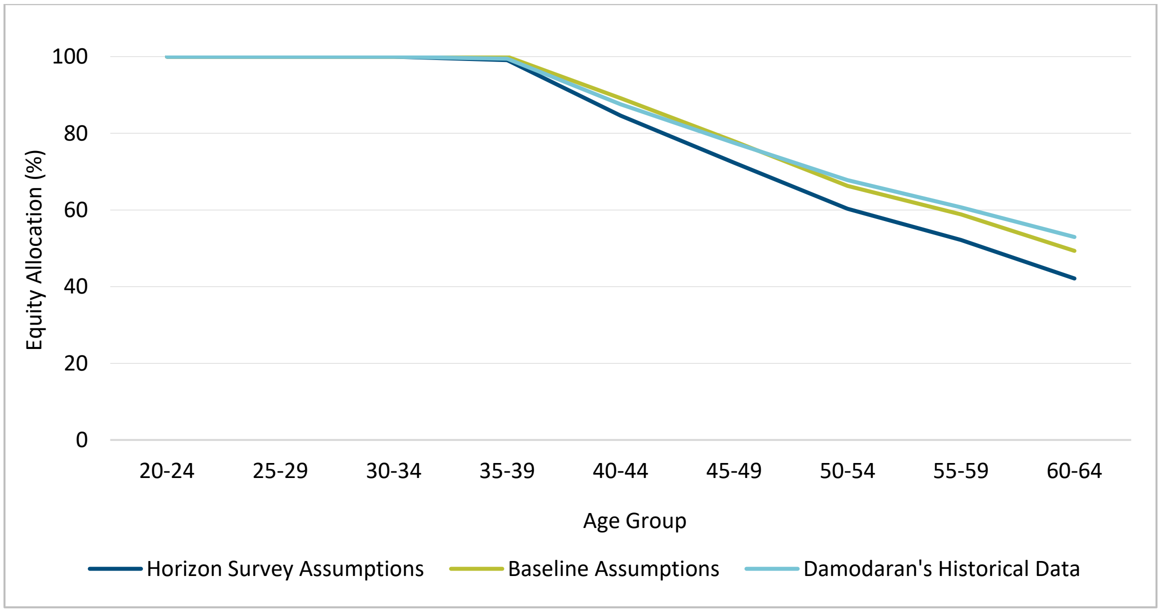
The optimal equity allocations under different capital assumptions are illustrated in Figure 13. The results under three assumptions look quite similar, particularly in the first 20-25 years of the pre-retirement period. This unanimity is due to the fact that the stock ratio is optimized primarily for human capital in younger age groups. While the optimal equity allocations start decreasing after the 35-39 age group, they are still similar for three scenarios and reach their lowest value for the Horizon Survey assumptions at about 42%. For the Horizon Survey assumptions, the standard deviations of stocks and bonds differ significantly compared to

[9] We do not provide any sensitivity analysis for temporary shocks. Since temporary shocks are equal in every age group, the optimal equity allocations are very similar for life-cycle funds under different levels of temporary shock.

our benchmark assumptions and Damodaran's dataset. Despite this fact, the largest difference between the Horizon survey assumptions and the other two scenarios is seen for the oldest age group, at about 10%.

Figure 13

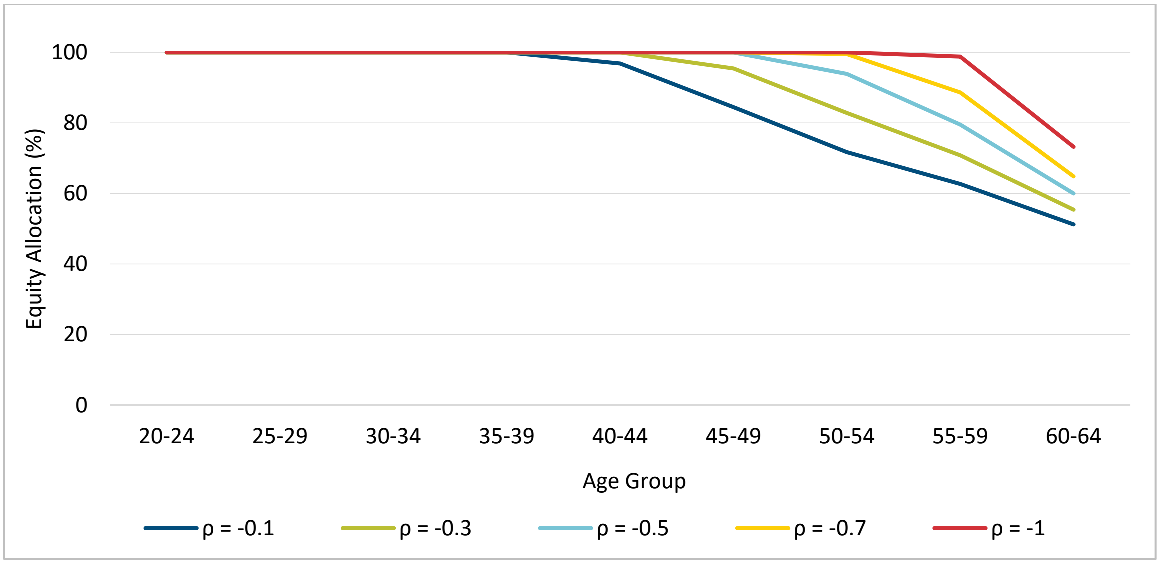
OPTIMAL EQUITY ALLOCATIONS (DIFFERENT CAPITAL MARKET ASSUMPTIONS)



Sixth, in addition to stochastic human capital, we examine optimal equity allocations for life-cycle funds under stochastic and correlated human capital. Portfolio distributions of life-cycle funds for different negative correlation coefficients (ρ) (-0.1, -0.3, -0.5, -0.7, and -1) are reported in Figure 14.[10] It is evident that, as the correlation between human capital and stock returns becomes more negative, the stock allocation for each age group increases. In other words, based on the perspective of classical portfolio optimization, the optimal stock ratio increases because the negative correlation between human capital and stocks provides diversification benefits.

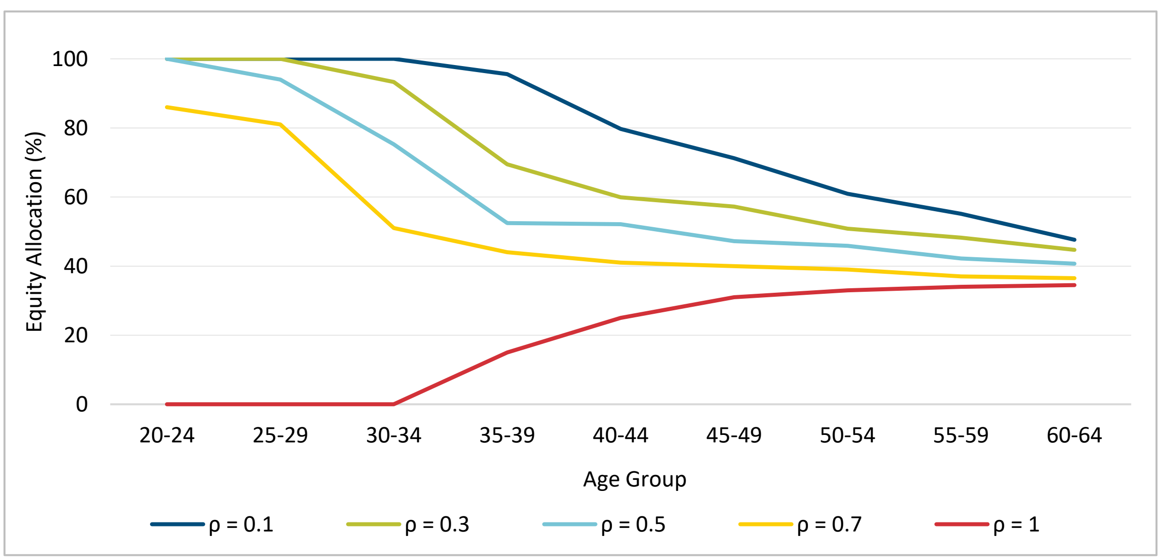
[10] It is worth mentioning that, generally, there is a positive correlation between employees' human capital and stock returns (Bikker et al., 2012; Blanchett and Straehl, 2015).

Figure 14
OPTIMAL EQUITY ALLOCATIONS (DIFFERENT NEGATIVE CORRELATION COEFFICIENTS)



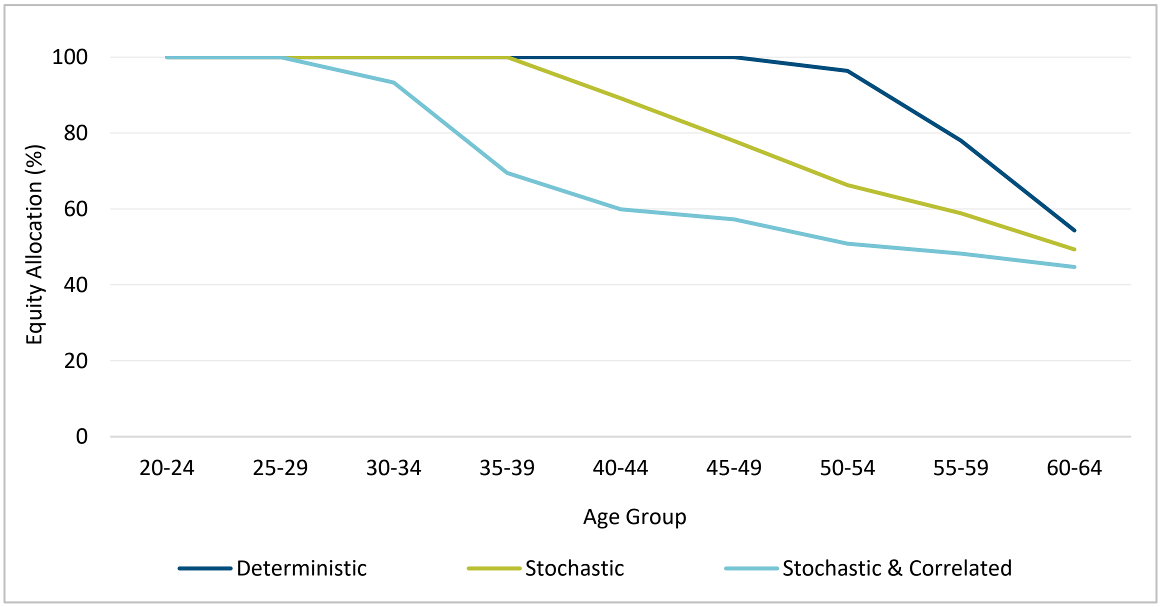
Seventh, we present life-cycle fund portfolios optimized for different positive correlation coefficients (ρ) (0.1, 0.3, 0.5, 0.7, and 1). As can be seen in Figure 15, as the positive correlation between human capital and stock return increases, the optimal equity allocation decreases up to the 70% correlation. In particular, in the case of perfect correlation (100%), portfolio distributions demonstrate increasing stock allocations with age, which is strikingly different from the distributions optimized for lower correlations when the equity share is typically falling with age. This finding is in line with the studies of Campbell and Viceira (2002) and Cocco et al. (2005), who reported that, with very high correlations between human capital and stock returns, optimized life-cycle funds could allocate low-risk assets for young investors.

Figure 15
OPTIMAL EQUITY ALLOCATIONS (DIFFERENT POSITIVE CORRELATION COEFFICIENTS)



Finally, the distribution of life-cycle fund portfolios modeled for deterministic, stochastic, or correlated human capital is presented in Figure 16. It emphasizes the importance of risky and correlated human capital in determining the optimal portfolio distribution. Here, the correlation between human capital and stock returns is assumed to be 30%, in addition to the previously reported assumptions. As shown in Figure 16, the optimal equity ratio to be allocated under the assumption of risk-free human capital is 100% up to the 45-49 age group and starts decreasing after the 35-39 age group if human capital is stochastic. For stochastic and correlated human capital, the optimal equity ratio starts decreasing in the 30-34 age group and reaches about 45% stock allocation at retirement.

Figure 16
OPTIMAL EQUITY ALLOCATIONS (DIFFERENT HUMAN CAPITAL ASSUMPTIONS)



In line with the studies of Cocco et al. (2005), Gomes et al. (2008), and Bagliano et al. (2019), our analysis clearly demonstrates the importance of stochastic and stochastic and correlated human capital in determining long-run portfolio distributions. For example, if an employee in the construction industry has a portfolio of stocks mostly belonging to this industry, then there is a positive and strong correlation between the human capital and stock returns for that employee. In this context, such an employee should have a very different stock allocation compared to the classical life-cycle funds offered in the U.S.

5.3 COMPARISON OF INVESTMENT STRATEGIES

In this subsection, we compare the investment performance of our proposed life-cycle funds with other investment approaches, such as 100% bonds, 100% stocks, the 60/40 investment strategy (60% stocks, 40% bonds), the 100-minus age strategy, and the contrarian approach.[11] We considered both retirement wealth distributions and expected utility analysis in the comparison of investment strategies.

Table 5 illustrates six different investment approaches in terms of the distribution of total financial wealth at retirement. To facilitate the interpretation of our results, the table reports the mean and percentiles of retirement wealth distribution as a multiple of the expected final annual salary, which is indicated in the bottom row of the table. Since the final salary is based on deterministic labor income, its distribution exhibits the same values regardless of percentiles.

Overall, as expected, the 100% stock investment strategy provides the highest average retirement wealth, equal to about 15 times the expected final salary. This strategy also has the highest retirement wealth for upper percentiles (90th and 75th) due to the high expected return on stocks. On the other hand, for the lowest percentiles (1st, 5th and 10th), only stocks investment strategy has the second-worst outcome after the contrarian approach due to its high volatility.

Our life-cycle fund strategy, modeled with deterministic labor income, provides the second highest average retirement wealth, equal to about 13 times the expected final salary. The investment performance of the life-cycle fund lags behind 100% stocks and contrarian investment strategies for the 90th and 75th percentiles, whereas for adverse outcomes, such as 1st, 5th, and 10th percentiles of wealth distribution, it provides the highest retirement income. Moreover, in terms of average retirement wealth, the life-cycle fund has similar investment performance to 100% stocks. The contrarian investment strategy has upside potential and closely follows the performance of 100% stocks up to the median of the distribution. On the other hand, its performance decreases significantly for the lower percentiles of distribution. While the investment performance of the 60/40 strategy and the 100-minus age is similar, the former outperforms the latter, except for the 1st and 5th percentiles. The only bonds strategy has the lowest volatility among other strategies; however, it accumulates insufficient wealth for the majority of percentiles.

[11] In our analysis, we adopted the contrarian investment strategy (20, 20), which means that, in the first 20 years, linear step increases for stocks and then holds 100% stocks in investors' portfolios. The detailed information on the contrarian approach and the 100-minus age rule has been given in the literature review part.

Table 5

COMPARISON OF INVESTMENT STRATEGIES (DETERMINISTIC LABOR INCOME)

Strategies	Mean	90%	75%	50%	25%	10%	5%	1%
100% Bonds	5.5	6.8	6.1	5.4	4.7	4.2	3.9	3.3
100% Stocks	14.8	27.7	18.4	11.6	7.5	5.2	4.2	2.5
60% Stocks / 40% Bonds	9.8	15.0	11.7	8.9	7.0	5.5	4.5	3.6
100-Minus Age	8.9	13.3	10.6	8.4	6.7	5.4	4.7	3.9
Life-cycle Strategy (Deterministic)	13.3	23.2	16.5	11.1	7.4	5.7	5.1	4.1
Contrarian Strategy	12.6	25.2	17.0	10.8	7.2	5.0	4.0	2.4
Final Income (Annual)	56,107	56,107	56,107	56,107	56,107	56,107	56,107	56,107

Table 6 compares the retirement wealth distribution of investment approaches modeled with stochastic labor income. In addition to traditional, 100-minus age, and contrarian investment strategies in the previous table, we add stochastic and stochastic and correlated life-cycle strategies to Table 6. The table reports the mean and percentiles of retirement wealth distribution as a multiple of the expected final annual salary, which is indicated in the bottom row of the table. Different from the previous table, the final salary is based on stochastic labor income, so its distribution significantly differs among percentiles.

Overall, the retirement wealth modeled with stochastic labor income is lower for all cases compared with Table 5. This result is expected due to the additional uncertainty in labor income (temporary and permanent shocks). When we compare the rankings of investment approaches, the results are quite similar to those in Table 5.

The stochastic life-cycle fund provides the second highest average retirement wealth, equal to about 12 times the expected final salary, whereas this value is slightly lower when human capital is both stochastic and correlated. While the wealth outcomes of the stochastic and correlated life-cycle strategy are lower than that of the stochastic life-cycle fund for the upper tail of the distribution, the former provides the highest retirement wealth for the 1st and 5th percentiles.

Table 6

COMPARISON OF INVESTMENT STRATEGIES (STOCHASTIC LABOR INCOME)

Strategies	Mean	90%	75%	50%	25%	10%	5%	1%
100% Bonds	4.8	6.1	5.4	4.7	4.1	3.7	3.4	2.9
100% Stocks	13.5	22.9	16.5	10.3	6.8	4.6	3.7	2.0
60% Stocks / 40% Bonds	8.7	13.4	10.4	7.9	6.1	4.9	4.2	3.1
100-Minus Age	7.9	11.6	9.4	7.4	5.9	4.8	4.3	3.2
Life-cycle Strategy (Stochastic)	12.3	18.8	14.3	9.8	6.6	5.1	4.4	3.3
Life-cycle Strategy (Stochastic & Correlated)	11.5	15.8	12.4	8.0	6.4	5.0	4.5	3.6
Contrarian Strategy	11.2	21.6	15.2	8.7	6.2	4.4	3.6	1.9
Final Income (Annual)	74,476	120,060	93,025	55,972	33,840	23,562	16,286	6,708

While the results in Tables 5 and 6 provide important information on the performance of various investment strategies, the expected utility analysis associated with a particular retirement wealth outcome is also of substantial interest. In this respect, we compared the investment performance of different approaches in terms of the expected utility analysis. We measured the utility of investors at retirement with the constant relative risk-aversion (CRRA) utility function.[12] In order to calculate the expected utility of each investment strategy, we conducted 10,000 simulations. The CRRA utility function can be expressed as:

$$E[U(w)] = \sum_{i=1}^{N} (\frac{1}{1-\gamma}\, w_i^{1-\gamma})$$

in which w_i represents the wealth accumulation at age 65 in each of N=10,000 simulations and γ is the risk aversion coefficient. In the case that $\gamma = 1$, the utility is defined as the natural logarithm of wealth accumulation. We provide the results for risk aversion coefficients from 1 to 10, where 1 and 2 represent aggressive investors and 10 refers to the most risk-averse investors.

The results associated with different investment strategies are presented in Table 7. As expected, the 100% stock investment strategy and contrarian investment approach have the two highest utility estimates for aggressive investors, with risk aversion coefficients of 1 and 2. While our proposed life-cycle strategies (stochastic and stochastic and correlated) lag behind only stocks and contrarian approaches for aggressive investors, they maximize the utility for most of the risk aversion coefficients. The stochastic and stochastic and correlated life-cycle funds have the two highest utility estimates for investors, with risk aversion coefficients of 4, 5, 6, and 7, whereas the latter maximizes the expected utility for coefficients of 8 and 9. While the 60/40 investment strategy has higher utility than the 100-minus age rule up to the risk aversion coefficient of 7, for highly risk-averse investors, the 100-minus age strategy outperforms the former one. The 100% bond strategy has the worst outcome up to the risk aversion coefficient of 5 and only maximizes the expected utility for the most risk averse investors.

Table 7

COMPARISON OF INVESTMENT STRATEGIES (EXPECTED UTILITY ANALYSIS)

Strategies	1	2	3	4	5	6	7	8	9	10
100% Bonds	7	7	7	7	7	6	5	3	2	1
100% Stocks	1	1	2	4	6	7	7	7	7	7
60% Stocks / 40% Bonds	5	5	5	5	4	3	3	5	5	5
100 - Minus Age	6	6	6	6	5	4	4	4	4	4
Life-cycle Strategy (Stochastic)	3	3	3	1	1	1	1	2	3	3
Life-cycle Strategy (Stochastic & Correlated)	4	4	4	2	2	2	2	1	1	2
Contrarian Strategy	2	2	1	3	3	5	6	6	6	6

[12] We also used the CRRA utility function to optimize our life-cycle funds in respect to the the sum of human capital and financial capital. In our baseline scenario, we used the risk aversion coefficient ($\gamma = 5$) in optimizing the stochastic and stochastic and correlated life-cycle funds. In this respect, it is also intuitive to use the CRRA utility function when comparing the financial wealth accumulation at retirement for different investment strategies and different risk aversion coefficients. The CRRA utility function has been adopted by many recent studies (Pfau, 2009; Poterba et al., 2009) in evaluating the performance of different investment approaches.

Overall, our optimized life-cycle funds have the potential to protect retirees from adverse outcomes, and they maximize the expected utility for most of the risk aversion coefficients.[13] These results are in line with the studies of Pfau (2009) and Poterba et al. (2009), despite the fact that we take additional types of investment strategies into account and optimize our life-cycle funds depending on stochastic and stochastic and correlated human capital.

5.4 THE OPTIMAL PORTFOLIO ALLOCATIONS (PRE - AND POST-RETIREMENT PERIODS)

This section presents life-cycle funds' portfolio allocation to stocks for the pre- and post-retirement stages in a detailed sensitivity analysis. First, the optimal equity ratios of life-cycle funds under stochastic and correlated human capital and social security benefits are analyzed. Then, the optimal portfolios under longevity risk are presented with different bequest motives. Finally, the portfolio allocations are analyzed under parameter uncertainty and different types of annuities.

The standard assumptions used in modeling life-cycle funds are as follows:

- The age at which the representative investor starts investment: 20;
- The retirement age of the representative investor: 65;
- The maximum age of the representative investor: 120;
- Risk aversion coefficient: 5;
- Discount rate: 5%;
- Contribution rate: 10%;
- The correlation between stocks and human capital: 20%;
- Financial wealth withdrawal: 4%;[14]
- The strength of the partial bequest motive: 0.5; and
- Utility Function: $U = \frac{(FC_{t+1}+HC_{t+1})^{1-\gamma}}{1-\gamma}$ for $\gamma \neq 1$

  $U = \ln{(FC_{t+1} + HC_{t+1})}$ for γ = 1

First, we present optimal stock allocations of life-cycle funds with different levels of social security benefits (37% (baseline), 57%, 77%, and 97%) in Figure 17. According to the SSA (2023), nearly 9 out of 10 people aged 65 and older are receiving social security benefits, and these benefits represent about 30% of the income of people over 65. In this respect, social security is the largest source of income for most retirees and should be included in the life-cycle fund design for the U.S. pension system. Our results suggest that investors receiving social security income should increase their allocation to stocks as they age during the post-retirement period. The main reason for this finding is that social security can act as a safe asset and provide a capacity for retirees to take additional risks in their portfolio allocation (Gomes et al., 2008). As the level of social security benefits increases, the optimal equity ratio rises to 100% for high levels of social security benefits during the final years of the post-retirement period.

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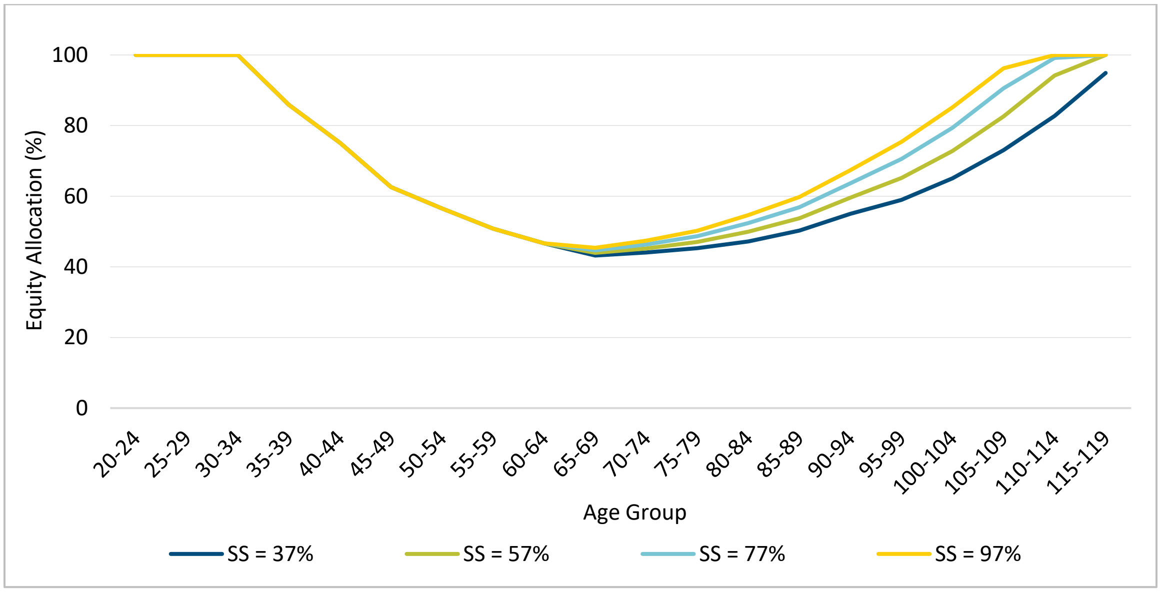
[13] It is noteworthy to mention that life-cycle funds designed to match the risk tolerance may have small welfare costs compared to traditional and contrarian approaches (Maurer et al. 2007, Gomes et al. 2008). Moreover, recent studies show that welfare loss associated with the adoption of rule of thumb rules (e.g., 100-minus age) in life-cycle funds is substantial, ranging from 3% to 9% of annual consumption. In this respect, optimizing life-cycle funds with respect to human capital risk, which captures the impact of labor market uncertainty, may also offer some advantages in terms of welfare gains (Bagliano et al., 2021).

[14] In our life-cycle fund optimization, we assumed that, during the pre-retirement period, people consume the portion of their income that is not invested in financial markets and, during the post-retirement period, they consume financial capital (which they accumulated during pre-retirement) to survive. In this respect, in our analysis, consumption is exogeneous for both pre- and post-retirement stages and, hence, excluded from the optimization problem. For the post-retirement period, we assumed 4% for the financial wealth withdrawal. We also tested the impact of different levels of withdrawals on asset allocation, and the results did not differ significantly.

Figure 17

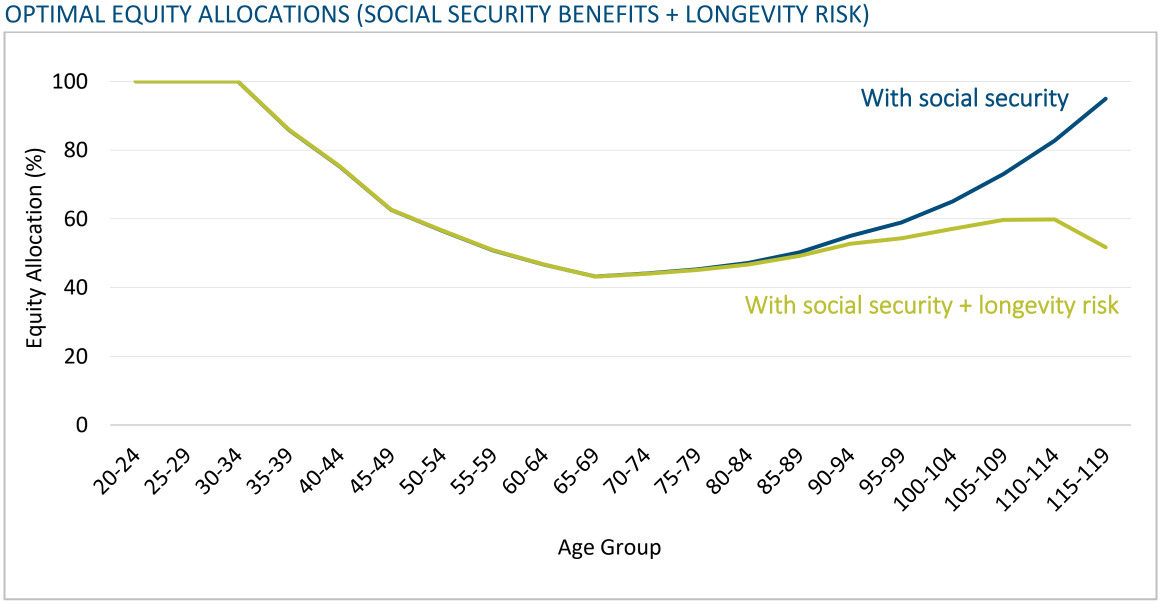
OPTIMAL EQUITY ALLOCATIONS (SOCIAL SECURITY BENEFITS)



Second, in Figure 18, the optimal equity allocations for life-cycle portfolio choice under social security benefits and longevity risk are presented. As can be seen from the figure, when we integrate longevity risk into the portfolio allocation problem with social security contributions, the optimal investment strategy follows a very similar path until the age of 90. On the other hand, retirement plan participants who live until the maximum life-expectancy should reduce equity exposure significantly after the age of 90; e.g., the optimal equity allocation decreases from 95% to 52% in the final years of the post-retirement period when we incorporate longevity risk into the social security benefits. This result is expected since survival probabilities, as shown in Figure 6, decrease noticeably after the age of 90.

Figure 18

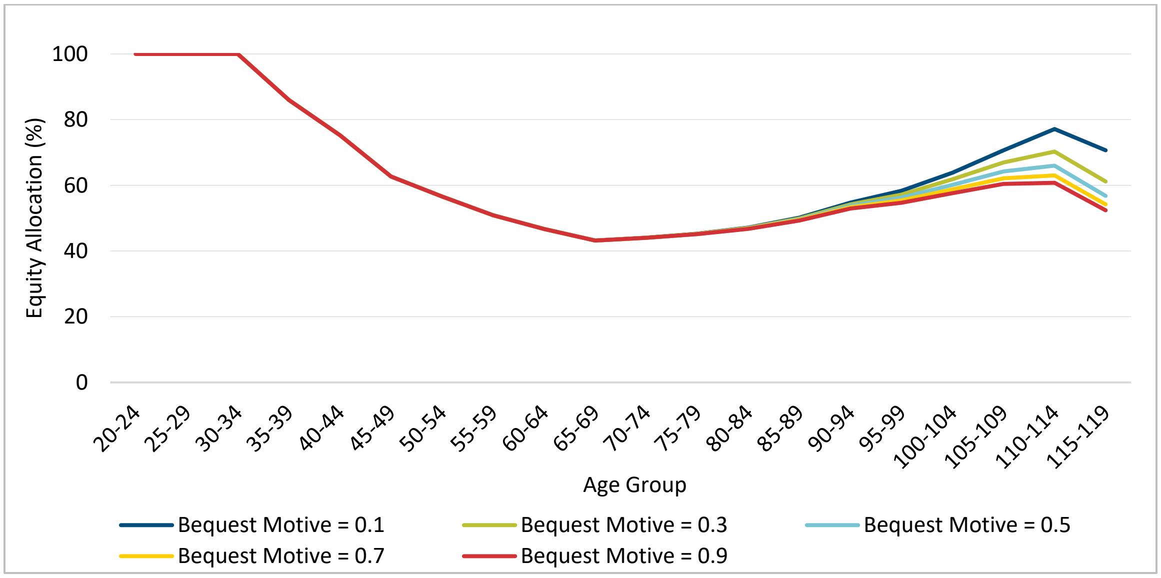
OPTIMAL EQUITY ALLOCATIONS (SOCIAL SECURITY BENEFITS + LONGEVITY RISK)



Third, the optimal equity allocations for life-cycle portfolio choice under social security contributions, longevity risk, and partial bequest motive are presented in Figure 19. The strength of the bequest motive can be between 0 and 1, and the results are illustrated for the bequest motives of 0.1, 0.3, 0.5 (baseline), 0.7, and 0.9. While, in the U.S., most households invest their financial capital with the motive of bequeathing, the bequest motive does not have a significant impact on the optimal portfolio allocation in the post-retirement period. As Figure 19 illustrates, when the bequest motive increases, the optimal equity allocation decreases; however, this effect is small and only becomes clear in the prior 15-20 years to the maximum life expectancy. According to Lee (2023), in the presence of mortality risk, most households with bequest motives are more reluctant to take additional risks in financial markets compared to households who mainly care about their own consumption. Therefore, they reduce equity exposure in their portfolios, particularly in the final years of the post-retirement period.

Figure 19

OPTIMAL EQUITY ALLOCATIONS (SOCIAL SECURITY BENEFITS + LONGEVITY RISK + PARTIAL BEQUEST MOTIVE)

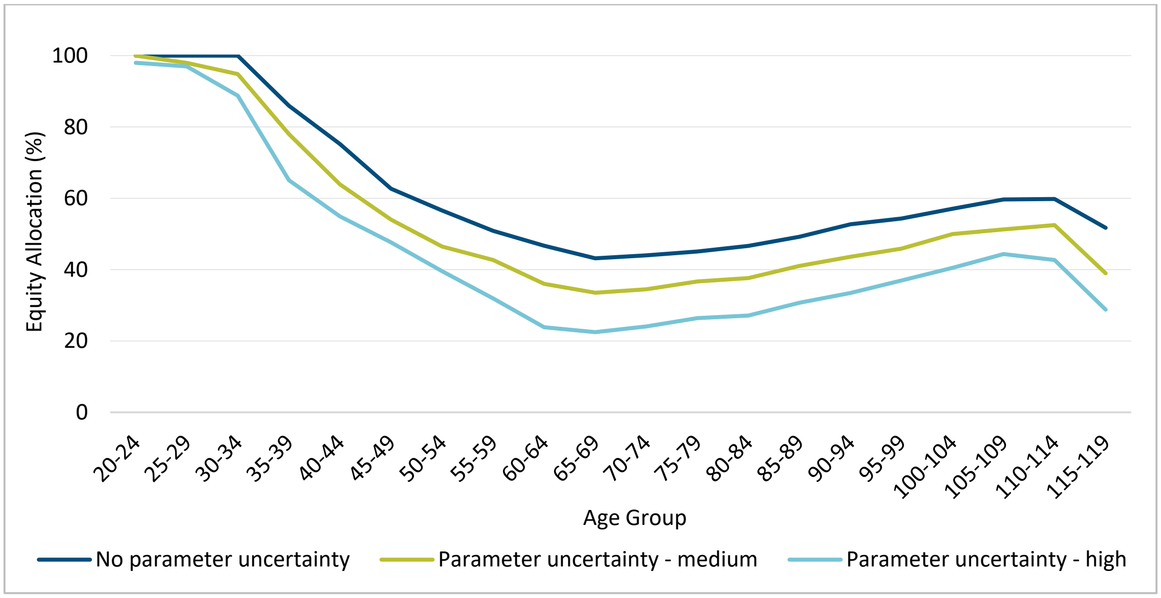


Fourth, in Figure 20, we present optimal equity allocations under social security contributions, longevity risk, partial bequest motive, and parameter uncertainty. Here, we define two different levels of parameter uncertainty – medium and high – based on the methodology in subsection 4.2. In medium parameter uncertainty, we assume that investors are confident in the parameters of stock returns (expected return and variance) as if they had observed them for 50 years. For the high parameter uncertainty, the parameters of stock returns are assumed to be observed for 20 years.

Figure 20 shows that parameter uncertainty lowers optimal equity allocation both in pre- and post-retirement stages compared to the no parameter uncertainty scenario. While the difference in optimal equity allocations between medium parameter uncertainty and no parameter uncertainty peaks at 12% in the final years of the post-retirement period, this ratio is about 23% for the difference between high parameter uncertainty and no parameter uncertainty cases. Our findings are in line with the study of Pastor and Stambaugh (2012), and emphasize the importance of parameter uncertainty in long-run portfolio optimization. To our knowledge, this analysis is the earliest in our literature search to investigate the effect of parameter uncertainty in life-cycle funds by also integrating other risks and parameters, such as human capital risk, social security benefits, longevity risk, and partial bequest motive.

Figure 20

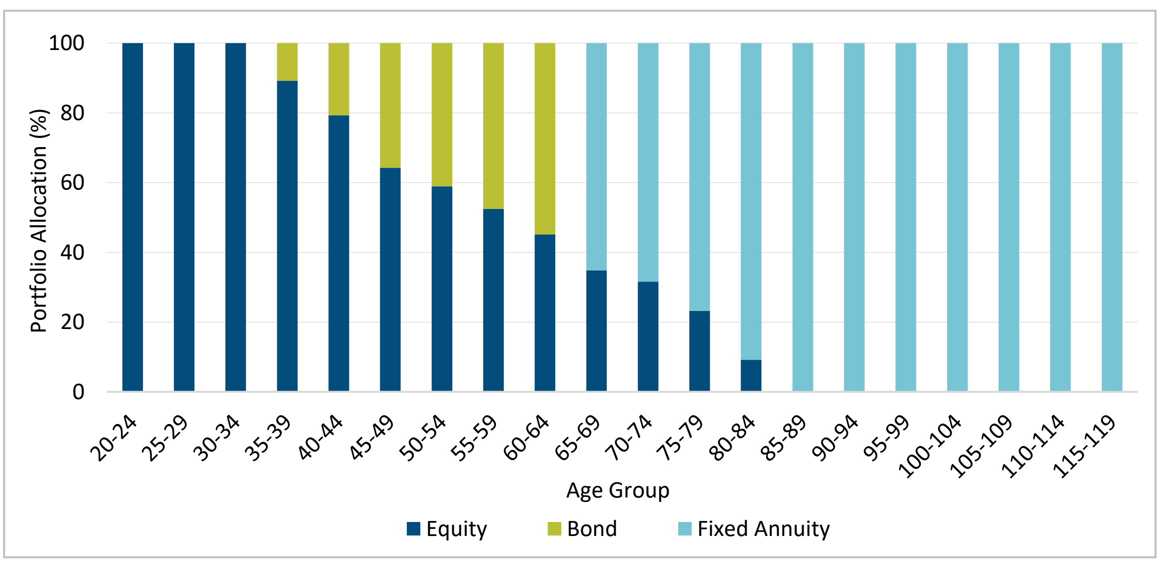
OPTIMAL EQUITY ALLOCATIONS (SOCIAL SECURITY BENEFITS + LONGEVITY RISK + PARTIAL BEQUEST MOTIVE + PARAMETER UNCERTAINTY)



Fifth, we present optimal portfolio allocations for life-cycle portfolio choice for the three-asset case by also adding fixed annuities in addition to stocks and bonds. In this respect, the optimal portfolio allocations for life-cycle portfolio choice under social security contributions, longevity risk, partial bequest motive, and fixed annuities are presented in Figure 21. As expected, the share of optimal equities relative to bonds decreases in the pre-retirement period. On the other hand, during the post-retirement period, the long-term portfolio optimization switches its allocation from bonds to fixed annuities. From the 85-89 age group, the optimal portfolio allocation only consists of fixed annuities, and there is no allocation to equities and bonds since, due to the longevity risk, fixed annuities have become an attractive option for investors at older ages.

Figure 21

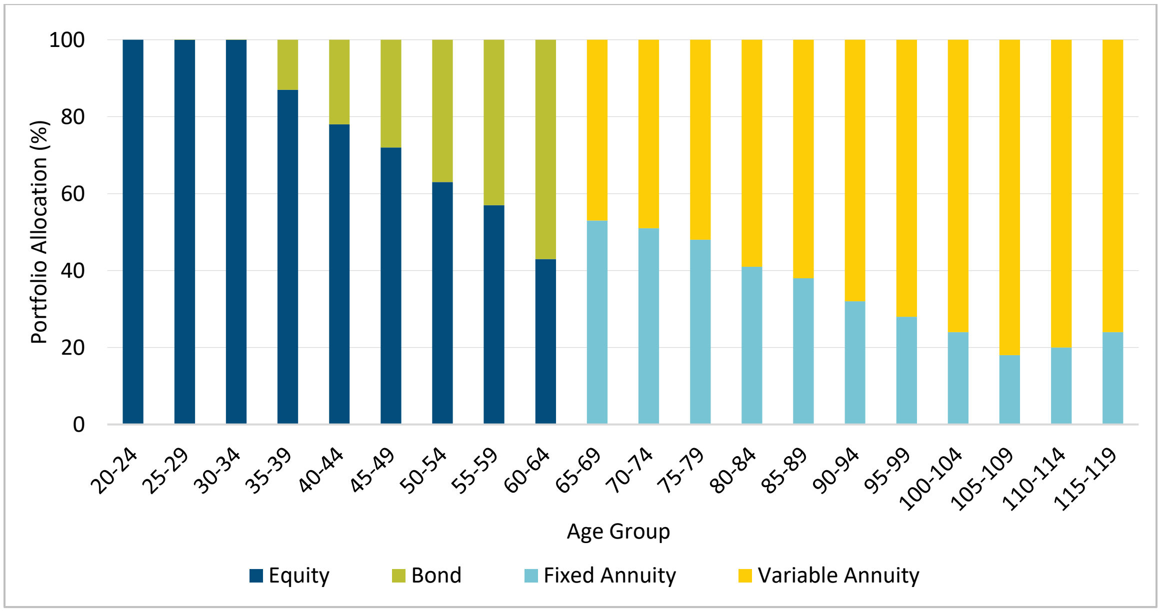
OPTIMAL PORTFOLIO ALLOCATIONS (SOCIAL SECURITY BENEFITS + LONGEVITY RISK + PARTIAL BEQUEST + FIXED ANNUITIES)



Sixth, in addition to fixed annuities, we add variable annuities to our portfolio optimization problem. The optimal portfolio allocations for life-cycle portfolio choice under social security contributions, longevity risk, partial bequest motive, fixed annuities, and variable annuities are presented in Figure 22. Similar to the previous figure, the share of optimal equities relative to bonds decreases in the pre-retirement period. On the other hand, during the post-retirement period, the optimal portfolio allocation consists of only fixed and variable annuities. While the share of fixed annuities relative to variable annuities decreases during the post-retirement period in general, the optimal share of fixed annuities increases slightly for the last 10 years of the post-retirement stage.

Figure 22

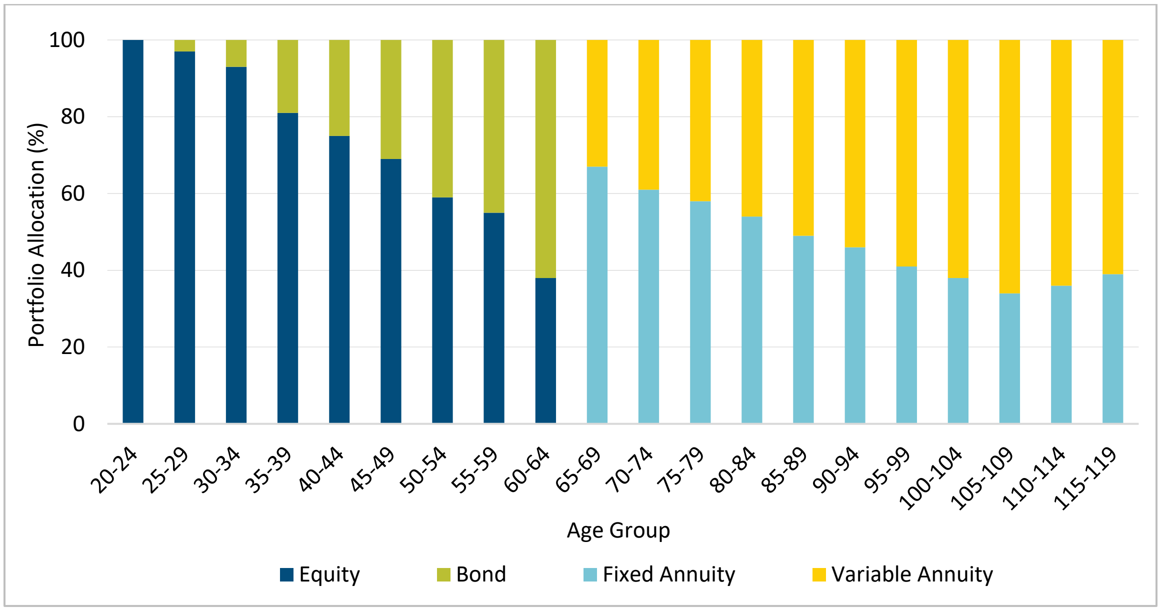
OPTIMAL PORTFOLIO ALLOCATIONS (SOCIAL SECURITY BENEFITS + LONGEVITY RISK + PARTIAL BEQUEST + FIXED ANNUITIES + VARIABLE ANNUITIES)



Finally, in Figure 23, the optimal portfolio allocations for life-cycle portfolio choice under social security benefits, longevity risk, partial bequest motive, fixed annuities, variable annuities, and parameter uncertainty are presented. Here, we adopt the assumption of medium parameter uncertainty. When we compare the results with the previous figure, the share of bonds relative to the equities increases due to the parameter uncertainty for the pre-retirement phase. Similarly, the share of fixed annuities relative to variable annuities increases due to the uncertainty in expected return and variance parameters for the post-retirement stage.

Figure 23

OPTIMAL PORTFOLIO ALLOCATIONS (SOCIAL SECURITY BENEFITS + LONGEVITY RISK + PARTIAL BEQUEST + FIXED ANNUITIES + VARIABLE ANNUITIES + PARAMETER UNCERTAINTY)



5.5 PORTFOLIO SUCCESS RATES

In this subsection, we illustrate the portfolio success rates (10,000 simulations) for six investment strategies for different payout periods and inflation-adjusted withdrawal rates. The portfolio success rate is defined as the number of portfolio successes divided by the number of simulations. A portfolio is counted as a success if it completes the payout period with an end-of-period value greater than or equal to zero despite annual withdrawals (Cooley et al., 2011).[15]

The investment strategies include the following: 100% bonds, 100% stocks, 60/40 investment strategy, 100-minus age investment strategy, life-cycle investment strategy (stochastic), and contrarian investment strategy. For each scenario, the labor income is assumed to be stochastic and the pre-retirement period is 45 years. The 100% bonds, 100% stocks, 60/40 investment strategy, and 100-minus age approach assume that investors choose the defined asset allocation scenario both in the pre- and post-retirement stages. The life-cycle investment strategy follows an asset allocation path optimized with assumptions in subsection 5.4. The contrarian investment strategy is the same as mentioned in subsection 5.3 for the pre-retirement stage and invests in 100% stocks for the post-retirement phase.

Table 8 illustrates the portfolio success rates of inflation-adjusted annual withdrawals ranging from 3% to 12%. The payout periods vary from 20 to 55 years for six different investment strategies. Overall, all investment strategies can support withdrawals with 100% success rates for 3% withdrawals. When we look at the rule of thumb withdrawal of 4%, the majority of portfolios still have high success rates, very close to 100%. One of the main findings is that portfolio success rates diminish quickly for retirement portfolios with the only bonds strategy. For example, a portfolio with 100% bonds supports a 50-year payout period with a

[15] On the other hand, some studies suggest different approaches designed to improve retirement income projections, such as focusing on the factors that influence participants' money's longevity and the decomposition of retirement income goal based on assumed spending elasticity (e.g., "needs" and "wants") (Milevsky, 2018; Blanchett ,2023).

6% withdrawal rate for 61% of the total simulations, whereas for a 7% withdrawal rate, this ratio is only 37%. The only bonds strategy is not sustainable for the majority of payout periods at 11% and 12% withdrawal rates. On the other hand, the 100% stock investment strategy has the highest portfolio success rates for the largest withdrawals.

Our proposed stochastic life-cycle fund strategy[16] has the highest portfolio success rates in the majority of payout periods for withdrawal rates ranging from 5% to 10%. As mentioned above, for very high withdrawal rates (11% and 12%), the only stocks strategy has the highest portfolio success rates. The contrarian investment strategy outperforms 100% stocks in most of the payout periods for withdrawal rates from 5% to 10%. While the 100-minus age and 60/40 investment strategies have similar portfolio success rates, between 3% and 8% withdrawal rates, the former outperforms the latter for 9% and 10% withdrawal rates, and the situation is vice versa for 11% and 12% withdrawal rates.

Table 8

PORTFOLIO SUCCESS RATES

	Withdrawal Rate as a % of Total Savings (Inflation-Adjusted)									
Payout Period	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
100% Bonds										
20 Years	100%	100%	98%	96%	95%	81%	65%	49%	29%	12%
30 Years	100%	100%	97%	81%	74%	52%	41%	30%	12%	1%
40 Years	100%	100%	93%	74%	58%	29%	23%	15%	0	0
50 Years	100%	99%	85%	61%	37%	14%	11%	5%	0	0
55 Years	100%	99%	81%	56%	25%	5%	2%	0	0	0
100% Stocks										
20 Years	100%	99%	96%	93%	89%	83%	76%	70%	62%	54%
30 Years	100%	97%	93%	91%	87%	81%	68%	62%	55%	48%
40 Years	100%	96%	91%	88%	85%	78%	58%	55%	45%	42%
50 Years	100%	95%	90%	86%	83%	75%	51%	44%	38%	35%
55 Years	100%	94%	90%	86%	82%	74%	47%	42%	37%	32%
60% Stocks/40% Bonds										
20 Years	100%	100%	97%	93%	90%	87%	80%	62%	56%	48%
30 Years	100%	100%	96%	93%	89%	85%	67%	58%	41%	33%
40 Years	100%	99%	95%	92%	89%	81%	53%	44%	31%	26%
50 Years	100%	98%	94%	91%	88%	74%	41%	34%	24%	18%
55 Years	100%	98%	94%	91%	88%	69%	36%	31%	20%	16%
100-Minus Age										
20 Years	100%	100%	97%	94%	92%	89%	85%	68%	55%	44%
30 Years	100%	100%	95%	93%	91%	86%	81%	55%	34%	29%
40 Years	100%	100%	94%	92%	90%	83%	75%	49%	22%	18%
50 Years	100%	99%	93%	91%	88%	79%	69%	41%	15%	12%
55 Years	100%	98%	93%	91%	88%	77%	66%	38%	13%	10%
Life-cycle Strategy										
20 Years	100%	100%	97%	95%	94%	90%	86%	69%	64%	51%
30 Years	100%	100%	96%	95%	93%	87%	81%	65%	56%	45%
40 Years	100%	100%	94%	94%	91%	85%	78%	62%	47%	35%
50 Years	100%	99%	94%	93%	90%	82%	77%	58%	40%	29%
55 Years	100%	98%	94%	93%	90%	81%	76%	56%	35%	25%

[16] While we do not illustrate results for stochastic & correlated life-cycle strategy, the main difference between stochastic and stochastic & correlated life-cycle strategies is that the former has higher portfolio success rates than the latter for the largest withdrawals (11% and 12%).

Withdrawal Rate as a % of Total Savings (Inflation-Adjusted)										
Contrarian Strategy										
20 Years	100%	100%	96%	94%	91%	88%	82%	70%	67%	52%
30 Years	100%	100%	95%	92%	89%	85%	77%	59%	55%	44%
40 Years	100%	99%	94%	91%	87%	81%	73%	56%	45%	36%
50 Years	100%	98%	92%	90%	85%	78%	71%	51%	32%	28%
55 Years	100%	97%	92%	89%	85%	76%	70%	49%	29%	24%

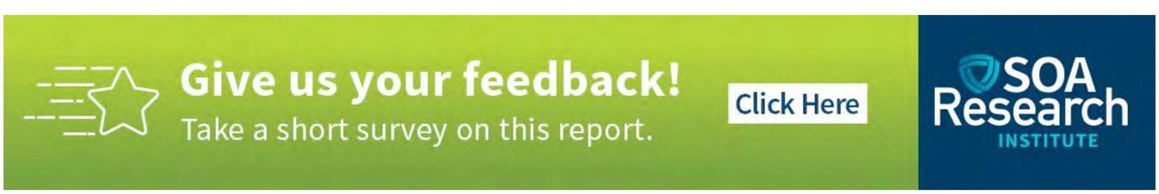
Section 6: Conclusion

This study aims to design a comprehensive life-cycle fund model in the context of demographic, economic, and financial structure of the U.S. In this respect, firstly, stochastic human capital is modeled according to the methodology of Campbell and Viceira (2002) and Cocco et al. (2005). Secondly, life-cycle funds' portfolio allocations are optimized for the pre-retirement stage based on the human capital that is both stochastic and correlated with stock returns. Thirdly, to address the impact of the model assumptions on optimal portfolio allocations, we conduct a sensitivity analysis for risk-aversion coefficients, discount rates, contribution rates, permanent shocks, capital market expectations, and correlation coefficients between human capital and stock returns. Finally, we estimate portfolio allocations of life-cycle funds by including social security benefits, longevity risk, bequest motive, parameter uncertainty, and annuities, for the pre- and post-retirement stages.

In line with the studies of Cocco et al. (2005), Gomes et al. (2008), and Bagliano et al. (2019), we find the following results: (i) the level of risk aversion, the positive correlation between human capital and stock returns, and permanent shocks have a significant effect on the portfolio allocation of life-cycle funds. On the other hand, discount rates, contribution rates, and capital market assumptions do not have a substantial effect on the asset allocation between equities and bonds; (ii) social security contributions act as a safe asset and increase the optimal equity ratio at the post-retirement phase; (iii) longevity risk becomes particularly important after the age of about 90, whereas partial bequest motive does not have a major impact on the portfolio allocation over the life cycle; (iv) parameter uncertainty affects optimal equity allocation at every age group. While the share of equity relative to bonds decreases until the retirement age (due to parameter uncertainty), the share of fixed annuities relative to variable annuities increases at the post-retirement phase; and (v) the proposed life-cycle funds outperform traditional, 100-minus age, and contrarian investment strategies, in most cases, in terms of the expected utility and portfolio success rates. Moreover, they achieve the highest retirement wealth accumulation for adverse outcomes.

This study contributes to life-cycle investing and human capital, as well as to the mean reversion and parameter uncertainty literature by modeling a comprehensive life-cycle fund structure for the U.S. pension system. The derived conclusions may provide important insights for retirement funds and asset management companies about the design of life-cycle funds, which are the most common default investment options offered by U.S. retirement plans.

For future studies, the correlation of stochastic human capital with stocks may be examined in detail for different industries and socioeconomic levels. Moreover, it is expected that more robust results will be obtained by adding real estate investments to the portfolio allocation problem, as they play an important role in household savings in the U.S.



Section 7: Acknowledgments

The researchers' deepest gratitude goes to those without whose efforts this project could not have come to fruition: the Project Oversight Group for their diligent work overseeing, reviewing and editing this report for accuracy and relevance.

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Appendix A: Simulation Exercise

This appendix shows optimal equity allocations in pre-retirement and pre- and post-retirement stages for different numbers of simulations (10,000, 100,000, and 1,000,000). The equity allocations in the pre-retirement period (see Figure A.1) are optimized for stochastic and correlated human capital and other assumptions mentioned in subsection 5.2. The equity allocations in the pre- and post-retirement periods (see Figure A.2) are optimized for stochastic and correlated human capital, social security, longevity risk, and parameter uncertainty assumptions mentioned in subsection 5.4.

As illustrated in the figures below, the results are close to each other among the same age groups, and particularly similar for 100,000 and 1,000,000 simulations.

Figure A.1
OPTIMAL EQUITY ALLOCATIONS FOR DIFFERENT NUMBER OF SIMULATIONS (PRE-RETIREMENT)

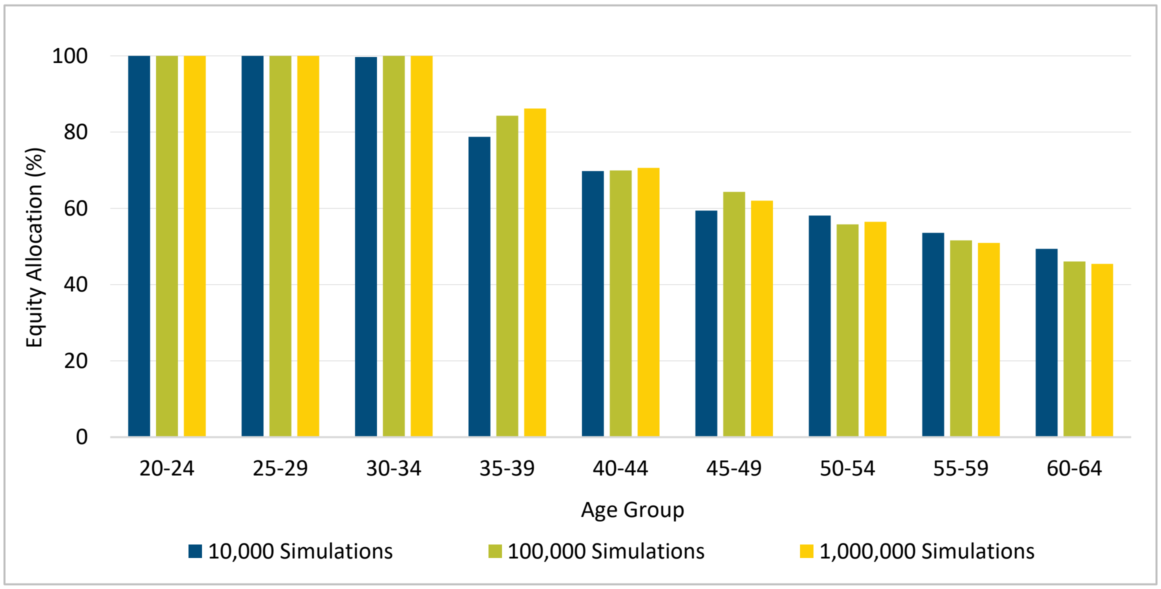
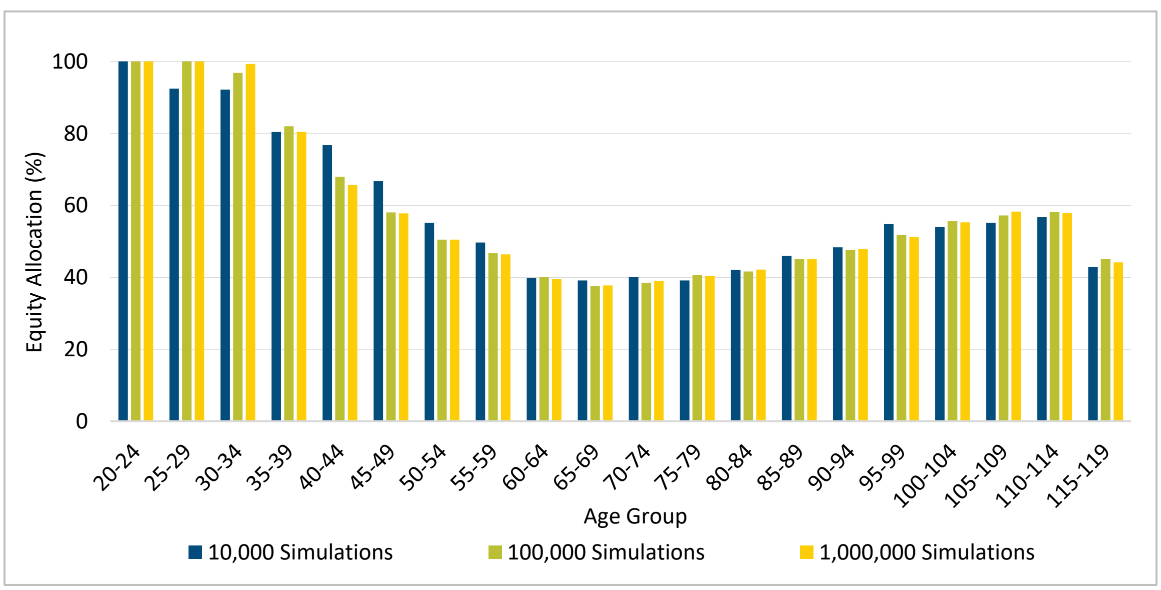


Figure A.2
OPTIMAL EQUITY ALLOCATIONS FOR DIFFERENT NUMBER OF SIMULATIONS (PRE- AND POST-RETIREMENT)

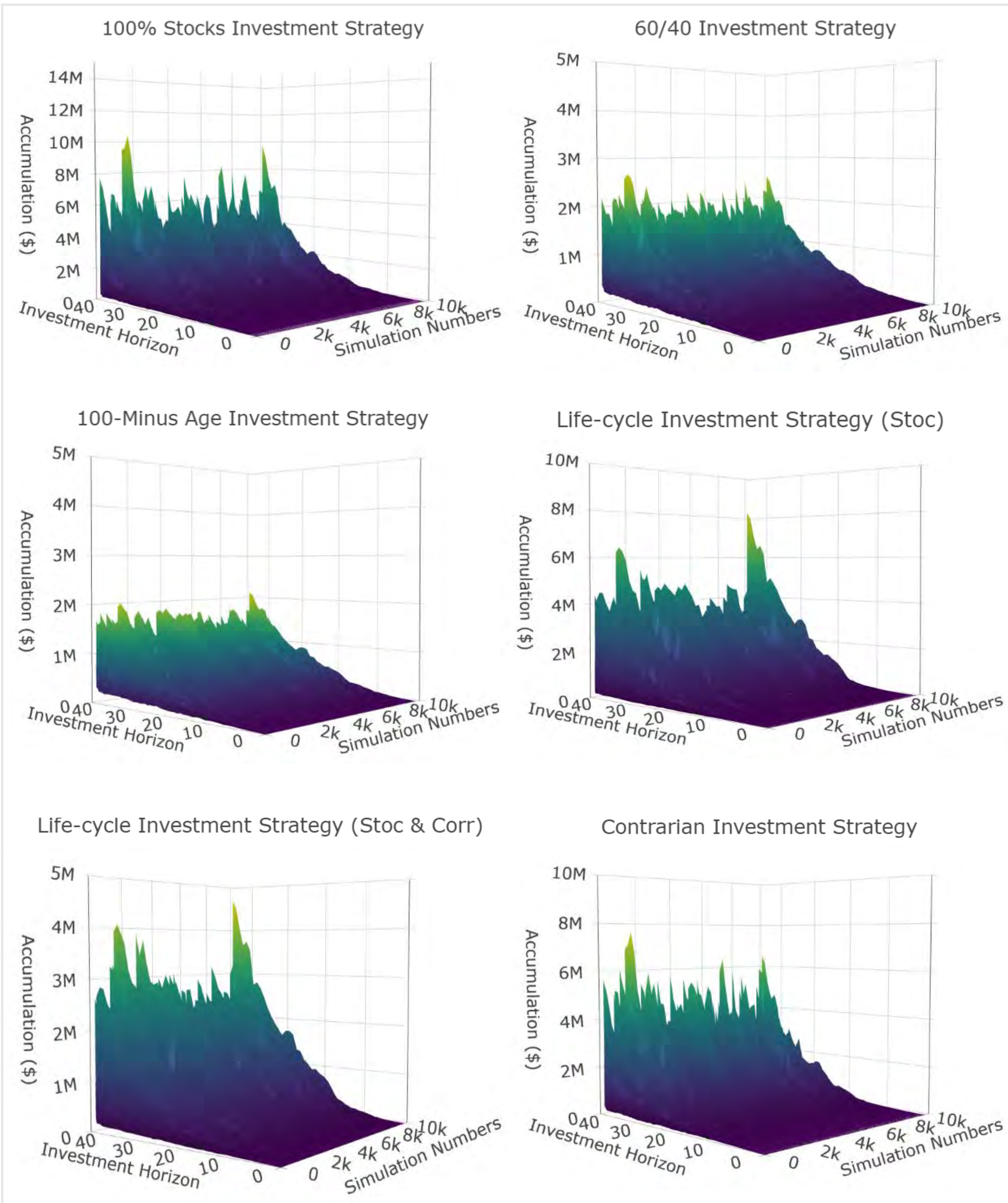


Appendix B: Simulated Wealth Accumulations

This appendix illustrates the simulated wealth accumulation paths (up to 10,000 simulations) for six investment strategies over different horizons (up to 45 years). The investment strategies include the following: 100% stocks, 60/40 investment strategy, 100-minus age investment strategy, life-cycle investment strategy (stochastic), life-cycle investment strategy (stochastic and correlated), and contrarian investment strategy. The simulated wealth accumulations are expressed in millions of dollars.

Figure B.1

SIMULATED WEALTH ACCUMULATION PATHS OVER DIFFERENT INVESTMENT HORIZONS



References

Aktuğ, E., Kuzubaş, T. U., & Torul, O. (2021). Heterogeneity in labor income profiles: evidence from Turkey. *Empirical Economics*, *60*(5), 2557-2587.

Azar, S. A. (2006). Measuring relative risk aversion. *Applied Financial Economics Letters*, *2*(5), 341-345.

Bagliano, F. C., Fugazza, C., & Nicodano, G. (2019). Life-cycle portfolios, unemployment and human capital loss. *Journal of Macroeconomics*, *60*, 325-340.

Bagliano, F. C., Fugazza, C., & Nicodano, G. (2021). Life-cycle welfare losses from rules-of-thumb asset allocation. *Economics Letters*, *198*, 109655.

Barberis, N. (2000). Investing for the long run when returns are predictable. *The Journal of Finance*, *55*(1), 225-264.

Basu, A. K., & Drew, M. E. (2009). Portfolio size effect in retirement accounts: What does it imply for life-cycle asset allocation funds? *Journal of Portfolio Management*, *35*(3), 61.

Benartzi, S., & Thaler, R. H. (2007). Heuristics and biases in retirement savings behavior. *Journal of Economic Perspectives*, *21*(3), 81-104.

Benzoni, L., Collin-Dufresne, P., & Goldstein, R. S. (2007). Portfolio choice over the life-cycle when the stock and labor markets are cointegrated. *The Journal of Finance*, *62*(5), 2123-2167.

Bikker, J. A., Broeders, D. W., Hollanders, D. A., & Ponds, E. H. (2012). Pension funds' asset allocation and participant age: a test of the life-cycle model. *Journal of Risk and Insurance, 79*(3), 595-618.

Blanchett, D. M., & Straehl, P. U. (2015). No portfolio is an island. *Financial Analysts Journal*, *71*(3), 15-33.

Blanchett, D. (2023). Redefining the optimal retirement income strategy. *Financial Analysts Journal*, *79*(1), 5-16.

Bodie, Z., Merton, R. C., & Samuelson, W. F. (1992). Labor supply flexibility and portfolio choice in a life-cycle model. *Journal of Economic Dynamics and Control*, *16*(3-4), 427-449.

Campbell, J. Y., & Viceira, L. M. (2002). *Strategic asset allocation: portfolio choice for long-term investors*. Clarendon Lectures in Economic.

Carvalho, C. M., Lopes, H. F., & McCulloch, R. E. (2018). On the long-run volatility of stocks. *Journal of the American Statistical Association*, *113*(523), 1050-1069.

Cederburg, S., & O'Doherty, M. S. (2019). Understanding the risk-return relation: The aggregate wealth proxy actually matters. *Journal of Business & Economic Statistics, 37*(4), 721-735.

Cocco, J. F., Gomes, F. J., & Maenhout, P. J. (2005). Consumption and portfolio choice over the life cycle. *The Review of Financial Studies*, *18*(2), 491-533.

Cooley, P. L., Hubbard, C. M., & Walz, D. T. (2011). Portfolio success rates: Where to draw the line. *Journal of Financial Planning*, *24*(4), 48-60.

Damodaran Database. (2023). Historical Returns on Stocks, Bonds and Bills: 1928-2023. https://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html

De Nardi, M., & Yang, F. (2014). Bequests and heterogeneity in retirement wealth. *European Economic Review*, *72*, 182-196.

Gomes, F. J., Kotlikoff, L. J., & Viceira, L. M. (2008). Optimal life-cycle investing with flexible labor supply: A welfare analysis of life-cycle funds. *American Economic Review*, *98*(2), 297-303.

Hoang, T. M. (2023). Long-horizon asset and portfolio returns revisited: Evidence from US markets. *Cogent Business & Management*, *10*(2), 2238147.

Hoevenaars, R. P., Molenaar, R. D., Schotman, P. C., & Steenkamp, T. B. (2014). Strategic asset allocation for long-term investors: Parameter uncertainty and prior information. *Journal of Applied Econometrics, 29*(3), 353-376.

Horizon Actuarial. (2023). Survey of Capital Market Assumptions. https://www.horizonactuarial.com/survey-of-capital-market-assumptions

Ibbotson, R. G., Milevsky, M. A., & Zhu, K. X. (2007). *Lifetime Financial Advice: Human Capital, Asset Allocation, and Insurance.* CFA Institute Research Foundation.

Izdorek, M. T., & Kaplan, D. P. (2024). *Lifetime Financial Advice: A Personalized Optimal Multilevel Approach.* CFA Institute Research Foundation.

Lee, M. (2023). Portfolio allocation over the life cycle with multiple late-in-life saving motives. *Journal of Empirical Finance, 74,* 101421.

Lee, S., & Tan, K. T. (2023). Bequest Motives and the Social Security Notch. *Review of Economic Dynamics*, *51*, 888-914.

Lussier, J. (2019). *Secure Retirement: Connecting Financial Theory and Human Behavior*. CFA Institute Research Foundation.

Maurer, R. H., Schlag, C., & Stamos, M. Z. (2007). Optimal life-cycle strategies in the presence of interest rate and inflation risk.

Maurer, R., Mitchell, O. S., Rogalla, R., & Kartashov, V. (2013). Lifecycle portfolio choice with systematic longevity risk and variable investment—linked deferred annuities. *Journal of Risk and Insurance*, *80*(3), 649-676.

Milevsky, M. A. (2016). It's time to retire ruin (probabilities). *Financial Analysts Journal*, *72*(2), 8-12.

Mitchell, O. S., & Lusardi, A. (Eds.). (2011). *Financial literacy: Implications for retirement security and the financial marketplace*. Oxford University Press, USA.

Moallemi, C. C., & Sağlam, M. (2017). Dynamic portfolio choice with linear rebalancing rules. *Journal of Financial and Quantitative Analysis*, *52*(3), 1247-1278.

Pástor, Ľ., & Stambaugh, R. F. (2012). Are stocks really less volatile in the long run? *The Journal of Finance*, *67*(2), 431-478.

Pfau, W. D. (2009). Lifecycle funds and wealth accumulation for retirement: Evidence for a more conservative asset allocation as retirement approaches. Financial Services Review, 19(1).

Pfau, W. D. (2010). An optimizing framework for the glide paths of life cycle asset allocation funds. *Applied Economics Letters*, *18*(1), 55-58.

Poterba, J. M., Rauh, J., Venti, S. F., & Wise, D. A. (2009). Lifecycle asset allocation strategies and the distribution of 401 (k) retirement wealth. In Developments in the Economics of Aging (pp. 15-50). University of Chicago Press.

Shoven, J. B., & Walton, D. B. (2023). *Target Retirement Fund: A Variant on Target Date Funds that uses Deferred Life Annuities rather than Bonds to Reduce Risk as Retirement Approaches* (No. w30817). National Bureau of Economic Research.

Siegel, J. J. (2021). *Stocks for the long run: The definitive guide to financial market returns & long-term investment strategies*. McGraw-Hill Education.

Simsek, K. D., Kim, M. J., Kim, W. C., & Mulvey, J. M. (2018). Optimal longevity risk management in the retirement stage of the life cycle. *The Journal of Retirement*, *5*(3), 73.

Social Security Administration (SSA). (2023). Actuarial Life Table. https://www.ssa.gov/oact/STATS/table4c6.html

Social Security Administration (SSA). (2023). Social Security Basic Facts. https://www.ssa.gov/news/press/factsheets/basicfact-alt.pdf

TSP. (2023). Investment Funds, Fund Options, Life-cycle Funds. https://www.tsp.gov/

U.S. Bureau of Labor Statistics. (2022). Median earnings of full-time wage and salary workers by age and sex. https://www.bls.gov/cps/earnings.htm#demographics

Vanguard. (2023). A look back on 20 years of Vanguard Target Retirement Funds. *Investing Insights.*

Viceira, L. (2001). Optimal portfolio choice for long-horizon investors with non-tradable labor income. *The Journal of Finance*, *56*(2), 433-470.

Viceira, L. (2009). *Pension Fund Design in Developing Economies.* Harvard Business School.

Warren, G. J. (2019). Choosing and using utility functions in forming portfolios. *Financial Analysts Journal*, *75*(3), 39-69.

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