GI ADV Model Solutions
Fall 2024

1.

Learning Objectives:

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5g) Calculate the price for a casualty per occurrence excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question tested a candidate's ability to analyze aspects of a pricing analysis for workers compensation excess of loss reinsurance. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the values of *a* and *b* for each hazard group.

Step 1: Calculate ELF[200,000] / ELF [1,000,000] for J and K using the provided ELF tables. We get 5.000 for J and 3.6316 for K.

Step 2: Calculate the natural logarithm of the step 1 amounts for J and K. We get 1.6094 for J and 1.2897 for K.

Step 3: Calculate the natural logarithm of 200,000 / 1,000,000. We get −1.6094

Step 4: Calculate *b* as (−step 2 amounts) divided by step 3 amount. We get 1.0000 for J and 0.8013 for K.

Step 5: Calculate *a* as ELF[200,000] divided $[200{,}000^{(-\text{step 4 amounts})}]$. We get 6,000 for J and 1,220.8056 for K.

1. Continued

(b) Calculate the loss cost rate for the treaty.

Step 1: Calculate the ELFs for amounts 100,000 and 400,000 for each of J and K as $a \times \text{amount}^{-b}$. We get

Loss Size	J	K
100,000	0.060	0.120
400,000	0.015	0.040

Step 2: Calculate the layer ELF each of J and K using amounts from step 1 as ELF[100,000] – ELF[400,000]

Step 3: Calculate the Treaty Loss for each State and Hazard combination as Standard Premium (SP) × Expected Loss Ratio (ELR) × layer ELF

State	Hazard	SP	Treaty Loss
X	J	70,000	1,575
X	K	120,000	4,839
Y	J	110,000	3,465
Y	K	100,000	5,646

Step 4: Treaty Loss Cost is total Treaty Loss divided total SP.
We get 15,525 / 400,000 = 3.9%

(c) Explain how excluding state X will affect the loss cost rate for the treaty.

Commentary on Question:
The model solution is an example of a full credit solution.

For state Y only, we get a loss cost of 9,111 / 210,000 = 4.3%.
Excluding state X would increase the loss cost from 3.9% to 4.3% for state Y only. This is an 11.8% increase in the rate.

2. Learning Objectives:

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6c) Understand and apply techniques for individual risk rating.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Friedland, 2nd Ed. (2022)

- Chapter 36: Individual Risk Rating and Funding Allocation for Self-Insurers

Commentary on Question:

This question tested a candidate's understanding of individual risk rating methods.

Solution:

(a) Compare schedule rating with judgement rating.

Commentary on Question:

The model solution is an example of a full credit solution.

For schedule rating, the insurance rate is determined entirely by the underwriter based on their subjective evaluation of the risk. In contrast, for schedule rating, the underwriter uses judgement to determine credits and surcharges to a schedule of risk characteristics that are applied to the manual rate.

(b) Insurance companies typically only use schedule rating for certain types of general insurance policies.

Describe these types of policies.

Commentary on Question:

There are several ways that these types of policies can be described. The model solution is an example of a full credit solution.

Commercial general insurance including lines of business such as commercial multi-peril and general liability.

(c) Identify three primary objectives typically used by insurers in this determination.

Commentary on Question:

There are more than three objectives that can be considered as primary. The model solution is an example of a full credit solution.

- Holding insureds responsible for claims
- Encouraging insureds to participate in risk control activities
- Enhancing market competitiveness

3. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark

Commentary on Question:

This question tested a candidate's understanding of the Clark's stochastic LDF model. This question included data and results from a completed analysis (using Clark's stochastic reserving model) in Excel. It required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Explain why the gamma distribution is also appropriate for use in Clark's model.

Any distribution that places probability from zero to infinity can be used as it will meet the requirement of increasing development

(b) Explain why the gamma distribution may not be the most reasonable choice.

The gamma distribution is light-tailed and may not fit a typical development pattern

(c) Recommend which of the three distributions should be used based upon fit to the data. Justify your recommendation including one numerical and one graphical argument.

Commentary on Question:

The model output was provided for the three distributions and included two model fit statistics (loglikelihood and scale). A review of only one of the model fit statistics was required to earn full credit for the numerical argument. As for the graphical argument for model fit, one should look at graphs of normalized residuals against months of development for each of the three distributions. The model solution is an example of a full credit solution.

3. Continued

The following table shows the scale fit statistics:

Model	Scale
Weibull	5,788
Gamma	4,432
Loglogistic	4,093

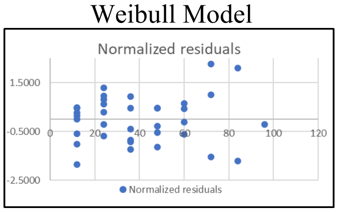
The lower the scale value, the better a model fits the dataset. The scale value of the Weibull model is considerably larger than the scale value for both the gamma and loglogistic models. As such, the gamma and loglogistic models have a better fit to the data.

A graph of the normalized residuals against the development period can give a visual check for model fit. The plot for a good model fit should not have a rising or decreasing pattern.

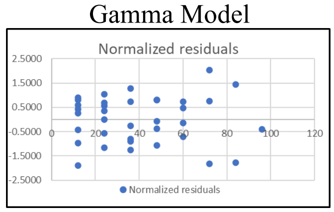
The following graphs plot the normalized residuals for each model against months of development.

The normalized residuals are calculated as the difference between the paid loss data increment and the mu-hat estimate all divided by the square root of sigma-square estimate for the data point times the mu-hat estimate.

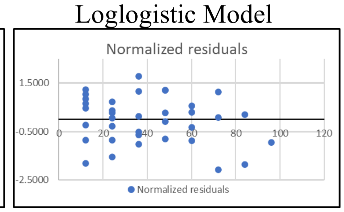
Weibull Model



Gamma Model



Loglogistic Model



Reviewing the three graphs of residuals, the Weibull graph indicates a slightly increasing pattern. The gamma and loglogistic graphs appear to have a more horizontal pattern. This would indicate that Weibull model is a poor fit, and the gamma and loglogistic models fit the data reasonably well.

Overall, the Weibull model is easily rejected by both the test statistic and the graph of residuals. Additionally, both the gamma and loglogistic models are reasonable by both the test statistic and the graph of residuals. However, the loglogistic model scale factor test statistic indicates the best fit. For these reasons, I recommend the loglogistic model.

4. **Learning Objectives:**

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Testing the Assumptions of Age-to-Age Factors, Venter

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the development factors (f_k) and complete the triangle using Mack's chain ladder approach.

The development factors (f_k) are calculated as follows:

- For $k = 1$, it equals the sum of paid claims at development year 2 divided by the sum of paid claims at development year 1, for accident years 1 through 9.
- For $k = 2$, it equals the sum of paid claims at development year 3 divided by the sum of paid claims at development year 2, for accident years 1 through 8.
- For $k = 3$, it equals the sum of paid claims at development year 4 divided by the sum of paid claims at development year 3, for accident years 1 through 7.

The pattern continues for $k = 4$ through 9.

The values in the triangle are represented by $c_{j,k}$ where j is the AY. To complete the triangle, the remaining $c_{j,k}$ values are calculated as follows:

$$c_{j,k} = c_{j,k-1} \times f_{k-1}$$

4. Continued

(b) Calculate the values of α_8^2 and α_9^2.

$$\alpha_8^2 = \left(\frac{1}{10-8-1}\right)\sum_{j=1}^{2} c_{j,8}\left(\frac{c_{j,9}}{c_{j,8}} - f_8\right)^2 \text{ and } \alpha_9^2 = \frac{(\alpha_8^2)^2}{\alpha_7^2}.$$

(c) Calculate the standard error of the reserve estimator for AYs 2 and 3.

VAR_k = variance of the AY k reserve estimator
SE_k = standard error of the AY k reserve estimator

$$VAR_2 = c_{2,10}^2 \times \frac{\alpha_9^2}{f_9^2} \times \left(\frac{1}{c_{2,9}} + \frac{1}{c_{1,9}}\right) \text{ and}$$

$$VAR_3 = c_{3,10}^2 \times \left[\frac{\alpha_8^2}{f_8^2} \times \left(\frac{1}{c_{3,8}} + \frac{1}{c_{1,8}+c_{2,8}}\right) + \frac{\alpha_9^2}{f_9^2} \times \left(\frac{1}{c_{3,9}} + \frac{1}{c_{1,9}}\right)\right].$$

So we get $SE_2 = \sqrt{VAR_2}$ and $SE_3 = \sqrt{VAR_3}$.

(d) Calculate a 95% confidence interval for the AY 8 reserve estimate using Mack's approach based on the lognormal distribution. (The 97.5 percentile of the normal distribution is 1.96.)

The confidence interval is calculated as $\mu \pm z\sqrt{\sigma^2}$.
The AY 8 reserve estimate is $R_8 = c_{8,10} - c_{8,3}$ with variance, Var_8.
Var_8 is provided with the data.
$\sigma^2 = \ln(1 + Var_8 / R_8^2), \mu = \ln(R_8 - \sigma^2 / 2),$ and $z = 1.96$.

(e) Explain why your assistant's approach is incorrect.

The variance of the sum of random variables is only the sum of the variances when the variables are uncorrelated. The individual estimators are correlated because they rely on the same estimates of the age-to-age factors.

(f) Explain why the correct value is larger than that obtained via your assistant's approach.

The correct value is larger because the correlations are positive, so the variance of the sum will exceed the sum of the variances.

4. Continued

(g) Venter restates one of Mack's assumptions as $E[q(w, d+1) \mid \text{data to } w+d] = f(d)c(w, d)$.

State the assumption in words.

The expected value of the incremental losses to emerge in the next period is proportional to the total losses emerged to date, by accident year.

(h) State a formula for each of the three alternative expressions including a verbal description of what they represent.

Formula 1: $E[q(w, d+1) \mid \text{data to } w+d] =$

Formula 2: $E[q(w, d+1) \mid \text{data to } w+d] =$

Formula 3: $E[q(w, d+1) \mid \text{data to } w+d] =$

	Formula as a function of $f(d)$	Verbal Description of Formula
Formula 1: E[q(w, d + 1) | data to w + d] =	$f(d)c(w, d) + g(d)$	linear with constant
Formula 2: E[q(w, d + 1) | data to w + d] =	$f(d)h(d)$	factor times parameter
Formula 3: E[q(w, d + 1) | data to w + d] =	$f(d)h(w)\, g(w+d)$	includes a calendar year effect

5. **Learning Objectives:**

7. The candidate will understand the application of game theory to the allocation of risk loads.

Learning Outcomes:

(7a) Allocate a risk load among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, Mango

Commentary on Question:

This question tested a candidate's ability to calculate property catastrophe risk loads based upon Mango's approach. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Explain why using a premium risk load based upon the Marginal Surplus method is problematic.

Commentary on Question:

The model solution is an example of a full credit solution.

The Marginal Surplus method uses a factor of standard deviation of the risk for the risk load. The square root operator is sub-additive. This means that the sum of the premiums calculated for each of the three accounts will be less than a premium calculated for the three accounts combined.

(b) Calculate the total premium to be received by WXY.

$L(J \mid K)$ is the loss for outcome J, portfolio K

p_J is the probility of outcome J

$$\text{Mean}(all) = E(L(total \mid all)) = \sum_{all\ J}\sum_{all\ K} p_J L(J \mid K)$$

$$\text{Variance} = \sum_{all\ J} p_J \left(\sum_{all\ K} L(J \mid K)\right)^2 - \text{Mean}(all)^2$$

$$\text{Premium} = \text{Mean}(all) + \lambda \times \text{Variance}$$

5. **Continued**

(c) Calculate the premium for each account using the Shapley method.

For each account K we have mean and variance as follows:

$$\text{Mean}(K) = E(L(total \mid K)) = \sum_{all\ J} p_J L(J \mid K)$$

$$\text{Var}(K) = \sum_{all\ J} p_J L(J \mid K)^2 - \text{Mean}(K)^2$$

For each account pairing (K_1, K_2) combined we have:

$$\text{Mean}(K_1 + K_2) = E(L(total \mid K)) = \sum_{all\ J} p_J \left[L(J \mid K_1) + L(J \mid K_2)\right]$$

$$\text{Var}(K_1 + K_2) = \sum_{all\ J} p_J \left[L(J \mid K_1) + L(J \mid K_2)\right]^2 - \text{Mean}(K_1 + K_2)^2$$

Covariances for each account pairing are calulated

$$\text{Cov}(K_1, K_2) = \left[\text{Var}(K_1 + K_2) - \text{Var}(K_1) - \text{Var}(K_2)\right] \times \tfrac{1}{2}$$

The Shapley values, SV, for each portfolio are calulated as:

SV(AA) = Var(AA) + Cov(AA, BB) + Cov(AA, CC)

SV(BB) = Var(BB) + Cov(AA, BB) + Cov(BB, CC)

SV(CC) = Var(CC) + Cov(AA, CC) + Cov(BB, CC)

Renewal Risk Load (RL) and premium for each account K is:

$$\text{RL}(K) = \text{SV}(K) \times \lambda$$

$$\text{Premium}(K) = \text{Mean}(K) + \text{RL}(K)$$

(d) Demonstrate that the Shapley method does not have the problem identified in part (a).

Commentary on Question:
The model solution is an example of a full credit solution.

From part (b), the premium for all accounts combined is 4,366.
From part (c), if we sum the premiums across the three accounts ,we get 4,366 (1,811 + 1,283 + 1,272 = 4,366).

The sum of premiums across the accounts using the Shapley method is equal to the total portfolio premium. Therefore, it does not have the problem that the Marginal Surplus method has as explained in part (a).

6. **Learning Objectives:**

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4a) Explain the mathematics of excess of loss coverages in graphical terms.

Sources:

The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Commentary on Question:

This question tested a candidate's knowledge of Lee's graphical approach to rating excess of loss coverages.

Solution:

(a) Provide the equation for the expected payment per ground-up claim on this contract using ***A***, ***B***, ***C***, ***D***, ***R***, ***S***, ***F*** and/or ***G***, for the following methods.

(i) Size method

(ii) Layer method

(i) Size method = $D + S \times G(S) - R \times G(R)$

(ii) Layer method = A

(b) Restate the formulas provided in part (a) using the areas from the graph (I, II, …).

(i) Size method = (IV + VIII) + (V + IX) – (VIII + IX)

(ii) Layer method = (IV + V)

7. Learning Objectives:

4. The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:

(4g) Estimate the premium asset for retrospectively rated polices for financial reporting.

Sources:

Estimating the Premium Asset on Retrospectively Rated Policies, Teng and Perkins

Discussion of Estimating the Premium Asset on Retrospectively Rated Policies, Feldblum

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

LR = Loss Ratio, EMLR = Emerged LR, ELR = Expected LR, TM = Tax Multiplier,
BPF = Basic Premium Factor, LCF = Loss Conversion Factor,
EPLE = Expected Percentage of Loss Emerged,
PLEMM = Expected Percentage of Losses Eliminated by Retro Min/Max,
CLCR = Cumulative Loss Capping Ratio, ILCR = Incremental Loss Capping Ratio

Solution:

(a) Calculate the incremental loss capping ratio by retro adjustment period using the Teng and Perkins methodology.

For each retro adjustment t period calculate:

- EMLR = Standard LR × Cumulative EPLE
- PLEMM = Charge at Retro Max –Savings at Retro Min
- CLCR = 1 – PLEMM – % of Losses Eliminated by a Per Accident Limit

For $t = 1$

- $ILCR_1 = CLCR_1$

For $t > 1$

$ILCR_t = (EMLR_t \times CLCR_t - EMLR_{t-1} \times CLCR_{t-1}) / (EMLR_t - EMLR_{t-1})$

(b) Calculate the implied PDLD ratios at each retro adjustment period based upon the retrospective rating parameters and the selected incremental loss capping ratios.

For retro adjustment period 1

- $PDLD_1 = (BPF \times TM / (ELR \times EPLE_1)) + (ILCR_1 \times LCF \times TM)$

For retro adjustment period $t > 1$

- $PDLD_t = ILCR_t \times LCF \times TM$

7. **Continued**

(c) Calculate the premium asset as of December 31, 2023, for the policy period subject to the second retrospective adjustment using the PDLD ratios from part (b).

Commentary on Question:
Amounts in millions

Premium Asset
= Estimated total premium – Premium booked
= [Expected future loss emergence × $CPDLD_2$ + Premium booked from prior adjustment] – Premium booked

For retro adjustment period 1
- $EPLE_1$ = Cumulative $EPLE_1$

For retro adjustment period $t > 1$
- $EPLE_t = EPLE_t - EPLE_{t-1}$

$$\text{CPDLD}_2 = \frac{\sum_{t=2}^{5} \text{EPLE}_t \times \text{PDLD}_t}{\sum_{t=2}^{5} \text{EPLE}_t} = 0.741$$

Premium Asset
= [72.65 × 0.741 + 302.38] – 298.62
= 57.56

8. **Learning Objectives:**

3. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(3a) Describe a risk margin analysis framework.

(3b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(3c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, Marshall, et al.

Commentary on Question:

This question tested a candidate's understanding of the framework for assessing risk margins as presented by Marshall et al.

Solution:

(a) Complete the following table using the information provided above.

Internal Systemic Risk			
Risk	**Risk Component**	**Risk Indicator**	**Which of I - VI are considered when scoring this risk indicator against best practice**
1			
2			
3			

Commentary on Question:

The model solution is an example of a full credit solution.

8. Continued

Internal Systemic Risk			
Risk	**Risk Component**	**Risk Indicator**	**Which of I - VI are considered when scoring this risk indicator against best practice**
1	Specification Error	Extent of monitoring and review of model and assumption performance	IV
2	Parameter Selection Error	Ability to identify and use best predictors	III
3	Data Error	Timeliness, consistency and reliability of information from business	V

(b) Identify two of these categories of ESR sources that are created from the information provided. Identify which of I through VI creates each.

Commentary on Question:
The model solution is an example of a full credit solution.

- Claim management process change risk from I
- Legislative risk from VI

(c) Select the appropriate CoV to be used for each line of business. Justify your selections.

Commentary on Question:
The model solution is an example of a full credit solution.

- Scales 1 and 2 are not reasonable because a higher score should be associated with a lower CoV.
- Scale 5 is not reasonable as the scale should not be linear.
- Scales 3 and 4 appear reasonable.
- CoVs for a long-tail line are generally higher than those for a short-tail line. Therefore, Scale 3 should be used for property and Scale 4 should be used for liability.
- With a weighted score of 3.9 for each line of business, we have:
  - Property CoV = 5.5%
  - Liability CoV = 9.5%

9. **Learning Objectives:**

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5k) Test for risk transfer in reinsurance contracts.

Sources:

Risk Transfer Testing of Reinsurance Contracts, Brehm and Ruhm

Insurance Risk Transfer and Categorization of Reinsurance Contracts, Gurenko, Itigin and Wiechert

Commentary on Question:

This question tested a candidate's understanding of testing for risk transfer in reinsurance contracts.

Solution:

(a) Complete the following table with the attributes listed above. *Note that an attribute may be included in more than one cell and a cell may include more than one attribute.*

Quantitative Test	Advantage(s)	Disadvantage(s)
Value-at-Risk (VaR)		
Tail Value-at-Risk (TVaR)		
Expected Reinsurer Deficit (ERD)		

Commentary on Question:

The model solution is an example of a full credit solution. Note that some of the attributes presented are subjective and could be included in different cells from what is included in the model solution. For example, it was also acceptable to include II as an advantage for TVaR. Also, while VI applies to ERD, arguments may be made for it either being an advantage or a disadvantage. However, some of the attributes could only be correctly placed in one cell. For example, I is an advantage for ERD only and III is a disadvantage for VaR only.

Quantitative Test	Advantage(s)	Disadvantage(s)
Value-at-Risk (VaR)	II	III, V, VII, VIII
Tail Value-at-Risk (TVaR)	IX	V, VII
Expected Reinsurer Deficit (ERD)	I, VI	V

9. Continued

(b) State the following:

(i) The accounting treatment for a reinsurance contract that is categorized as not transferring sufficient insurance risk.

(ii) A type of reinsurance coverage deemed to transfer sufficient risk transfer despite being not "reasonably self-evident" and not fulfilling quantitative risk transfer tests.

(i) Deposit accounting (*or accounted for as a financial instrument*)

(ii) A reinsurance coverage that assumes substantially all the risks from the primary contract (*e.g., straight quota share*).

(c) Compare the risk measurement in the ERD test with that in the Risk Coverage Ratio (RCR) test.

Commentary on Question:
The model solution is an example of a full credit solution.

Both measure tail risk. However, ERD is a risk/premium measure, while RCR is the corresponding risk/return measure.

(d) Show the formula for RCR (in percent form) that includes ERD as a term in the formula. Define all terms in the formula, excluding ERD.

RCR (% form) = ERD / (E[G]/P)

- E[G] = expected economic gain across all possibilities
- P = reinsurance premium

10. **Learning Objectives:**

2. The candidate will understand the considerations in the development of losses for excess limits and layers.

Learning Outcomes:

(2a) Estimate ultimate claims for excess limits and layers.

(2b) understand the difference in development patterns and trends for excess limits and layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Appendix G

Commentary on Question:

This question tested a candidate's knowledge regarding the development of excess limits and layers. This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

(a) Calculate the IBNR as of December 31, 2023 by AY for the 200,000 limit using Siewert's formula.

Commentary on Question:

CDF = Cumulative Development Factor

Step 1: Calculate the first three CDFs for unlimited.

For t = 12, 24 and 36,

$CDF_t(\text{Unlimited}) = CDF_t(100k) \times R_t(100k) / R_{72}(100k)$

Step 2: Calculate the first three CDFs for 200k limits.

For t = 12, 24 and 36,

$CDF_t(200k) = CDF_t(\text{Unlimited}) \times R_{72}(200k) / R_t(200k)$

Step 3: Calculate IBNR by AY for 200k limits.

AY2023: $[CDF_{12}(200k) - 1] \times$ Reported 200k limit (AY2023, 12 months)

AY2022: $[CDF_{24}(200k) - 1] \times$ Reported 200k limit (AY2022, 24 months)

AY2021: $[CDF_{36}(200k) - 1] \times$ Reported 200k limit (AY2021, 36 months)

(b) Explain why actuarial judgement is needed when using Siewert's formula based on the results in part (a).

Commentary on Question:

The model solution is an example of a full credit solution.

10. Continued

Actuarial judgement is needed when using Siewert's formulas because the estimated relativities at alternative limits can result in unusual cumulative development factors. The following results from part (a) are unusual:

- The calculated CDF at 200k limit for 24 months is lower than that for 36 months.
- The calculated CDF at 200k limit for 24 months is lower than that CDF at 100k limit for 24 months.

(c) Calculate the IBNR as of December 31, 2023 by AY for the 200,000 limit using the ILF method.

Step 1: Calculate the ILF trend for 200k limit over the base limit as
(claims trend 200k limit + 1)/ (claims trend 100k limit + 1)

Step 2: Calculate the ILF trend period in years for each AY as the period from the average date of loss for the AY to the date of the cost level for the ILF.
So, it is 0.5 for AY2021, 1.5 for AY2022 and 2.5 for AY2023.

Step 3: Calculate the trended ILF factor for each AY as the
ILF × [(1 + ILF trend *from step 1*)^(AY trend period *from step 2*)]

Step 4: Calculate the ultimate claims at 200k limit for each AY as
AY trended ILF *from step 3* × AY Ultimate 100k *provided in the 2nd table*

Step 5: Calculate the IBNR at 200k limit for each AY as
AY Ultimate 200k *from step 4* – AY Reported as of December 31, 2023

11. **Learning Objectives:**

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6b) Develop rates for claims made contracts.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 35: Claims-Made Ratemaking

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

Determine the following:

(i) (*0.5 points*) Average annual accident year trend rate

(ii) (*1 point*) Accident year reporting pattern as a percent of total

(iii) (*1.5 points*) Step factor at each year of claims-made maturity

(iv) (*0.5 points*) Tail factor applicable to coverage following a first-year claims-made maturity policy

(v) (*0.5 points*) Tail factor applicable to coverage following a third-year claims-made maturity policy

Commentary on Question:

Comments are included within the solution as italicized text.

(i) Average annual accident year trend rate

One should look at the trend in claims over several AYs. The model solution uses AY 6 to AY10 as they represent all the completed AYs in the information provided. Claims ($C_{AY\text{-}Lag,\ RY}$) for AYs 6 to 10 are as follows:

$C(AY6) = C_{0,6}+C_{1,7}+C_{2,8}+C_{3,9}+C_{4,10}$

$= 541.93+451.63+451.63+180.65+180.65 = 1,806.49$

$C(AY7) = C_{0,7}+C_{1,8}+C_{2,9}+C_{3,10}+C_{4,11}$

$= 585.30+487.75+487.75+195.09+195.09 = 1,950.98$

and similarly

C(AY8) = 2,107.06, C(AY9) = 2,275.59 and C(AY10) = 2,457.67

AY claims year-over-year changes = $C(AYt) / C(AYt-1) - 1$

11. Continued

This provides the following year-over-year changes:

7.998%, 8%, 7.998%, 8%

Therefore, the average annual accident year trend rate = 8%

Note that one could also look at the changes to AY lag claims such as $C_{0,7}/C_{0,6} - 1 = 8\%$, $C_{1,8}/C_{1,7} - 1 = 7.998\%$, *etc. to reveal the 8% trend.*

(ii) Accident year reporting pattern as a percent of total

AY reporting pattern for AYx is given by $C_{0,x}/C(AYx)$, $C_{1,x+1}/CY(AYx)$, *etc.*

Using AY6 as an example to calculate the pattern:

AY Lag 0 report % = 541.93 / 1,806.49 = 30%,

AY Lag 1 report % = 451.63 / 1,806.49 = 25%, etc. so we have

AY Lag	0	1	2	3	4
Pattern	**30.0%**	**25.0%**	**25.0%**	**10.0%**	**10.0%**

(iii) Step factor at each year of claims-made maturity

SF(1) = 1st year step factor, SF(2) = 2nd year step factor, etc.

Using RY 10 as an example.

$SF(1) = C_{0,10}/(C_{0,10}+C_{1,10}+C_{2,10}+C_{3,10}+C_{4,10}) = 0.33$

$SF(2) = SF(1) + C_{1,10}/(C_{0,10}+C_{1,10}+C_{2,10}+C_{3,10}+C_{4,10}) = 0.59$

$SF(3) = SF(2) + C_{2,10}/(C_{0,10}+C_{1,10}+C_{2,10}+C_{3,10}+C_{4,10}) = 0.83$

$SF(4) = SF(3) + C_{3,10}/(C_{0,10}+C_{1,10}+C_{2,10}+C_{3,10}+C_{4,10}) = 0.92$

$SF(mature) = SF(4) + C_{4,10}/(C_{0,10}+C_{1,10}+C_{2,10}+C_{3,10}+C_{4,10}) = 1.00$

(iv) Tail factor applicable to coverage following a first-year claims-made maturity policy

Using RY 10 as an example.

Tail factor 1st year = $(C_{1,11}+C_{2,12}+C_{3,13}+C_{4,14}) / C_{0,10}$

= (614.42+614.42+245.76+245.76) / 737.71 = 2.33

(v) Tail factor applicable to coverage following a third-year claims-made maturity policy

Using RY 10 as an example.

Tail factor 3rd year

$= (C_{1,11}+C_{2,11}+C_{3,11}+C_{2,12}+C_{3,12}+C_{4,12}+C_{3,13}+C_{4,13}+C_{4,14}) /$

$(C_{0,10}+C_{1,10}+C_{2,10}) = 1.73$

12. **Learning Objectives:**

6. The candidate will understand and apply specialized ratemaking techniques.

Learning Outcomes:

(6a) Price for deductible options and increased limits.

Sources:

Fundamentals of General Insurance Actuarial Analysis, 2nd Ed. (2022), Friedland

- Chapter 34: Actuarial Pricing for Deductibles and Increased Limits

Commentary on Question:

This question tested a candidate's knowledge of policy limits and deductibles.

Solution:

(a) Identify two other reasons insurers use deductibles in their policies.

Commentary on Question:

There are more than two other reasons. The model solution is an example of a full credit solution.

- Encourage insureds to adhere to some measure of risk control
- Eliminate the processing costs associated with small claims

(b) Provide an example of an action taken by an insured that would be considered:

(i) Moral hazard

(ii) Morale hazard

Commentary on Question:

There are many possible examples for this. The model solution is an example of a full credit solution.

(i) Moral Risk:
An insured fraudulently puts forth a claim for a stolen item when the item was not stolen because the insured sold it.

(ii) Morale Risk:
An insured does not properly safeguard their property against theft because they know they are insured.

(c) Describe a problem with the use of a percentage deductible for property insurance.

Commentary on Question:

The model solution is an example of a full credit solution.

12. Continued

It can provide an incentive for insureds to purchase insurance with a lower sum insured than the full value of the property. This is because those who purchase insurance policies with a smaller total insured value will automatically have a lower deductible and a lower premium. However, they will be underinsured if there is a total loss.

(d) Describe how a coinsurance clause in a property policy limits claims.

Commentary on Question:
The model solution is an example of a full credit solution.

It creates a penalty for underinsuring a property. The penalty is based on the percentage that the policy limit is below the property value.

13. **Learning Objectives:**

5. The candidate will understand several aspects of reinsurance including the various types of reinsurance, the issues encountered when performing a reserve analysis on reinsurance, how to apply the fundamental techniques of reinsurance pricing and risk transfer testing of reinsurance contracts.

Learning Outcomes:

(5g) Calculate the price for a casualty per occurrence excess treaty.

Sources:

Basics of Reinsurance Pricing, Clark

Commentary on Question:

This question required the candidate to respond in Excel. An example of a full credit solution is in the Excel solutions spreadsheet. The solution in this file is for explanatory purposes only.

Solution:

Estimate the experience rating loss cost, including ALAE, as a percentage of the subject premium.

Step 1: Determine the trend period (in years) for each claim as the difference between the accident date of the claim and the average accident date of the policy period being priced (accident year 2025, so July 1, 2025)

Step 2: Calculate trended losses and trended ALAE separately for each claim. The trended loss uses the loss trend (5%) over the trend period for the claim multiplied by the untrended loss. The trended ALAE uses the ALAE trend (10%) over the trend period for the claim multiplied by the untrended ALAE.

Step 3: Calculate the layer trended loss and ALAE combined for each claim. For each claim, sum the trended loss and the trended ALAE. The amount in the layer is that combined trended loss and ALAE amount above the excess attachment point (200,000) limited by the size of the layer (800,000) for each claim.

Step 4: Calculate the total developed trended loss and ALAE in the layer. For each claim, apply the applicable development factor (based on the accident year of the claim) to the trended loss and ALAE in the layer. Sum this amount for all six of the claims to get the total developed trended loss and ALAE in the layer (4,082,185).

13. **Continued**

Step 5: Calculate the experience rating loss cost.

This is the total developed trended loss and ALAE in the layer (step 4 amount) divided by the total premium over the experience period (30,000,000 because it is 10,000,000 for each of the three years in the experience period). This is equal to 6.80%

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