

## QFI-Quantitative Finance Formula Package Spring and Fall 2022

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula package was developed by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not in the formula package.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

In sources where some equations are numbered and others are not, the page number is provided instead.

## Chapter 2

$$(2.10) \quad \begin{bmatrix} 1 \\ S(t) \\ C(t) \end{bmatrix} = \begin{bmatrix} (1+r\Delta) & (1+r\Delta) \\ S_1(t+\Delta) & S_2(t+\Delta) \\ C_1(t+\Delta) & C_2(t+\Delta) \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$

$$(2.46) \quad R_1(t+1) = \frac{S_1(t+1)}{S(t)}$$

$$(2.47) \quad R_2(t+1) = \frac{S_2(t+1)}{S(t)}$$

$$(2.48) \quad 0 = ((1+r) - R_1)\psi_1 + ((1+r) - R_2)\psi_2$$

$$(2.66) \quad S_t = \frac{1}{1+r} [\mathbb{Q}_{up}(S_t + \sigma\sqrt{\Delta}) + \mathbb{Q}_{down}(S_t - \sigma\sqrt{\Delta})]$$

$$(2.67) \quad C_t = \frac{1}{(1+r)} [\mathbb{Q}_{up}C_{t+\Delta}^{up} + \mathbb{Q}_{down}C_{t+\Delta}^{down}]$$

$$(2.68) \quad C_T = \max[S_T - C_0, 0]$$

$$(2.70) \quad S = \frac{1+d}{1+r} [S^u\mathbb{Q}_{up} + S^d\mathbb{Q}_{down}]$$

$$(2.71) \quad C = \frac{1}{1+r} [C^u\mathbb{Q}_{up} + C^d\mathbb{Q}_{down}]$$

$$(page\ 27) \quad \mathbb{E}^{\mathbb{Q}} \left[ \frac{C_{t+\Delta}}{C_t} \right] \approx 1 + r\Delta$$

## Chapter 3

$$(3.37) \quad \sum_{i=1}^n f\left(\frac{t_i + t_{i-1}}{2}\right)(t_i - t_{i-1}) \rightarrow \int_0^T f(s)ds$$

$$(3.49) \quad \int_0^T g(s)df(s) \approx \sum_{i=1}^n g\left(\frac{t_i + t_{i-1}}{2}\right)(f(t_i) - f(t_{i-1}))$$

## Chapter 4

$$(4.20) \quad f(x) = \sum_{i=0}^{\infty} \frac{1}{i!} f^{(i)}(x_0)(x - x_0)^i$$

$$(4.23) \quad dF(t) = F_S dS_t + F_r dr_t + F_t dt$$

$$(4.24) \quad dF(t) = F_s dS_t + F_r dr_t + F_t dt + \frac{1}{2} F_{ss} dS_t^2 + \frac{1}{2} F_{rr} dr_t^2 + F_{sr} dS_t dr_t$$

## Chapter 5

$$(5.11) \quad \mathbb{E}[S_t | I_u] = \int_{-\infty}^{\infty} S_t f(S_t | I_u) dS_t, \quad u < t$$

$$(5.18) \quad P(\Delta F(t) = +a\sqrt{\Delta}) = p$$

$$(5.19) \quad P(\Delta F(t) = -a\sqrt{\Delta}) = 1 - p$$

$$(5.37) \quad P(\Delta N_t = 1) \approx \lambda\Delta$$

$$(5.38) \quad P(\Delta N_t = 0) \approx 1 - \lambda\Delta$$

$$(5.39) \quad P(\Delta N_t = n) = \frac{e^{-\lambda\Delta}(\lambda\Delta)^n}{n!}$$

$$(5.40) \quad f(x) = \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right), \quad x \geq 0$$

$$(5.41) \quad F(x) = 1 - \exp(-x/\theta), \quad x \geq 0$$

$$(5.42) \quad f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$$

$$(5.44) \quad P(X_{t+s} \leq x_{t+s} | x_t, \dots, x_1) = P(X_{t+s} \leq x_{t+s} | x_t)$$

$$(5.45) \quad r_{t+\Delta} - r_t = \mathbb{E}[(r_{t+\Delta} - r_t) | I_t] + \sigma(I_t, t) \Delta W_t$$

$$(5.48) \quad dr_t = \mu(r_t, t) dt + \sigma(r_t, t) dW_t$$

$$(5.49) \quad \begin{bmatrix} r_{t+\Delta} \\ R_{t+\Delta} \end{bmatrix} = \begin{bmatrix} \alpha_1 r_t + \beta_1 R_t \\ \alpha_2 r_t + \beta_2 R_t \end{bmatrix} + \begin{bmatrix} \sigma_1 W_{t+\Delta}^1 \\ \sigma_2 W_{t+\Delta}^2 \end{bmatrix}$$

## Chapter 6

$$(6.3) \quad \mathbb{E}_t[S_T] = \mathbb{E}[S_T | I_t], \quad t < T$$

$$(6.4) \quad \mathbb{E}|S_t| < \infty$$

$$(6.5) \quad \mathbb{E}_t[S_T] = S_t, \quad \text{for all } t < T$$

$$(6.9) \quad \mathbb{E}_t^{\mathbb{Q}}[e^{-ru} B_{t+u}] = B_t, \quad 0 < u < T - t \quad (\text{The text formula is incorrect, the right-hand side should be } B_t)$$

$$(6.10) \quad \mathbb{E}_t^{\mathbb{Q}}[e^{-ru} S_{t+u}] = S_t, \quad 0 < u$$

$$(6.29) \quad V^1 = \sum_{i=1}^n |X_{t_i} - X_{t_{i-1}}|$$

$$(6.30) \quad V^2 = \sum_{i=1}^n |X_{t_i} - X_{t_{i-1}}|^2$$

$$(6.31) \quad V^4 = \sum_{i=1}^n |X_{t_i} - X_{t_{i-1}}|^4$$

$$(6.36) \quad V^2 \leq \max_i |X_{t_i} - X_{t_{i-1}}| V^1$$

$$(6.37) \quad \max_i |X_{t_i} - X_{t_{i-1}}| \rightarrow 0$$

$$(6.38) \quad V^4 \leq \max_i |X_{t_i} - X_{t_{i-1}}|^2 V^2$$

$$(6.44) \quad \Delta X_t \sim N(\mu\Delta, \sigma^2\Delta)$$

$$(6.46) \quad X_{t+T} = X_0 + \int_0^{t+T} dX_u$$

$$(6.49) \quad X_t + \mathbb{E}_t \left[ \int_t^{t+T} dX_u \right] = X_t + \mu T \quad (\text{This is a correction to the text formula})$$

$$(6.50) \quad Z_t = X_t - \mu t$$

$$(6.53) \quad \mathbb{E}_t[Z_{t+T}] = X_t + \mathbb{E}_t[(X_{t+T} - X_t)] - \mu(t+T) \quad (\text{This is a correction to the text formula})$$

$$(6.54) \quad \mathbb{E}_t[Z_{t+T}] = X_t - \mu t \quad (\text{This is a correction to the text formula})$$

$$(6.55) \quad \mathbb{E}_t[Z_{t+T}] = Z_t$$

$$(6.64) \quad I_t \subseteq I_{t+1} \subseteq \dots \subseteq I_{T-1} \subseteq I_T$$

$$(6.65) \quad M_t = \mathbb{E}^{\mathbb{P}}[Y_T | I_t]$$

$$(6.66) \quad \mathbb{E}^{\mathbb{P}}[M_{t+s}|I_t] = M_t$$

$$(6.70) \quad G_T = f(S_T)$$

$$(6.71) \quad B_T = e^{\int_t^T r_s ds}$$

$$(6.72) \quad M_t = \mathbb{E}^{\mathbb{P}} \left[ \frac{G_T}{B_T} | I_t \right]$$

$$(6.105) \quad e^{-rt} S_t = A_t + Z_t$$

$$(6.106) \quad M_{t_k} = M_{t_0} + \sum_{i=1}^k H_{t_{i-1}} [Z_{t_i} - Z_{t_{i-1}}]$$

$$(6.108) \quad \mathbb{E}_{t_0}[M_{t_k}] = M_{t_0} \quad (\text{This is a correction to the text formula})$$

## Chapter 7

$$(7.23) \quad \Delta W_k = [S_k - S_{k-1}] - \mathbb{E}_{k-1}[S_k - S_{k-1}]$$

$$(7.26) \quad W_k = \sum_{i=1}^k \Delta W_i$$

$$(7.28) \quad \mathbb{E}_{k-1}[W_k] = W_{k-1}$$

$$(7.29) \quad \mathbb{V}^k = \mathbb{E}_0[\Delta W_k^2]$$

$$(7.30) \quad \mathbb{V} = \mathbb{E}_0 \left[ \sum_{k=1}^n \Delta W_k \right]^2 = \sum_{k=1}^n \mathbb{V}^k$$

$$(7.31) \quad \mathbb{V} > A_1 > 0$$

$$(7.33) \quad \mathbb{V} < A_2 < \infty$$

$$(7.34) \quad \mathbb{V}_{max} = \max_k [\mathbb{V}^k, k = 1, \dots, n]$$

$$(7.35) \quad \frac{\mathbb{V}^k}{\mathbb{V}_{max}} > A_3, \quad 0 < A_3 < 1$$

$$(7.36) \quad \mathbb{E}[\Delta W_k]^2 = \sigma_k^2 h$$

$$(7.56) \quad S_k - S_{k-1} = \mathbb{E}_{k-1}[S_k - S_{k-1}] + \sigma_k \Delta W_k$$

$$(7.63) \quad \mathbb{E}_{k-1}(S_k - S_{k-1}) \approx a(I_{k-1}, kh)h$$

$$(7.64) \quad S_{kh} - S_{(k-1)h} \approx a(I_{k-1}, kh)h + \sigma_k [W_{kh} - W_{(k-1)h}]$$

## Chapter 8

$$(8.7) \quad dN_t = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases}$$

$$(8.8) \quad M_t = N_t - \lambda t$$

$$(8.9) \quad \mathbb{E}[M_t] = 0$$

$$(8.10) \quad \mathbb{E}[M_t]^2 = \lambda t$$

$$(8.21) \quad \sigma_k \Delta W_k = \begin{cases} \omega_1 & \text{with probability } p_1 \\ \omega_2 & \text{with probability } p_2 \\ \vdots & \vdots \\ \omega_m & \text{with probability } p_m \end{cases}$$

$$(8.22) \quad \mathbb{E}[\sigma_k \Delta W_k]^2 = \sigma_k^2 h$$

$$(8.23) \quad \sum_{i=1}^m p_i \omega_i^2 = \sigma_k^2 h$$

$$(8.29) \quad p_i(h) = \bar{p}_i h^{q_i}$$

$$(8.33) \quad q_i + 2r_i = 1$$

$$(8.34) \quad c_i = \bar{\omega}_i^2 \bar{p}_i$$

$$(8.58) \quad J_t = (N_t - \lambda t)$$

$$(8.60) \quad dS_t = a(S_t, t)dt + \sigma_1(S_t, t)dW_t + \sigma_2(S_t, t)dJ_t$$

$$(8.61) \quad \mathbb{V}[X_t] = \mathbb{E}[X_t - \mathbb{E}[X_t]]^2$$

$$(8.62) \quad \text{Higher-order (centered) moments are } \mathbb{E}[X_t - \mathbb{E}[X_t]]^k, \quad k > 2$$

$$(8.72) \quad \mathbb{E}[\sigma_2 \Delta J_k]^2 = h \left[ \sum_{i=1}^m \omega_i^2 \bar{p}_i \right]$$

$$(8.73) \quad \mathbb{E}[\sigma_2 \Delta J_k]^n = h \left[ \sum_{i=1}^m \omega_i^n \bar{p}_i \right]$$

$$(8.75) \quad t_0 = 0 < t_1 < \dots < t_n = T$$

$$(8.76) \quad n\Delta = T$$

$$(8.77) \quad S_i = S_{t_i}, \quad i = 0, 1, \dots, n$$

$$(8.78) \quad S_{i+1} = \begin{cases} u_i S_i & \text{with probability } p_i \\ d_i S_i & \text{with probability } 1 - p_i \end{cases}$$

$$(8.79) \quad u_i = e^{\sigma \sqrt{\Delta}}, \quad \text{for all } i$$

$$(8.80) \quad d_i = e^{-\sigma \sqrt{\Delta}}, \quad \text{for all } i$$

$$(8.81) \quad p_i = \frac{1}{2} \left[ 1 + \frac{\mu}{\sigma} \sqrt{\Delta} \right], \quad \text{for all } i$$

$$(8.91) \quad \log \frac{S_{i+n}}{S_i} = Z \log u + (n - Z) \log d$$

$$(8.92) \quad \log \frac{S_{i+n}}{S_i} = Z \log \frac{u}{d} + n \log d$$

$$(8.96) \quad \mathbb{E} \left[ \log \frac{S_{i+n}}{S_i} \right] = \log \frac{u}{d} np + n \log d$$

$$(8.97) \quad \mathbb{V} \left[ \log \frac{S_{i+n}}{S_i} \right] = \left[ \log \frac{u}{d} \right]^2 np(1 - p)$$

$$(8.98) \quad n = \frac{T}{\Delta}$$

$$(8.99) \quad \log \frac{u}{d} np + n \log d \approx \mu T$$

$$(8.100) \quad \left[ \log \frac{u}{d} \right]^2 np(1 - p) \approx \sigma^2 T$$

$$(8.102) \quad [\log S_{i+n} - \log S_i] \sim \mathcal{N}(\mu(n\Delta), \sigma^2 \Delta)$$

$$(8.103) \quad [\log S_{i+n} - \log S_i] \sim \text{Poisson}$$

## Chapter 9

$$(9.37) \quad \lim_{n \rightarrow \infty} \mathbb{E} \left[ \sum_{k=1}^n \sigma(S_{k-1}, k) [W_k - W_{k-1}] - \int_0^T \sigma(S_u, u) dW_u \right]^2 = 0$$

$$(9.38) \quad S_k - S_{k-1} = a(S_{k-1}, k)h + \sigma(S_{k-1}, k)[W_k - W_{k-1}], \quad k = 1, 2, \dots, n$$

$$(9.39) \quad \mathbb{E} \left[ \int_0^T \sigma(S_t, t)^2 dt \right] < \infty$$

$$(9.41) \quad \sum_{k=1}^n \sigma(S_{k-1}, k)[W_k - W_{k-1}] \rightarrow \int_0^T \sigma(S_t, t) dW_t \text{ as } n \rightarrow \infty (h \rightarrow 0)$$

$$(9.73) \quad \int_0^T x_t dx_t = \frac{1}{2} [x_T^2 - T]$$

$$(9.74) \quad \lim_{n \rightarrow \infty} \mathbb{E} \left[ \sum_{i=0}^{n-1} \Delta x_{t_{i+1}}^2 - T \right]^2 = 0$$

$$(9.76) \quad \text{If } \int_0^T (dx_t)^2 \text{ exists, then } \lim_{n \rightarrow \infty} \mathbb{E} \left[ \sum_{i=0}^{n-1} \Delta x_{t_{i+1}}^2 - \int_0^T (dx_t)^2 \right]^2 = 0$$

$$(9.77) \quad \int_0^T dt = T$$

$$(9.78) \quad \int_0^T (dx_t)^2 = \int_0^T dt$$

$$(9.79) \quad (dW_t)^2 = dt$$

$$(9.85) \quad \mathbb{E}_t \left[ \int_0^{t+\Delta} \sigma_u dW_u \right] = \int_0^t \sigma_u dW_u \quad (\text{This is a correction to the text formula})$$

$$(9.132) \quad \mathbb{E} \left[ \int_0^T f(W_t, t) dW_t \int_0^T g(W_t, t) dW_t \right] = \mathbb{E} \left[ \int_0^T f(W_t, t) g(W_t, t) dt \right]$$

(This is a correction to the text formula)

$$(9.133) \quad \mathbb{E} \left[ \int_0^T f(W_t, t) dW_t \right]^2 = \mathbb{E} \left[ \int_0^T f(W_t, t)^2 dt \right] \quad (\text{This is a correction to the text formula})$$

## Chapter 10

(page 170)  $dS_t = a_t dt + \sigma_t dW_t, \quad t \geq 0$

$$(10.36) \quad dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \sigma_t^2 \frac{\partial^2 F}{\partial S_t^2} dt$$

$$(10.37) \quad dF_t = \left[ a_t \frac{\partial F}{\partial S_t} + \frac{\partial F}{\partial t} + \frac{1}{2} \sigma_t^2 \frac{\partial^2 F}{\partial S_t^2} \right] dt + \frac{\partial F}{\partial S_t} \sigma_t dW_t \quad (\text{This is a correction to the text formula})$$

$$(10.64) \quad \int_0^t F_s dS_u = [F(S_t, t) - F(S_0, 0)] - \int_0^t \left[ F_u + \frac{1}{2} F_{ss} \sigma_u^2 \right] du$$

$$(10.69) \quad dF = F_t dt + F_{s_1} dS_1 + F_{s_2} dS_2 + \frac{1}{2} [F_{s_1 s_1} dS_1^2 + F_{s_2 s_2} dS_2^2 + 2F_{s_1 s_2} dS_1 dS_2]$$

$$(10.72) \quad dS_1(t)^2 = [\sigma_{11}^2(t) + \sigma_{12}^2(t)] dt$$

$$(10.73) \quad dS_2(t)^2 = [\sigma_{21}^2(t) + \sigma_{22}^2(t)] dt$$

$$(10.74) \quad dS_1(t) dS_2(t) = [\sigma_{11}(t) \sigma_{21}(t) + \sigma_{12}(t) \sigma_{22}(t)] dt$$

$$(10.79) \quad Y(t) = \sum_{i=1}^n N_i(t)P_i(t)$$

$$(10.80) \quad dY(t) = \sum_{i=1}^n N_i(t)dP_i(t) + \sum_{i=1}^n dN_i(t)P_i(t) + \sum_{i=1}^n dN_i(t)dP_i(t)$$

$$(10.81) \quad dS_t = a_t dt + \sigma_t dW_t + dJ_t, \quad t \geq 0$$

$$(10.82) \quad E[\Delta J_t] = 0$$

$$(10.83) \quad \Delta J_t = \Delta N_t - \left[ \lambda_t h \left( \sum_{i=1}^k a_i p_i \right) \right]$$

$$(10.84) \quad a_t = \alpha_t + \lambda_t \left( \sum_{i=1}^k a_i p_i \right)$$

$$(10.85) \quad dF(S_t, t) = \left[ F_t + \lambda_t \sum_{i=1}^k (F(S_t + a_i, t) - F(S_t, t)) p_i + \frac{1}{2} F_{ss} \sigma^2 \right] dt + F_s dS_t + dJ_F$$

$$(10.86) \quad dJ_F = [F(S_t, t) - F(S_t^-, t)] - \lambda_t \left[ \sum_{i=1}^k (F(S_t + a_i, t) - F(S_t, t)) p_i \right] dt$$

$$(10.87) \quad S_t^- = \lim_{s \rightarrow t} S_s, \quad s < t$$

## Chapter 11

$$(11.24) \quad dS_t = \mu S_t dt + \sigma S_t dW_t, \quad t \in [0, \infty)$$

$$(11.30) \quad S_t = S_0 e^{\{(a - \frac{1}{2}\sigma^2)t + \sigma W_t\}}$$

$$(11.34) \quad dS_t = r S_t dt + \sigma S_t dW_t, \quad t \in [0, \infty)$$

$$(11.38) \quad S_T = \left[ S_0 e^{(r - \frac{1}{2}\sigma^2)T} \right] \left[ e^{\sigma W_T} \right]$$

$$(11.42) \quad Z_t = e^{\sigma W_t}$$

$$(11.50) \quad x_t = \mathbb{E}[Z_t] = e^{\frac{1}{2}\sigma^2 t}$$

$$(11.56) \quad S_t = e^{-r(T-t)} \mathbb{E}_t[S_T]$$

$$(11.72) \quad dS_t = \mu S_t dt + \sigma \sqrt{S_t} dW_t, \quad t \in [0, \infty)$$

$$(11.74) \quad dS_t = \lambda(\mu - S_t) dt + \sigma S_t dW_t$$

$$(11.78) \quad dS_t = -\mu S_t dt + \sigma dW_t$$

$$(11.79) \quad dS_t = \mu dt + \sigma_t dW_{1t}$$

$$(page 192) \quad d\sigma_t = \lambda(\sigma_0 - \sigma_t) dt + \alpha \sigma_t dW_{2t}$$

$$(page 193) \quad dN_t = \begin{cases} 1 & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt \end{cases}$$

$$(11.83) \quad \frac{dS_t}{S_t} = (\mu - \lambda \kappa) dt + \sigma dW_t + (e^J - 1) dN_t$$

$$(11.84) \quad \frac{dS_t}{S_t} = (\mu - \lambda^* \kappa^*) dt + \sigma d\tilde{W}_t + (e^J - 1) dN_t$$

$$(page 194) \quad S_t = S_0 e^{(r-q+\omega)t + X(t; \sigma, \nu, \theta)}$$

$$(page 194) \quad f(x; \sigma, \nu, \theta) = \int_0^\infty \frac{1}{\sigma \sqrt{2\pi g}} \exp\left(-\frac{(x - \theta g)^2}{2\sigma^2 g}\right) \frac{g^{t/\nu-1} e^{-g/\nu}}{\nu^{t/\nu} \Gamma(t/\nu)} dg$$

## Chapter 12

$$(12.3) \quad P_t = \theta_1 F(S_t, t) + \theta_2 S_t$$

$$(12.4) \quad dP_t = \theta_1 dF_t + \theta_2 dS_t$$

$$(12.5) \quad dS_t = a(S_t, t)dt + \sigma(S_t, t)dW_t, \quad t \in [0, \infty)$$

$$(12.6) \quad dF_t = F_t dt + \frac{1}{2} F_{ss} \sigma_t^2 dt + F_s dS_t$$

$$(12.7) \quad dF_t = \left[ F_s a_t + \frac{1}{2} F_{ss} \sigma_t^2 + F_t \right] dt + F_s \sigma_t dW_t$$

$$(12.10) \quad \theta_1 = 1$$

$$(12.11) \quad \theta_2 = -F_s$$

$$(12.12) \quad dP_t = F_t dt + \frac{1}{2} F_{ss} \sigma_t^2 dt$$

$$(12.16) \quad r(F(S_t, t) - F_s S_t) = F_t + \frac{1}{2} F_{ss} \sigma_t^2$$

$$(12.17) \quad -rF + rF_s S_t + F_t + \frac{1}{2} F_{ss} \sigma_t^2 = 0, \quad 0 \leq S_t, \quad 0 \leq t \leq T$$

$$(12.20) \quad P_t = \theta_1 F(S_t, t) + \theta_2 S_t$$

$$(12.23) \quad P_t = F(S_t, t) - F_s(S_t, t) S_t$$

$$(12.24) \quad dP_t = dF(S_t, t) - F_s dS_t - S_t dF_s - dF_s(S_t, t) dS_t$$

$$(12.26) \quad dP_t = dF(S_t, t) - F_s dS_t - S_t \left[ \left[ F_{st} + F_{ss} \mu S_t + \frac{1}{2} F_{sss} \sigma_t^2 S_t^2 \right] dt + F_{ss} \sigma_t dW_t \right] - F_{ss} \sigma_t^2 S_t^2 dt$$

$$(12.28) \quad dP_t = dF(S_t, t) - F_s dS_t - S_t [F_{ss}(\mu - r) S_t dt] - F_{ss} \sigma_t^2 S_t^2 dW_t$$

$$(page 202) \quad \mathbb{E}^Q [S_t^2 F_{SS}(\sigma dW_t + (\mu - r)\Delta)] \approx 0$$

$$(page 202) \quad dW_t^* = \sigma dW_t + (\mu - r)dt$$

$$(12.29) \quad a_0 F + a_1 F_s S_t + a_2 F_t + a_3 F_{ss} = 0, \quad 0 \leq S_t, \quad 0 \leq t \leq T$$

$$(12.30) \quad F(S_T, T) = G(S_T, T)$$

## Chapter 13

$$(13.1) \quad a(S_t, t) = \mu S_t$$

$$(13.2) \quad \sigma(S_t, t) = \sigma S_t, \quad t \in [0, \infty)$$

$$(13.3) \quad -rF + rF_s S_t + F_t + \frac{1}{2} \sigma^2 F_{ss} S_t^2 = 0, \quad 0 \leq S_t, \quad 0 \leq t \leq T$$

$$(13.4) \quad F(T) = \max[S_T - K, 0]$$

$$(13.6) \quad F(S_t, t) = S_t N(d_1) - K e^{-r(T-t)} N(d_2)$$

$$(13.7) \quad d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$(13.8) \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

$$(13.9) \quad N(d_i) = \int_{-\infty}^{d_i} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx, \quad i = 1, 2$$

$$(13.12) \quad a(S_t, t) = \mu S_t$$

$$(13.13) \quad \sigma(S_t, t) = \sigma(S_t, t) S_t, \quad t \in [0, \infty)$$



$$(13.14) \quad -rF + rF_s S_t + F_t + \frac{1}{2}\sigma(S_t, t)^2 F_{ss} S_t^2 = 0, \quad 0 \leq S_t, \quad 0 \leq t \leq T$$

$$(13.15) \quad F(T) = \max[S_T - K, 0]$$

$$(13.34) \quad rF - rF_s S_t - \delta - F_t - \frac{1}{2}F_{ss}\sigma_t^2 = 0$$

$$(13.35) \quad F(S_{1T}, S_{2T}, T) = \max[0, \max(S_{1T}, S_{2T}) - K] \text{ (multi-asset option)}$$

$$(13.36) \quad F(S_{1T}, S_{2T}, T) = \max[0, (S_{1T} - S_{2T}) - K] \text{ (spread call option)}$$

$$(13.37) \quad F(S_{1T}, S_{2T}, T) = \max[0, (\theta_1 S_{1T} + \theta_2 S_{2T}) - K] \text{ (portfolio call option)}$$

$$(13.38) \quad F(S_{1T}, S_{2T}, T) = \max[0, (S_{1T} - K_1), (S_{2T} - K_2)] \text{ (dual strike call option)}$$

(This is a correction to the text formula)

$$(13.47) \quad \frac{\Delta F}{\Delta t} + rS \frac{\Delta F}{\Delta S} + \frac{1}{2}\sigma^2 S^2 \frac{\Delta^2 F}{\Delta S^2} \approx rF$$

$$(13.48) \quad \frac{\Delta F}{\Delta t} \approx \frac{F_{ij} - F_{i,j-1}}{\Delta t}$$

$$(13.49) \quad \frac{\Delta F}{\Delta S} \approx \frac{F_{ij} - F_{i-1,j}}{\Delta S}$$

$$(13.50) \quad rS \frac{\Delta F}{\Delta S} \approx rS_j \frac{F_{i+1,j} - F_{ij}}{\Delta S}$$

$$(13.51) \quad \frac{\Delta^2 F}{\Delta S^2} \approx \left[ \frac{F_{i+1,j} - F_{ij}}{\Delta S} - \frac{F_{ij} - F_{i-1,j}}{\Delta S} \right] \frac{1}{\Delta S}$$

## Chapter 14

$$(14.3) \quad \mathbb{P}\left(\bar{z} - \frac{1}{2}\Delta < z_t < \bar{z} + \frac{1}{2}\Delta\right) = \int_{\bar{z}-\frac{1}{2}\Delta}^{\bar{z}+\frac{1}{2}\Delta} \frac{1}{\sqrt{2\pi}} e^{-\frac{z_t^2}{2}} dz_t$$

$$(14.6) \quad d\mathbb{P}(\bar{z}) = \mathbb{P}\left(\bar{z} - \frac{1}{2}dz_t < z_t < \bar{z} + \frac{1}{2}dz_t\right)$$

$$(14.7) \quad \int_{-\infty}^{\infty} d\mathbb{P}(z_t) = 1$$

$$(14.8) \quad \mathbb{E}[z_t] = \int_{-\infty}^{\infty} z_t d\mathbb{P}(z_t)$$

$$(14.9) \quad \mathbb{E}[z_t - E[z_t]]^2 = \int_{-\infty}^{\infty} [z_t - \mathbb{E}[z_t]]^2 d\mathbb{P}(z_t)$$

$$(14.29) \quad \mathbb{E}_t\left[\frac{1}{1+R_t} S_{t+1}\right] = S_t$$

$$(14.31) \quad \mathbb{E}_t^{\mathbb{Q}}\left[\frac{1}{1+r_t} S_{t+1}\right] = S_t$$

$$(14.41) \quad z_t \sim N(0, 1)$$

$$(14.42) \quad d\mathbb{P}(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t)^2} dz_t$$

$$(14.43) \quad \xi(z_t) = e^{z_t \mu - \frac{1}{2}\mu^2}$$

$$(14.44) \quad [d\mathbb{P}(z_t)][\xi(z_t)] = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t^2) + \mu z_t - \frac{1}{2}\mu^2} dz_t$$

$$(14.45) \quad d\mathbb{Q}(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z_t - \mu)^2} dz_t$$

$$(14.47) \quad d\mathbb{Q}(z_t) = \xi(z_t)d\mathbb{P}(z_t)$$

$$(14.48) \quad \xi(z_t)^{-1}d\mathbb{Q}(z_t) = d\mathbb{P}(z_t)$$

$$(14.53) \quad f(z_{1t}, z_{2t}) = \frac{1}{2\pi\sqrt{|\Omega|}} e^{-\frac{1}{2} \begin{bmatrix} z_{1t} - \mu_1 & z_{2t} - \mu_2 \end{bmatrix} \Omega^{-1} \begin{bmatrix} z_{1t} - \mu_1 \\ z_{2t} - \mu_2 \end{bmatrix}}$$

(This is a correction to the text formula)

$$(14.54) \quad \Omega = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$$

$$(14.55) \quad |\Omega| = \sigma_1^2\sigma_2^2 - \sigma_{12}^2$$

$$(14.56) \quad d\mathbb{P}(z_{1t}, z_{2t}) = f(z_{1t}, z_{2t})dz_{1t}dz_{2t}$$

$$(14.57) \quad \xi(z_{1t}, z_{2t}) = \exp \left\{ - \begin{bmatrix} z_{1t} & z_{2t} \end{bmatrix} \Omega^{-1} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \mu_1 & \mu_2 \end{bmatrix} \Omega^{-1} \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \right\}$$

(This is a correction to the text formula)

$$(14.58) \quad d\mathbb{Q}(z_{1t}, z_{2t}) = \xi(z_{1t}, z_{2t})d\mathbb{P}(z_{1t}, z_{2t})$$

$$(14.59) \quad d\mathbb{Q}(z_{1t}, z_{2t}) = \frac{1}{2\pi\sqrt{|\Omega|}} \exp \left\{ -\frac{1}{2} \begin{bmatrix} z_{1t} & z_{2t} \end{bmatrix} \Omega^{-1} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} \right\} dz_{1t}dz_{2t}$$

(This is a correction to the text formula)

$$(14.60) \quad \xi(z_t) = e^{-z_t' \Omega^{-1} \mu + \frac{1}{2} \mu' \Omega^{-1} \mu} \quad (\text{This is a correction to the text formula})$$

$$(14.69) \quad \frac{d\mathbb{Q}(z_t)}{d\mathbb{P}(z_t)} = \xi(z_t)$$

$$(14.74) \quad d\mathbb{Q}(z_t) = \xi(z_t)d\mathbb{P}(z_t)$$

$$(14.75) \quad d\mathbb{P}(z_t) = \xi(z_t)^{-1}d\mathbb{Q}(z_t)$$

$$(14.76) \quad \xi_t = e^{(\int_0^t X_u dW_u - \frac{1}{2} \int_0^t X_u^2 du)}, \quad t \in [0, T]$$

$$(14.77) \quad \mathbb{E} \left[ e^{\int_0^t X_u^2 du} \right] < \infty, \quad t \in [0, T]$$

$$(14.83) \quad \mathbb{E} \left[ \int_0^t \xi_s X_s dW_s \middle| I_u \right] = \int_0^u \xi_s X_s dW_s$$

$$(14.84) \quad W_t^* = W_t - \int_0^t X_u du, \quad t \in [0, T] \quad (\text{This is a correction to the text formula})$$

$$(14.85) \quad \mathbb{Q}(A) = \mathbb{E}^{\mathbb{P}}[1_A \xi_T]$$

$$(14.86) \quad dW_t^* = dW_t - X_t dt$$

$$(14.93) \quad d\mathbb{Q} = \xi_T d\mathbb{P}$$

$$(14.122) \quad A_1 \cup A_2 \cup \dots \cup A_n = \Omega$$

$$(14.123) \quad 1_{A_1} + 1_{A_2} + \dots + 1_{A_n} = 1_{\Omega}$$

$$(14.127) \quad \mathbb{E}^{\mathbb{P}}[Z_t 1_{A_i}] = \mathbb{Q}(A_i)$$

$$(14.138) \quad \mathbb{E}^{\mathbb{P}}[g(X_t)] = \int_{\Omega} g(x) f(x) dx$$

(page 249)  $g(X_t) = Z_t h(X_t)$

$$(14.140) \quad \mathbb{E}^{\mathbb{P}}[g(X_t)] = \int_{\Omega} h(x) \tilde{f}(x) dx = \mathbb{E}^{\mathbb{Q}}[h(X_t)]$$

## Chapter 15

$$(15.2) \quad Y_t \sim N(\mu t, \sigma^2 t)$$

$$(15.4) \quad M(\lambda) = \mathbb{E}[e^{Y_t \lambda}]$$

$$(15.10) \quad M(\lambda) = e^{(\lambda \mu t + \frac{1}{2} \sigma^2 \lambda^2 t)}$$

$$(15.15) \quad S_t = S_0 e^{Y_t}, \quad t \in [0, \infty)$$

$$(15.25) \quad \mathbb{E}[S_t | S_u, u < t] = S_u e^{\mu(t-s) + \frac{1}{2} \sigma^2 (t-s)}$$

$$(15.30) \quad Z_t = e^{-rt} S_t$$

$$(15.31) \quad \mathbb{E}^{\mathbb{Q}}[e^{-rt} S_t | S_u, u < t] = e^{-ru} S_u$$

$$(15.32) \quad \mathbb{E}^{\mathbb{Q}}[Z_t | Z_u, u < t] = Z_u$$

$$(15.38) \quad \mathbb{E}^{\mathbb{Q}}[e^{-r(t-u)} S_t | S_u, u < t] = S_u e^{-r(t-u)} e^{\rho(t-u) + \frac{1}{2} \sigma^2 (t-u)} \text{ where under } \mathbb{Q}, Y_t \sim N(\rho t, \sigma^2 t)$$

(This is a correction with  $t - u$  replacing  $t - s$ )

$$(15.39) \quad \rho = r - \frac{1}{2} \sigma^2$$

$$(15.42) \quad \mathbb{E}^{\mathbb{Q}}[e^{-rt} S_t | S_u, u < t] = e^{-ru} S_u$$

$$(15.51) \quad dS_t = r S_t dt + \sigma S_t dW_t^*$$

$$(15.58) \quad C_t = \mathbb{E}_t^{\mathbb{Q}}[e^{-r(T-t)} \max\{S_T - K, 0\}]$$

$$(15.88) \quad dS_t = \mu_t dt + \sigma_t dW_t$$

$$(15.90) \quad d[e^{-rt} S_t] = e^{-rt} [\mu_t - r S_t] dt + e^{-rt} \sigma_t dW_t$$

$$(15.92) \quad dW_t^* = dX_t + dW_t$$

$$(15.97) \quad dX_t = \left[ \frac{\mu_t - r S_t}{\sigma_t} \right] dt$$

$$(15.98) \quad d[e^{-rt} S_t] = e^{-rt} \sigma_t dW_t^*$$

$$(15.111) \quad d[e^{-rt} F(S_t, t)] = e^{-rt} \sigma_t F_s dW_t^*$$

## Chapter 17

$$\text{(page 282)} \quad R_{t_1} = (1 + r_{t_1} \Delta)$$

$$\text{(page 282)} \quad R_{t_2} = (1 + r_{t_2} \Delta)$$

$$\text{(page 282)} \quad B_{t_1}^s = B(t_1, t_3)$$

$$\text{(page 282)} \quad B_{t_1} = B(t_1, T)$$

$$\text{(page 282)} \quad B_{t_3} = B(t_3, T)$$

$$(17.6) \quad \begin{bmatrix} 1 \\ 0 \\ B_{t_1}^s \\ B_{t_1} \\ C_{t_1} \end{bmatrix} = \begin{bmatrix} R_{t_1} R_{t_2}^u & R_{t_1} R_{t_2}^u & R_{t_1} R_{t_2}^d & R_{t_1} R_{t_2}^d \\ (F_{t_1} - L_{t_2}^u) & (F_{t_1} - L_{t_2}^u) & (F_{t_1} - L_{t_2}^d) & (F_{t_1} - L_{t_2}^d) \\ 1 & 1 & 1 & 1 \\ B_{t_3}^{uu} & B_{t_3}^{ud} & B_{t_3}^{du} & B_{t_3}^{dd} \\ C_{t_3}^{uu} & C_{t_3}^{ud} & C_{t_3}^{du} & C_{t_3}^{dd} \end{bmatrix} \begin{bmatrix} \psi^{uu} \\ \psi^{ud} \\ \psi^{du} \\ \psi^{dd} \end{bmatrix}$$

$$(17.13) \quad 1 = R_{t_1} R_{t_2}^u \psi^{uu} + R_{t_1} R_{t_2}^u \psi^{ud} + R_{t_1} R_{t_2}^d \psi^{du} + R_{t_1} R_{t_2}^d \psi^{dd}$$

$$(17.14) \quad \mathbb{Q}_{ij} = (1 + r_{t_1})(1 + r_{t_2}^i)\psi^{ij}$$

$$(17.15) \quad 1 = \mathbb{Q}_{uu} + \mathbb{Q}_{ud} + \mathbb{Q}_{du} + \mathbb{Q}_{dd}$$

$$(17.16) \quad \mathbb{Q}_{ij} > 0$$

$$(17.18) \quad B_{t_1}^s = \mathbb{E}^{\mathbb{Q}} \left[ \frac{1}{(1 + r_{t_1})(1 + r_{t_2})} \right]$$

$$(17.21) \quad B_{t_1} = \mathbb{E}^{\mathbb{Q}} \left[ \frac{B_{t_3}}{(1 + r_{t_1})(1 + r_{t_2})} \right] \quad (\text{This is a correction to the text formula})$$

$$(17.22) \quad 0 = \mathbb{E}^{\mathbb{Q}} \left[ \frac{1}{(1 + r_{t_1})(1 + r_{t_2})} [F_{t_1} - L_{t_2}] \right]$$

$$(17.23) \quad C_{t_1} = \mathbb{E}^{\mathbb{Q}} \left[ \frac{1}{(1 + r_{t_1})(1 + r_{t_2})} C_{t_3} \right]$$

$$(17.31) \quad F_{t_1} = \frac{1}{\mathbb{E}^{\mathbb{Q}} \left[ \frac{1}{(1 + r_{t_1})(1 + r_{t_2})} \right]} \mathbb{E}^{\mathbb{Q}} \left[ \frac{L_{t_2}}{(1 + r_{t_1})(1 + r_{t_2})} \right]$$

$$(17.36) \quad B_{t_1}^s = \psi^{uu} + \psi^{ud} + \psi^{du} + \psi^{dd}$$

$$(17.38) \quad \pi_{ij} = \frac{1}{B_{t_1}^s} \psi^{ij}$$

$$(17.39) \quad 1 = \pi_{uu} + \pi_{ud} + \pi_{du} + \pi_{dd}$$

$$(17.46) \quad F_{t_1} = \mathbb{E}^{\pi} [L_{t_2}]$$

$$(17.52) \quad C_{t_3} = N \max[L_{t_2} - K, 0]$$

$$(17.53) \quad C_{t_1} = \mathbb{E}^{\mathbb{Q}} \left[ \frac{1}{(1 + r_{t_1})(1 + r_{t_2})} \max[L_{t_2} - K, 0] \right]$$

$$(17.55) \quad C_{t_1} = B_{t_1}^s \mathbb{E}^{\pi} [\max[L_{t_2} - K, 0]]$$

$$(17.56) \quad \frac{C_t}{S_t} = \mathbb{E}_t^{\mathbb{S}} \left( \frac{C_T}{S_T} \right)$$

$$(17.57) \quad \frac{C(K)}{S_0} = \mathbb{E}^{\mathbb{S}} \left( \frac{(S_T - K)^+}{S_T} \right)$$

$$(\text{page 291}) \quad y = \log \left( \frac{S_T}{K} \right)$$

$$(\text{page 291}) \quad \frac{C(K)}{S_0} = \int_0^{\infty} (1 - F(y)) e^{-y} dy$$

$$(17.61) \quad \frac{C(K)}{S_0} = P(\ln S - \ln K > Y)$$

$$(17.63) \quad F(t; T, S) = \frac{1}{S - T} \left( \frac{P(t, T)}{P(t, S)} - 1 \right)$$

$$(17.64) \quad 0 = T_0 < T_1 < T_2 < \dots < T_M$$

$$(17.65) \quad \Delta_i = T_{i+1} - T_i, i = 0, 1, 2, \dots, M - 1$$

$$(17.66) \quad L_n(t) = \frac{P(t, T_n) - P(t, T_{n+1})}{\Delta_n P(t, T_{n+1})}$$

$$(17.71) \quad P(T_i, T_{n+1}) = \prod_{j=1}^n \frac{1}{1 + \Delta_j L_j(T_i)}$$

$$(17.72) \quad P(t, T_n) = P(t, T_l) \prod_{j=l}^{n-1} \frac{1}{1 + \Delta_j L_j(T_i)}, \quad T_{l-1} < t \leq T_l$$

$$(17.75) \quad B_t^* = P(t, T_l) \prod_{j=0}^{l-1} (1 + \Delta_j L_j(T_j))$$

$$(17.77) \quad D_n(t) = \frac{\prod_{j=\ell}^{n-1} \frac{1}{1 + \Delta_j L_j(t)}}{\prod_{j=0}^{l-1} (1 + \Delta_j L_j(T_j))}$$

$$(17.78) \quad dL_n(t) = \mu_n(t) L_n(t) dt + L_n(t) \sigma_n^\tau(t) dW_t, \quad 0 \leq t \leq T_n, \quad n = 1, 2, \dots, M$$

$$(17.103) \quad \mu_n(t) = \sum_{j=\ell}^n \frac{\Delta_j L_j(t) \sigma_n^\tau(t) \sigma_j(t)}{1 + \Delta_j L_j(t)}$$

$$(17.104) \quad dL_n(t) = \left( \sum_{j=\ell}^n \frac{\Delta_j L_j(t) \sigma_n^\tau(t) \sigma_j(t)}{1 + \Delta_j L_j(t)} \right) L_n(t) dt + L_n(t) \sigma_n^\tau(t) dW_t, \quad 0 \leq t \leq T_n, \quad n = 1, \dots, M$$

$$(17.105) \quad V_t = \mathbb{E}_t^{\mathbb{Q}} \left[ e^{-\int_t^{T+\delta} r_u du} (F_t - L_T) N \delta \right]$$

$$(17.110) \quad V_t = \mathbb{E}_t^\pi [B(t, T + \delta) (F_t - L_T) N \delta]$$

$$(17.111) \quad V_t = B(t, T + \delta) \mathbb{E}_t^\pi [(F_t - L_T) N \delta]$$

$$(17.112) \quad F_t = \mathbb{E}_t^\pi [L_T]$$

(page 296)  $C_T = \max[L_{T-\delta} - K, 0]$

$$(17.113) \quad C_t = \mathbb{E}_t^{\mathbb{Q}} \left[ e^{-\int_t^{T+\delta} r_u du} \max[L_{T-\delta} - K, 0] \right]$$

## Chapter 18

$$(18.3) \quad B(t, T) = e^{-R(t, T)(T-t)}, \quad t < T$$

$$(18.12) \quad B(t, T) = \mathbb{E}_t^{\mathbb{Q}} \left[ e^{-\int_t^T r_s ds} \right]$$

$$(18.20) \quad R(t, T) = \frac{-\log \mathbb{E}_t^{\mathbb{Q}} \left[ e^{-\int_t^T r_s ds} \right]}{T-t}$$

$$(18.33) \quad B(t, T) = e^{-\int_t^T F(t, s) ds}$$

$$(18.40) \quad F(t, T) = \lim_{\Delta \rightarrow 0} \frac{\log B(t, T) - \log B(t, T + \Delta)}{\Delta}$$

(page 311)  $F(t, T, U) = \frac{\log B(t, T) - \log B(t, U)}{U - T}$

## Chapter 19

$$(19.14) \quad dB_t = \mu(t, T, B_t) B_t dt + \sigma(t, T, B_t) B_t dV_t^T, \quad \text{where } B_t = B(t, T)$$

$$(19.15) \quad dB_t = r_t B_t dt + \sigma(t, T, B_t) B_t dW_t^T$$

$$(19.21) \quad dF(t, T) = \sigma(t, T, B(t, T)) \left[ \frac{\partial \sigma(t, T, B(t, T))}{\partial T} \right] dt - \left[ \frac{\partial \sigma(t, T, B(t, T))}{\partial T} \right] dW_t$$

(This is a correction to the text formula)

$$(19.22) \quad dF(t, T) = a(F(t, T), t) dt + b(F(t, T), t) dW_t$$

$$(19.25) \quad r_t = F(t, t)$$

(page 325)  $F(t, T) = F(0, T) + \int_0^t b(s, T) \left[ \int_s^T b(s, u) du \right] ds + \int_0^t b(s, T) dW_s$

$$(19.26) \quad r_t = F(0, t) + \int_0^t b(s, t) \left[ \int_s^t b(s, u) du \right] ds + \int_0^t b(s, t) dW_s$$

$$(19.33) \quad dF(t, T) = b^2(T-t)dt + bdW_t$$

$$(19.34) \quad dB(t, T) = r_t B(t, T)dt - b(T-t)B(t, T)dW_t \quad (\text{This is a correction to the text formula})$$

$$(19.35) \quad r_t = F(0, t) + \frac{1}{2}b^2t^2 + bW_t$$

$$(19.36) \quad dr_t = (F_t(0, t) + b^2t)dt + bdW_t$$

$$(19.37) \quad F_t(0, t) = \frac{\partial F(0, t)}{\partial t}$$

## Chapter 20

$$(20.5) \quad B^1 = B(t, T_1)$$

$$(20.6) \quad B^2 = B(t, T_2)$$

$$(20.7) \quad dB^1 = \mu(B^1, t)B^1 dt + \sigma_1(B^1, t)B^1 dW_t$$

$$(20.8) \quad dB^2 = \mu(B^2, t)B^2 dt + \sigma_2(B^2, t)B^2 dW_t$$

$$(20.9) \quad dr_t = a(r_t, t)dt + b(r_t, t)dW_t$$

$$(20.10) \quad \mathcal{P} = \theta_1 B^1 - \theta_2 B^2$$

$$(20.11) \quad \theta_1 = \frac{\sigma_2}{B^1(\sigma_2 - \sigma_1)} \mathcal{P} \quad (\text{This is a correction to the text formula})$$

$$(20.12) \quad \theta_2 = \frac{\sigma_1}{B^2(\sigma_2 - \sigma_1)} \mathcal{P} \quad (\text{This is a correction to the text formula})$$

$$(20.13) \quad d\mathcal{P} = \theta_1 dB^1 - \theta_2 dB^2$$

$$(20.15) \quad (\theta_1 \sigma_1 B^1 - \theta_2 \sigma_2 B^2) = \left( \frac{\sigma_2}{B^1(\sigma_2 - \sigma_1)} \sigma_1 B^1 - \frac{\sigma_1}{B^2(\sigma_2 - \sigma_1)} \sigma_2 B^2 \right) \mathcal{P} = 0$$

(This is a correction to the text formula)

$$(20.16) \quad d\mathcal{P} = (\theta_1 \mu_1 B^1 - \theta_2 \mu_2 B^2) dt$$

$$(20.17) \quad d\mathcal{P} = \frac{(\sigma_2 \mu_1 - \sigma_1 \mu_2)}{(\sigma_2 - \sigma_1)} \mathcal{P} dt$$

$$(20.18) \quad r_t \mathcal{P} dt = \frac{(\sigma_2 \mu_1 - \sigma_1 \mu_2)}{(\sigma_2 - \sigma_1)} \mathcal{P} dt \quad (\text{This is a correction to the text formula})$$

$$(20.19) \quad \frac{\mu_1 - r_t}{\sigma_1} = \frac{\mu_2 - r_t}{\sigma_2}$$

$$(20.20) \quad \frac{\mu_i - r_t}{\sigma_i} = \lambda(r_t, t)$$

$$(20.21) \quad dB(t, T) = B_r dr_t + B_t dt + \frac{1}{2} B_{rr} b(r_t, T)^2 dt \quad (\text{This is a correction to the text formula})$$

$$(20.23) \quad dB(t, T) = \left( B_r a(r_t, t) + B_t + \frac{1}{2} B_{rr} b(r_t, T)^2 \right) dt + b(r_t, t) B_r dW_t \quad (\text{This is a correction to the text formula})$$

$$(20.31) \quad B_r (a(r_t, t) - b(r_t, t) \lambda_t) + B_t + \frac{1}{2} B_{rr} b(r_t, T)^2 - r_t B = 0$$

$$(20.33) \quad dr_t = (a(r_t, t) - b(r_t, t) \lambda_t) dt + b(r_t, t) \widetilde{W}_t$$

$$(20.39) \quad B(0, T) = e^{\frac{1}{\alpha}(1-e^{-\alpha T})(R-r) - TR - \frac{b^2}{4\alpha^3}(1-e^{-\alpha T})^2}$$

$$(20.40) \quad R = \kappa - \frac{b\lambda}{\alpha} - \frac{b^2}{\alpha^2}$$

$$(20.48) \quad B(t, T) = A(t, T)e^{-C(t, T)r}$$

$$(20.49) \quad A(t, T) = \left( 2 \frac{\gamma e^{1/2(\alpha + \lambda + \gamma)T}}{(\alpha + \lambda + \gamma)(e^{\gamma T} - 1) + 2\gamma} \right)^{2 \frac{\alpha \kappa}{b^2}}$$

$$(20.50) \quad C(t, T) = 2 \frac{e^{\gamma T} - 1}{(\alpha + \lambda + \gamma)(e^{\gamma T} - 1) + 2\gamma}$$

$$(20.51) \quad \gamma = \sqrt{(\alpha + \lambda)^2 + 2b^2}$$

## Chapter 21

$$(21.41) \quad B(t, T) = \mathbb{E}_t^{\mathbb{Q}} \left[ e^{-\int_t^T r_s ds} \right]$$

$$(21.42) \quad dr_t = (a(r_t, t) - \lambda_t b(r_t, t))dt + b(r_t, t)dW_t$$

$$(21.43) \quad dr_t = a(r_t, t)dt + b(r_t, t)dW_t^*$$

$$(21.45) \quad B(t, T) = \mathbb{E}_t^{\mathbb{P}} \left[ \exp \left( - \int_t^T r_s ds \right) \exp \left( \int_t^T \left[ \lambda_s(r_s, s)dW_t^* - \frac{1}{2} \lambda_s(r_s, s)^2 ds \right] \right) \right]$$

$$(21.68) \quad dS_t = a(S_t, t)dt + \sigma(S_t, t)dW_t, \quad t \in [0, \infty)$$

$$(21.69) \quad dS_t = a(S_t)dt + \sigma(S_t)dW_t, \quad t \in [0, \infty)$$

$$(21.70) \quad dS_t = a(S_t)dt + \sigma(S_t)dW_t, \quad t \in [0, \infty)$$

$$(21.71) \quad \mathbb{E}[f(S_{t+h})|I_t] = \mathbb{E}[f(S_{t+h})|S_t], \quad h > 0$$

$$(21.90) \quad \widehat{f}(t, r_t) = \mathbb{E}^{\mathbb{P}} \left[ e^{-\int_t^u q(r_s)ds} f(r_u) | r_t \right] \quad (\text{This is a correction to the text formula})$$

$$(21.91) \quad \frac{\partial \widehat{f}}{\partial t} = q(r_t)\widehat{f} - A\widehat{f} \quad (\text{This is a correction to the text formula})$$

$$(21.92) \quad A\widehat{f} = a_t \frac{\partial \widehat{f}}{\partial r_t} + \frac{1}{2} \sigma_t^2 \frac{\partial^2 \widehat{f}}{\partial r_t^2}$$

## Frequently Asked Questions in Quantative Finance, Wilmott (QFIQ-113-17)

### Jensen's Inequality (103-105)

If  $f(\cdot)$  is a convex function,  $E[f(x)] \geq f(E[x])$ .

### Girsanov's Theorem (113-115)

$$dS = \mu S dt + \sigma S dW_t$$

$$dS = r S dt + \sigma S d\tilde{W}_t$$

$$\tilde{W}_t = W_t + \int_0^t \gamma_s ds$$

$$d\tilde{W}_t = dW_t + \gamma_t dt$$

### Volatility Smile (167-173)

$$\frac{\partial^2 V}{\partial \sigma^2} = S \sqrt{T-t} \frac{d_1 d_2 e^{-D(T-t)} e^{-d_1^2/2}}{\sqrt{2\pi}\sigma}$$

# Fixed Income Securities: Valuation, Risk, and Risk Management, Veronesi

## Chapter 2

$$(2.35) \quad d = \frac{100 - P_{bill}(t, T)}{100} \times \frac{360}{n}$$

$$(2.36) \quad P_{bill}(t, T) = 100 \times \left[ 1 - \frac{n}{360} \times d \right]$$

$$(2.39) \quad \text{Price floating rate bond with spread } s = 100 + \frac{s}{2} \times \sum_{t=0.5}^T Z(0, t)$$

(This is a correction to the text formula)

$$(2.52) \quad Z(0, T) = e^{-r(0, T)T}$$

$$(2.53) \quad r(0, T) = \theta_0 + (\theta_1 + \theta_2) \frac{1 - e^{-T/\lambda}}{T/\lambda} - \theta_2 e^{-T/\lambda}$$

$$(2.54) \quad P_c^{i, NS \text{ model}} = 100 \times \left( \frac{c^i}{2} \sum_{j=1}^{n^i} Z(0, T_j^i) + Z(0, T_{n^i}^i) \right) \quad (\text{This is a correction to the text formula})$$

## Chapter 3

$$(3.17) \quad D = -\frac{1}{P} \frac{dP}{dr}$$

$$(3.19) \quad MD = -\frac{1}{P} \frac{dP}{dy} = \frac{1}{(1 + \frac{y}{2})} \sum_{j=1}^n w_j \times T_j$$

$$(3.20) \quad D^{Mc} = \sum_{j=1}^n w_j \times T_j$$

$$(3.21) \quad \text{Dollar duration} = D_P^{\$} = -\frac{dP}{dr}$$

$$(3.22) \quad D_P^{\$} = P \times D_P$$

$$(3.23) \quad D_W^{\$} = \sum_{i=1}^n N_i D_i^{\$}$$

$$(3.26) \quad \text{Price value of a basis point} = PV01(\text{or } PVBP) = -D_P^{\$} \times dr$$

$$(3.44) \quad D_E = \frac{A}{A-L} \times D_A - \frac{L}{A-L} \times D_L$$

## Chapter 4

$$(4.1) \quad \text{Convexity} = C = \frac{1}{P} \frac{d^2 P}{dr^2}$$

$$(4.2) \quad \frac{dP}{P} = -D \times dr + \frac{1}{2} \times C \times dr^2$$

$$(4.6) \quad \text{Convexity of portfolio} = C_W = \sum_{i=1}^n w_i C_i, \text{ where (4.7) } w_i = \frac{N_i \times P_i}{W}$$



$$(4.8) \quad C = \sum_{i=1}^n w_i \times C_{z,i}, \text{ where } C_{z,i} = (T_i - t)^2 \text{ and}$$

$$(4.9) \quad w_i = \frac{c/2 \times P_z(t, T_i)}{P_c(t, T)} \text{ for } i = 1, \dots, n-1$$

$$(4.10) \quad w_n = \frac{(1 + c/2) \times P_z(t, T_n)}{P_c(t, T)}$$

$$(4.14) \quad \text{Dollar convexity} = C^s = \frac{d^2 P}{dr^2}$$

$$(4.36) \quad \max_{a_{11}, \dots, a_{1n}} \text{Var}(\Delta \phi_i^{PCA}) = \sum_{k=1}^n \sum_{\ell=1}^n a_{1k} a_{1\ell} \sigma_{kl} \text{ under the restriction}$$

$$(4.37) \quad \sum_{j=1}^n a_{1j}^2 = 1$$

$$(4.40) \quad \mathbf{M} \mathbf{v}_i = \lambda_i \mathbf{v}_i$$

## Chapter 5

$$(5.5) \quad F(t, T_1, T_2) = \frac{1}{\left(1 + \frac{f_n(t, T_1, T_2)}{n}\right)^{n \times (T_2 - T_1)}}$$

$$(5.6) \quad F(t, T_1, T_2) = e^{-f(t, T_1, T_2)(T_2 - T_1)}$$

$$(5.14) \quad f(0, T, T + \Delta) = r(0, T) + (T + \Delta) \times \frac{r(0, T + \Delta) - r(0, T)}{\Delta}$$

$$(5.21) \quad r(0, T_n) = \frac{1}{T_n} \sum_{i=1}^n f(0, T_{i-1}, T_i) \times \Delta$$

$$(5.25) \quad \text{Value of FRA at } t = V^{FRA}(t) = V^{fixed}(t) - V^{floating}(t) = N \times [M \times Z(t, T_2) - Z(t, T_1)]$$

$$(5.34) \quad P_c^{fwd}(0, T, T^*) = \frac{c}{2} \times \sum_{i=1}^n P_z^{fwd}(0, T, T_i) + P_z^{fwd}(0, T, T_n)$$

$$(5.40) \quad V^{swap}(t_i; c, T) = 100 - \left( \frac{c}{2} \times 100 \times \sum_{j=i+1}^M Z(T_i, T_j) + Z(T_i, T_M) \times 100 \right)$$

$$(5.43) \quad c = n \times \left( \frac{1 - Z(0, T_M)}{\sum_{j=1}^M Z(0, T_j)} \right)$$

$$(5.51) \quad f_n^s(0, T, T^*) = n \times \frac{1 - F(0, T, T^*)}{\sum_{j=1}^m F(0, T, T_j)} \quad (\text{This is a correction to the text formula})$$

## Chapter 6

$$(6.4) \quad \text{P\&L from futures at } t = k \times \text{Contract size} \times [P^{fut}(t, T) - P^{fut}(t - dt, T)]$$

## Chapter 14

$$(14.27) \quad dr_t = \gamma(\bar{r} - r_t)dt$$

$$(14.29) \quad dr_t = \theta_t dt + \sigma dX_t$$

$$(14.34) \quad r_t \sim \mathcal{N}(\mu(r_0, t), \sigma^2(t)) \quad \text{where}$$

$$(14.35) \quad \mu(r_0, t) = \bar{r} + (r_0 - \bar{r})e^{-\gamma t}$$

$$(14.36) \quad \sigma^2(t) = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma t})$$

$$(14.37) \quad dP_t = \frac{1}{2} \left( \frac{d^2 F}{dX^2} \right) dt + \left( \frac{dF}{dX} \right) dX_t$$

$$(14.39) \quad dP_t = \left\{ \left( \frac{\partial F}{\partial t} \right) + \frac{1}{2} \left( \frac{\partial^2 F}{\partial X^2} \right) \right\} dt + \left( \frac{\partial F}{\partial X} \right) dX_t$$

## Chapter 15

$$(15.9) \quad dr_t = \gamma(\bar{r} - r_t)dt + \sigma dX_t$$

$$(15.22) \quad \frac{\left( \frac{\partial Z_1}{\partial t} + \frac{1}{2} \frac{\partial^2 Z_1}{\partial r^2} \sigma^2 - r_t Z_1 \right)}{\partial Z_1 / \partial r} = \frac{\left( \frac{\partial Z_2}{\partial t} + \frac{1}{2} \frac{\partial^2 Z_2}{\partial r^2} \sigma^2 - r_t Z_2 \right)}{\partial Z_2 / \partial r}$$

$$(15.28) \quad Z(r, t; T) = e^{A(t; T) - B(t; T)r}$$

$$(15.29) \quad B(t; T) = \frac{1}{\gamma^*} \left( 1 - e^{-\gamma^*(T-t)} \right)$$

$$(15.30) \quad A(t; T) = (B(t; T) - (T - t)) \left( \bar{r}^* - \frac{\sigma^2}{2(\gamma^*)^2} \right) - \frac{\sigma^2 B(t; T)^2}{4\gamma^*}$$

$$(15.31) \quad P_c(r, t; T) = \frac{100 \times c}{2} \sum_{i=1}^n Z(r, t; T_i) + 100 \times Z(r, t; T_n)$$

$$(15.34) \quad Z(r_t, \tau) = Z(r_t, t; T)$$

$$(15.35) \quad A(\tau) = A(0; T - t)$$

$$(15.36) \quad B(\tau) = B(0; T - t)$$

$$(15.39) \quad D_Z(\tau) = -\frac{1}{Z} \frac{\partial Z}{\partial r} = B(\tau) = \frac{1}{\gamma^*} \left( 1 - e^{-\gamma^* \tau} \right)$$

$$(15.44) \quad V(r_0, 0) = Z(r_0, 0; T_B) \mathcal{N}(d_1) - K Z(r_0, 0; T_O) \mathcal{N}(d_2) \quad (\text{This is a correction to the text formula})$$

$$(15.45) \quad d_1 = \frac{1}{S_Z(T_O; T_B)} \log \left( \frac{Z(r_0, 0; T_B)}{K Z(r_0, 0; T_O)} \right) + \frac{S_Z(T_O; T_B)}{2} \quad (\text{This is a correction to the text formula})$$

$$(15.46) \quad d_2 = d_1 - S_Z(T_O; T_B) \quad (\text{This is a correction to the text formula})$$

$$(15.47) \quad S_Z(T_O; T_B) = B(T_O; T_B) \times \sqrt{\frac{\sigma^2}{2\gamma^*} (1 - e^{-2\gamma^* T_O})} \quad (\text{This is a correction to the text formula})$$

$$(15.48) \quad V(r_0, 0) = K Z(r_0, 0; T_O) \mathcal{N}(-d_2) - Z(r_0, 0; T_B) \mathcal{N}(-d_1) \quad (\text{This is a correction to the text formula})$$

$$(\text{page 547}) \quad P_c(r_K^*, T_0, T_B) = K$$

$$(\text{page 547}) \quad K_i = Z(r_K^*, T_0; T_i), \quad i = 1, 2, \dots, n$$

$$(15.51) \quad \text{Call} = \sum_{i=1}^n c(i)(Z(r_0, 0; T_i)\mathcal{N}(d_1(i)) - K_i Z(r_0, 0; T_O)\mathcal{N}(d_2(i)))$$

$$(15.52) \quad d_1(i) = \frac{1}{S_Z(T_O; T_i)} \log \left( \frac{Z(r_0, 0; T_i)}{K_i Z(r_0, 0; T_O)} \right) + \frac{S_Z(T_O; T_i)}{2}$$

$$(15.53) \quad d_2(i) = d_1(i) - S_Z(T_O; T_i)$$

$$(15.55) \quad \text{Put} = \sum_{i=1}^n c(i)(K_i Z(r_0, 0; T_O)\mathcal{N}(-d_2(i)) - Z(r_0, 0; T_i)\mathcal{N}(-d_1(i)))$$

$$(15.66) \quad dr_t = \gamma(\bar{r} - r_t)dt + \sqrt{\alpha r_t}dX_t$$

$$(15.67) \quad E[r_t|r_0] = \bar{r} + (r_0 - \bar{r})e^{-\gamma t}$$

$$(15.68) \quad \text{Var}[r_t|r_0] = r_0 \frac{\alpha}{\gamma} (e^{-\gamma t} - e^{-2\gamma t}) + \frac{\bar{r}\alpha}{2\gamma} (1 - e^{-\gamma t})^2$$

$$(15.70) \quad Z(r, t; T) = e^{A(t; T) - B(t; T)r}$$

$$(15.71) \quad B(t; T) = \frac{2(e^{\psi_1(T-t)} - 1)}{(\gamma^* + \psi_1)(e^{\psi_1(T-t)} - 1) + 2\psi_1}$$

$$(15.72) \quad A(t; T) = 2 \frac{\bar{r}^* \gamma^*}{\alpha} \log \left( \frac{2\psi_1 e^{(\psi_1 + \gamma^*) \frac{(T-t)}{2}}}{(\gamma^* + \psi_1)(e^{\psi_1(T-t)} - 1) + 2\psi_1} \right), \text{ and } \psi_1 = \sqrt{(\gamma^*)^2 + 2\alpha}$$

## Chapter 16

$$(16.8) \quad C_t = Z_1(r_t, t) - \Delta Z_2(r_t, t)$$

$$(16.9) \quad P_t = \Delta Z_{2,t} + C_t$$

$$(16.10) \quad dP_t = dZ_{1,t}$$

$$(16.18) \quad \left( \frac{1}{\Pi} \frac{\partial \Pi}{\partial t} \right) + \frac{1}{2} \left( \frac{1}{\Pi} \frac{\partial^2 \Pi}{\partial r^2} \right) \sigma^2 = r$$

## Chapter 17

$$(17.1) \quad dr_t = m(r_t, t)dt + s(r_t, t)dX_t$$

$$(17.2) \quad rV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial r} m^*(r, t) + \frac{1}{2} \frac{\partial^2 V}{\partial r^2} s(r, t)^2$$

$$(17.3) \quad V(r_T, T) = g(r_T, T)$$

$$(17.4) \quad R(r)V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial r} m^*(r, t) + \frac{1}{2} \frac{\partial^2 V}{\partial r^2} s(r, t)^2$$

$$(17.5) \quad V(r_t, t) = E^* \left[ e^{-\int_t^T R(r_u) du} g(r_T, T) | r_t \right]$$

$$(17.6) \quad dr_t = m^*(r_t, t)dt + s(r_t, t)dX_t$$

$$(17.7) \quad V(r_t, t) = E^* \left[ e^{-\int_t^T r_u du} g(r_T, T) | r_t \right]$$

$$(17.17) \quad f_n^s(0, T, T^*) = \frac{1 - F(0, T, T^*)}{\Delta \sum_{j=1}^n F(0, T, T_j)}$$

$$(17.20) \quad \text{Standard error} = \frac{\text{Standard deviation}}{\sqrt{J}}$$

$$(17.21) \quad \text{Standard deviation} = \sqrt{\frac{1}{J} \sum_{j=1}^J (V^j(r_0, 0) - \widehat{V}(r_0, 0))^2}$$

$$(17.24) \quad \frac{\partial V}{\partial r} \approx \frac{\widehat{V}(r_0 + \delta) - \widehat{V}(r_0 - \delta)}{2\delta}$$

$$(17.26) \quad \frac{\partial^2 V}{\partial r^2} \approx \frac{\widehat{V}(r_0 + \delta) + \widehat{V}(r_0 - \delta) - 2 \times \widehat{V}(r_0)}{\delta^2}$$

$$(17.27) \quad rV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial r} m^*(r, t) + \frac{1}{2} \frac{\partial^2 V}{\partial r^2} s^2(r, t) \quad (\text{This is a correction to the text formula})$$

$$(17.28) \quad \frac{\partial V}{\partial t} \approx \frac{\widehat{V}(r_0, t + \delta) - \widehat{V}(r_0, t)}{\delta}$$

$$(17.29) \quad \frac{\partial V}{\partial t} = rV - \frac{\partial V}{\partial r} m^*(r, t) - \frac{1}{2} \frac{\partial^2 V}{\partial r^2} s^2(r, t) \quad (\text{This is a correction to the text formula})$$

## Chapter 18

$$(18.7) \quad \text{Risk premium} = E \left[ \frac{dZ}{Z} \right] / dt - r = -B(t; T)(\gamma(\bar{r} - r) - \gamma^*(\bar{r}^* - r))$$

$$(18.8) \quad \lambda(r, t) = \frac{1}{\sigma} (\gamma(\bar{r} - r) - \gamma^*(\bar{r}^* - r))$$

$$(18.9) \quad \lambda(r, t) = \lambda_0 + \lambda_1 r$$

$$(18.13) \quad \text{Risk premium} = E \left[ \frac{dZ}{Z} \right] / dt - r = \sigma_Z \times \lambda(r, t)$$

$$(18.16) \quad \text{Risk premium} = E \left[ \frac{dZ}{Z} \right] / dt - r = \sigma_Z \lambda(r, t), \text{ where}$$

$$(18.17) \quad \sigma_Z = \frac{1}{Z} \frac{\partial Z}{\partial r} s(r, t) \text{ and}$$

$$(18.18) \quad \lambda(r, t) = \frac{1}{s(r, t)} (m(r, t) - m^*(r, t))$$

$$(18.26) \quad V(r_{t+\delta}) \approx V(r_t) + \frac{\partial V}{\partial r} (r_{t+\delta} - r_t)$$

$$(\text{page 634}) \quad \text{Standard deviation of } (V(r_{t+\delta}) - V(r_t)) \approx \frac{\partial V}{\partial r} \times \text{Standard deviation of } (r_{t+\delta} - r_t)$$

$$(18.28) \quad r_\delta - r_0 \sim N(\mu(r_0, \delta), \sigma^2(\delta))$$

$$(18.29) \quad \mu(r_0, \delta) = (r_0 - \bar{r}) \times (e^{-\gamma\delta} - 1); \quad \sigma(\delta) = \sqrt{\frac{\sigma^2}{2\gamma} (1 - e^{-2\gamma\delta})}$$

$$(18.32) \quad Z(t, T) = E_t \left[ e^{-\rho(T-t)} \frac{Q_t Y_t^h}{Q_T Y_T^h} \right]$$

$$(18.33) \quad Z(t, T) = E_t \left[ e^{-\rho(T-t) - (q_T - q_t) - h(y_T - y_t)} \right]$$

$$(18.34) \quad Z(i, t, T) = e^{A(t; T) - B(t; T)(i+c)}$$

$$(18.35) \quad B(t; T) = \frac{1}{\gamma} (1 - e^{-\gamma(T-t)})$$

$$(18.36) \quad A(t; T) = (B(t; T) - (T - t)) \left( \bar{r}^* - \frac{\sigma^2}{2\gamma^2} \right) - \frac{\sigma^2 B(t; T)^2}{4\gamma}$$

$$(18.37) \quad c = \left( \rho + hg - \frac{1}{2} h^2 \sigma_y^2 \right) - h\sigma_y \sigma_q \rho_{qy} - \frac{1}{2} \sigma_q^2$$

$$(18.38) \quad \bar{r}^* = \bar{r} - \frac{1}{\gamma} (h\sigma_i\sigma_y\rho_{yi} + \sigma_i\sigma_y\rho_{iq})$$

$$(18.39) \quad \bar{r} = \bar{i} + c$$

$$(18.40) \quad r_t = i_t + c$$

$$(18.41) \quad \text{Risk natural (true) dynamics: } dr_t = \gamma(\bar{r} - r_t)dt + \sigma_i dX_i$$

$$(18.42) \quad \text{Risk neutral dynamics: } dr_t = \gamma(\bar{r}^* - r_t)dt + \sigma_i dX_i$$

$$(18.43) \quad \lambda = \frac{\gamma}{\sigma_i} (\bar{r} - \bar{r}^*) = h\sigma_y\rho_{yi} + \sigma_y\rho_{iq}$$

## Chapter 19

$$(19.7) \quad dr_t = \theta_t dt + \sigma dX_t$$

$$(19.8) \quad Z(r, 0; T) = e^{A(0;T) - T \times r}$$

$$(19.9) \quad A(0, T) = - \int_0^T (T-t)\theta_t dt + \frac{T^3}{6}\sigma^2$$

$$(19.13) \quad \theta_t = \frac{\partial f(0, t)}{\partial t} + \sigma^2 \times t$$

$$(19.14) \quad \text{Payoff at } T_O = \max(Z(T_O; T_B) - K, 0)$$

$$(19.15) \quad V(r_0, 0) = Z(r_0, 0; T_B)\mathcal{N}(d_1) - KZ(r_0, 0; T_O)\mathcal{N}(d_2)$$

Note: For this, and many of the following formulas, the text has  $Z(0, r_0; \cdot)$  while  $Z(r_0, 0; \cdot)$  is correct

$$(19.16) \quad d_1 = \frac{1}{S_Z(T_O; T_B)} \log \left( \frac{Z(r_0, 0; T_B)}{KZ(r_0, 0; T_O)} \right) + \frac{S_Z(T_O; T_B)}{2}$$

$$(19.17) \quad d_2 = d_1 - S_Z(T_O; T_B)$$

$$(19.18) \quad S_Z(T_O; T_B)^2 = \sigma^2 T_O (T_B - T_O)^2$$

$$(19.19) \quad \text{Payoff at } T_O = \max(K - Z(T_O; T_B), 0)$$

$$(19.20) \quad V(r_0, 0) = KZ(r_0, 0; T_O)\mathcal{N}(-d_2) - Z(r_0, 0; T_B)\mathcal{N}(-d_1)$$

$$(19.25) \quad Z(r, 0; T) = e^{A(0;T) - B(0;T) \times r}$$

$$(19.26) \quad B(0; T) = \frac{1}{\gamma^*} (1 - e^{-\gamma^* T})$$

$$(19.27) \quad A(0; T) = - \int_0^T B(0; T-t)\theta_t dt + \frac{\sigma^2}{2(\gamma^*)^2} \left( T + \frac{1 - e^{-2\gamma^* T}}{2\gamma^*} - 2B(0; T) \right)$$

(This is a correction to the text formula)

$$(19.28) \quad \theta_t = \frac{\partial f(0, t)}{\partial t} + \gamma^* f(0, t) + \frac{\sigma^2}{2\gamma^*} \times (1 - e^{-2\gamma^* t})$$

$$(19.29) \quad \sigma_t(\tau) = \frac{B(\tau)}{\tau} \sigma$$

$$(19.30) \quad V(r_0, 0) = Z(r_0, 0; T_B)\mathcal{N}(d_1) - KZ(r_0, 0; T_O)\mathcal{N}(d_2)$$

$$(19.31) \quad d_1 = \frac{1}{S_Z(T_O; T_B)} \log \left( \frac{Z(r_0, 0; T_B)}{KZ(r_0, 0; T_O)} \right) + \frac{S_Z(T_O; T_B)}{2}$$

$$(19.32) \quad d_2 = d_1 - S_Z(T_O; T_B)$$

$$(19.33) \quad S_Z(T_O; T_B)^2 = B(T_O; T_B)^2 \frac{\sigma^2}{2\gamma^*} (1 - e^{-2\gamma^* T_O})$$

$$(19.34) \quad V(r_0, 0) = KZ(r_0, 0; T_O)\mathcal{N}(-d_2) - Z(r_0, 0; T_B)\mathcal{N}(-d_1)$$

$$(19.36) \quad \text{Payoff of call option at } T_O = \max(P_c(r_{T_O}, T_O; T_B) - K, 0)$$

$$(19.37) \quad Call = \sum_{i=1}^n c(i) (Z(r_0, 0; T_i) \mathcal{N}(d_1(i)) - K_i Z(r_0, 0; T_O) \mathcal{N}(d_2(i)))$$

$$(19.38) \quad d_1(i) = \frac{1}{S_Z(T_O; T_i)} \log \left( \frac{Z(r_0, 0; T_i)}{K_i Z(r_0, 0; T_O)} \right) + \frac{S_Z(T_O; T_i)}{2}$$

$$(19.39) \quad d_2(i) = d_1(i) - S_Z(T_O; T_i)$$

$$(page\ 664) \quad Z(r_t, t; T) = e^{A(t; T) - B(t; T)r_t}$$

$$(19.40) \quad A(t; T) = - \int_t^T B(u; T) \theta_u du + \frac{\sigma^2}{2(\gamma^*)^2} \left( (T-t) + \frac{1 - e^{-2\gamma^*(T-t)}}{2\gamma^*} - 2B(t; T) \right)$$

(This is a correction to the text formula)

$$(19.41) \quad A(t; T) = \log \left( \frac{Z(r_0, 0; T)}{Z(r_0, 0; t)} \right) + B(t; T) f(0, t) - \frac{\sigma^2}{4\gamma^*} B(t; T)^2 (1 - e^{-2\gamma^* t})$$

$$(19.42) \quad A(t; T) = \log \left( \frac{Z(r_0, 0; T)}{Z(r_0, 0; t)} \right) + (T-t) f(0, t) - \frac{\sigma^2}{2} (T-t)^2 t$$

$$(19.44) \quad V(r_0, 0) = M \times (KZ(r_0, 0; T - \Delta) \mathcal{N}(-d_2) - Z(r_0, 0; T) \mathcal{N}(-d_1))$$

$$(19.45) \quad d_1 = \frac{1}{S_Z(T - \Delta; T)} \log \left( \frac{Z(r_0, 0; T)}{KZ(r_0, 0; T - \Delta)} \right) + \frac{S_Z(T - \Delta; T)}{2}$$

$$(19.46) \quad d_2 = d_1 - S_Z(T - \Delta; T)$$

$$(19.47) \quad CF(T_j) = \Delta \times N \times \max(r_n(T_{j-1}, T_j) - r_K, 0)$$

$$(19.48) \quad Cap = \sum_{j=2}^n M \times (KZ(r_0, 0; T_{j-1}) \mathcal{N}(-d_2(j)) - Z(r_0, 0; T_j) \mathcal{N}(-d_1(j)))$$

$$(19.49) \quad d_1(j) = \frac{1}{S_Z(T_{j-1}; T_j)} \log \left( \frac{Z(r_0, 0; T_j)}{KZ(r_0, 0; T_{j-1})} \right) + \frac{S_Z(T_{j-1}; T_j)}{2}$$

$$(19.50) \quad d_2(j) = d_1(j) - S_Z(T_{j-1}; T_j)$$

$$(19.55) \quad dy_t = \left( \theta_t + \frac{\partial \sigma_t / \partial t}{\sigma_t} y_t \right) dt + \sigma_t dX_t$$

$$(19.57) \quad dy_t = (\theta_t - \gamma_t y_t) dt + \sigma_t dX_t$$

$$(19.58) \quad dr_t = (\theta_t - \gamma_t r_t) dt + \sqrt{\sigma_t^2 + \alpha_t r_t} dX_t$$

$$(19.59) \quad Z(r_t, t; T) = e^{A(t; T) - B(t; T)r_t}$$

$$(19.60) \quad \frac{\partial B(t; T)}{\partial t} = B(t; T) \gamma_t + \frac{1}{2} B(t; T)^2 \alpha_t - 1$$

$$(19.61) \quad \frac{\partial A(t; T)}{\partial t} = B(t; T) \theta_t - \frac{1}{2} B(t; T)^2 \sigma_t^2$$

## Chapter 20

$$(20.3) \quad \text{Caplet}(0; T_{i+1}) = N \times \Delta \times Z(0, T_{i+1}) \times [f_n(0, T_i, T_{i+1})\mathcal{N}(d_1) - r_K \mathcal{N}(d_2)]$$

$$(20.4) \quad d_1 = \frac{1}{\sigma_f \sqrt{T_i}} \log \left( \frac{f_n(0, T_i, T_{i+1})}{r_K} \right) + \frac{1}{2} \sigma_f \sqrt{T_i}$$

$$(20.5) \quad d_2 = d_1 - \sigma_f \sqrt{T_i}$$

$$(20.6) \quad \text{Floorlet}(0; T_{i+1}) = N \times \Delta \times Z(0, T_{i+1}) \times [r_K \mathcal{N}(-d_2) - f_n(0, T_i, T_{i+1})\mathcal{N}(-d_1)]$$

$$(20.7) \quad \text{Cap}(0; T) = \sum_{i=1}^n \text{Caplets}(0; T_i)$$

$$(20.17) \quad V(0, T_O; T_S) = N \times \Delta \times \left[ \sum_{i=1}^n Z(0; T_i) \right] \times [r_K \mathcal{N}(-d_2) - f_n^s(0, T_O, T_S)\mathcal{N}(-d_1)]$$

$$(20.18) \quad d_1 = \frac{1}{\sigma_f^s \sqrt{T_O}} \ln \left( \frac{f_n^s(0, T_O, T_S)}{r_K} \right) + \frac{1}{2} \sigma_f^s \sqrt{T_O}; \quad d_2 = d_1 - \sigma_f^s \sqrt{T_O}$$

(This is a correction to the text formula)

$$(20.19) \quad V(0, T_O; T_S) = N \times \Delta \times \left[ \sum_{i=1}^n Z(0; T_i) \right] \times [f_n^s(0, T_O, T_S)\mathcal{N}(d_1) - r_K \mathcal{N}(d_2)]$$

## Chapter 21

$$(21.2) \quad V(r, t; T) = E^* \left[ e^{-\int_t^T r_u du} g_T \right]$$

$$(21.3) \quad dr_t = m^*(r_t, t)dt + s(r_t, t)dX_t$$

$$(21.4) \quad \tilde{V}(r, t; T) = \frac{V(r, t; T)}{Z(r, t; T)}$$

$$(21.5) \quad 0 = \frac{\partial \tilde{V}}{\partial t} + \frac{\partial \tilde{V}}{\partial r} (m^*(r, t) + \sigma_Z(r, t)s(r, t)) + \frac{1}{2} \frac{\partial^2 \tilde{V}}{\partial r^2} s(r, t)^2$$

$$(21.6) \quad \sigma_Z(r, t) = \frac{1}{Z} \frac{\partial Z}{\partial r} s(r, t)$$

$$(21.7) \quad \frac{dZ}{Z} = \mu_Z(r, t)dt + \sigma_Z(r, t)dX_t$$

$$(21.8) \quad \tilde{V}(r, t; T) = E_f^*[g_T]$$

$$(21.9) \quad dr_t = (m^*(r, t) + \sigma_Z(r, t)s(r, t))dt + s(r, t)dX_t$$

$$(21.10) \quad V(r, t; T) = Z(r, t; T)E_f^*[g_T]$$

$$(21.11) \quad E_f^*[\max(g_T - K, 0)] = F(0, T)\mathcal{N}(d_1) - K\mathcal{N}(d_2)$$

$$(21.12) \quad d_1 = \frac{1}{\sigma_T} \log \left( \frac{F(0, T)}{K} \right) + \frac{1}{2} \sigma_T$$

$$(21.13) \quad d_2 = d_1 - \sigma_T$$

$$(21.14) \quad \text{Call} = Z(0, T) \times [F(0, T)\mathcal{N}(d_1) - K\mathcal{N}(d_2)]$$

$$(21.15) \quad \text{Put} = Z(0, T) \times [K\mathcal{N}(-d_2) - F(0, T)\mathcal{N}(-d_1)]$$

$$(21.27) \quad V^{fwd}(0; T) = Z(0, T)N\Delta E_f^*[r_n(\tau, T) - K]$$

$$(21.28) \quad \frac{df_n(t, \tau, T)}{f_n(t, \tau, T)} = \sigma_f(t)dX_t$$

$$(21.29) \quad r_n(\tau, T) \sim \text{LogN} \left( f_n(0, \tau, T), \int_0^\tau \sigma_f(t)^2 dt \right)$$

$$(21.32) \quad \text{Caplet}(0; T_{i+1}) = N\Delta Z(0, T_{i+1})[f_n(0, T_i, T_{i+1})\mathcal{N}(d_1) - r_K\mathcal{N}(d_2)]$$

$$(21.33) \quad d_1 = \frac{1}{\sigma_f\sqrt{T_i}} \log\left(\frac{f_n(0, T_i, T_{i+1})}{r_K}\right) + \frac{1}{2}\sigma_f\sqrt{T_i}$$

$$(21.34) \quad d_2 = d_1 - \sigma_f\sqrt{T_i}$$

$$(21.37) \quad \frac{df_n(t, T_i, T_{i+1})}{f_n(t, T_i, T_{i+1})} = \left(\sum_{j=\bar{i}}^i \frac{\Delta f_n(t, T_j, T_{j+1})\sigma_f^{i+1}(t)\sigma_f^{j+1}(t)}{1 + \Delta f_n(t, T_j, T_{j+1})}\right) dt + \sigma_f^{i+1}(t)dX_t$$

$$(21.38) \quad \frac{df_n(t, T_i, T_{i+1})}{f_n(t, T_i, T_{i+1})} = -\left(\sum_{j=i}^{\bar{i}-1} \frac{\Delta f_n(t, T_j, T_{j+1})\sigma_f^{i+1}(t)\sigma_f^{j+1}(t)}{1 + \Delta f_n(t, T_j, T_{j+1})}\right) dt + \sigma_f^{i+1}(t)dX_t$$

$$(21.39) \quad \sigma_f^{Fwd}(T_{i+1})^2 \times (T_i - t) = S_i^2 \times (T_1 - t) + S_{i-1}^2 \times \Delta + \dots + S_1^2 \times \Delta$$

$$\text{(page 722)} \quad S_1 = \sigma_f^{Fwd}(0.25)$$

$$\text{(page 722)} \quad S_i = \sqrt{\frac{T_i}{\Delta} \left(\sigma_f^{Fwd}(T_{i+1})\right)^2 - \sum_{j=1}^{i-1} S_j^2} \quad (\text{This is a correction to the text formula})$$

$$(21.42) \quad f_n^s(t + \delta, T_i, T_{i+1}) = f_n^s(t, T_i, T_{i+1})e^{m_{i+1}^s(t)\delta + S(T_{i+1}-t)\sqrt{\delta}\epsilon_t^s}$$

$$\text{(page 723)} \quad m_{i+1}^s(t) = \sum_{j=\bar{i}}^i \frac{\Delta f_n^s(t, T_j, T_{j+1})S(T_{i+1}-t)S(T_{j+1}-t)}{1 + \Delta f_n^s(t, T_j, T_{j+1})} - \frac{1}{2}S(T_{i+1}-t)^2$$

$$(21.54) \quad df(t, T) = m(t, T)dt + \sigma_f(t, T)dX_t$$

$$(21.55) \quad m(t, T) = \sigma_f(t, T) \int_t^T \sigma_f(t, \tau)d\tau$$

$$(21.59) \quad f(0, \tau, T) = f^{fut}(0, \tau, T) - \int_0^\tau \frac{\sigma_Z(t, T)^2 - \sigma_Z(t, \tau)^2}{2(T-\tau)} dt \quad (\text{This is a correction to the text formula})$$

$$(21.60) \quad f(0, \tau, T) = f^{fut}(0, \tau, T) - \frac{1}{2}\sigma^2\tau T$$



## Chapter 22

$$(22.3) \quad dP_t = \left\{ \left( \frac{\partial F}{\partial t} \right) + \left( \frac{\partial F}{\partial \phi_1} \right) m_{1,t} + \left( \frac{\partial F}{\partial \phi_2} \right) m_{2,t} + \frac{1}{2} \left( \frac{\partial^2 F}{\partial \phi_1^2} \right) s_{1,t}^2 + \frac{1}{2} \left( \frac{\partial^2 F}{\partial \phi_2^2} \right) s_{2,t}^2 \right\} dt \\ + \left( \frac{\partial F}{\partial \phi_1} \right) s_{1,t} dX_{1,t} + \left( \frac{\partial F}{\partial \phi_2} \right) s_{2,t} dX_{2,t}$$

$$(22.13) \quad R(\phi_1, \phi_2)V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \phi_1} m_{1,t}^* + \frac{\partial V}{\partial \phi_2} m_{2,t}^* + \frac{1}{2} \frac{\partial^2 V}{\partial \phi_1^2} s_{1,t}^2 + \frac{1}{2} \frac{\partial^2 V}{\partial \phi_2^2} s_{2,t}^2$$

$$(22.19) \quad Z(\phi_{1,t}, \phi_{2,t}, t; T) = e^{A(t;T) - B_1(t;T)\phi_{1,t} - B_2(t;T)\phi_{2,t}}$$

$$(22.20) \quad B_i(t; T) = \frac{1}{\gamma_i^*} \left( 1 - e^{\gamma_i^*(T-t)} \right)$$

$$(22.21) \quad A(t; T) = (B_1(t; T) - (T-t)) \left( \bar{\phi}_1^* - \frac{\sigma_1^2}{2(\gamma_1^*)^2} \right) - \frac{\sigma_1^2}{4\gamma_1^*} B_1(t; T)^2 \\ + (B_2(t; T) - (T-t)) \left( \bar{\phi}_2^* - \frac{\sigma_2^2}{2(\gamma_2^*)^2} \right) - \frac{\sigma_2^2}{4\gamma_2^*} B_2(t; T)^2$$

$$(22.27) \quad Z(r_t, r_{\ell,t}, t; T) = e^{A_{\tau_\ell}(\tau) - B_{\tau_\ell,1}(\tau)r_t - C_{\tau_\ell}(\tau)r_{\ell,t}}, \text{ where } \tau = T - t$$

$$(22.28) \quad A_{\tau_\ell}(\tau) = A(\tau) - C(\tau) \times \frac{A(\tau_\ell)}{C(\tau_\ell)}$$

$$(22.29) \quad B_{\tau_\ell,1}(\tau) = B_1(\tau) - C(\tau) \times \frac{B_1(\tau_\ell)}{C(\tau_\ell)}$$

$$(22.30) \quad C_{\tau_\ell}(\tau) = C(\tau) \times \frac{\tau_\ell}{C(\tau_\ell)}$$

$$(22.31) \quad r_t(\tau) = -\frac{A_{\tau_\ell}(\tau)}{\tau} + \frac{B_{\tau_\ell,1}(\tau)}{\tau} r_t + \frac{C_{\tau_\ell}(\tau)}{\tau} r_{\ell,t}$$

$$(22.34) \quad \sigma_{\ell,1} = \sigma_1 \frac{1 - e^{-\gamma_1^* \tau_\ell}}{\tau_\ell}; \quad \sigma_{\ell,2} = \sigma_2 \frac{1 - e^{-\gamma_2^* \tau_\ell}}{\tau_\ell}$$

$$(22.38) \quad \text{Vasicek volatility of } dr_t(\tau) = \sigma_t(\tau) = \frac{\sigma}{\gamma^*} \frac{1 - e^{-\gamma^* \tau}}{\tau}$$

$$(22.39) \quad \text{Volatility of } dr_t(\tau) = \sigma_t(\tau) = \sqrt{\sigma_1^2 \left( \frac{B_1(\tau)}{\tau} \right)^2 + \sigma_2^2 \left( \frac{B_2(\tau)}{\tau} \right)^2}$$

$$(22.41) \quad V(\phi_{1,0}, \phi_{2,0}, 0) = Z(\phi_{1,0}, \phi_{2,0}, 0; T_B) \mathcal{N}(d_1) - KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O) \mathcal{N}(d_2)$$

$$(22.42) \quad d_1 = \frac{1}{S_Z(T_O)} \log \left( \frac{Z(\phi_{1,0}, \phi_{2,0}, 0; T_B)}{KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O)} \right) + \frac{S_Z(T_O)}{2}$$

$$(22.43) \quad d_2 = d_1 - S_Z(T_O) \quad (\text{This is a correction to the text formula})$$

$$(22.44) \quad V(\phi_{1,0}, \phi_{2,0}, 0) = -Z(\phi_{1,0}, \phi_{2,0}, 0; T_B) \mathcal{N}(-d_1) + KZ(\phi_{1,0}, \phi_{2,0}, 0; T_O) \mathcal{N}(-d_2)$$

$$(22.46) \quad d\phi_{1,t} = m_1(\phi_{1,t}, \phi_{2,t}, t)dt + s_1(\phi_{1,t}, \phi_{2,t}, t)dX_{1,t}$$

$$(22.47) \quad d\phi_{2,t} = m_2(\phi_{1,t}, \phi_{2,t}, t)dt + s_2(\phi_{1,t}, \phi_{2,t}, t)dX_{2,t}$$

$$(22.48) \quad dP_t = \left\{ \left( \frac{\partial F}{\partial t} \right) + \left( \frac{\partial F}{\partial \phi_1} \right) m_{1,t} + \left( \frac{\partial F}{\partial \phi_2} \right) m_{2,t} + \frac{1}{2} \left( \frac{\partial^2 F}{\partial \phi_1^2} \right) s_{1,t}^2 + \frac{1}{2} \left( \frac{\partial^2 F}{\partial \phi_2^2} \right) s_{2,t}^2 + \left( \frac{\partial^2 F}{\partial \phi_1 \phi_2} \right) s_{1,t} s_{2,t} \rho \right\} dt \\ + \left( \frac{\partial F}{\partial \phi_1} \right) s_{1,t} dX_{1,t} + \left( \frac{\partial F}{\partial \phi_2} \right) s_{2,t} dX_{2,t}$$

$$(22.49) \quad R(\phi_1, \phi_2)V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \phi_1} m_{1,t}^* + \frac{\partial V}{\partial \phi_2} m_{2,t}^* + \frac{1}{2} \frac{\partial^2 V}{\partial \phi_1^2} s_{1,t}^2 + \frac{1}{2} \frac{\partial^2 V}{\partial \phi_2^2} s_{2,t}^2 + \frac{\partial^2 V}{\partial \phi_1 \phi_2} s_{1,t} s_{2,t} \rho$$

$$(22.61) \quad V(\phi_{1,t}, \phi_{2,t}, t) = E^* \left[ e^{-\int_t^T R(\phi_{1,u}, \phi_{2,u}) du} g_T | \phi_{1,t}, \phi_{2,t} \right]$$

$$(22.62) \quad d\phi_{1,t} = m_{1,t}^* dt + s_{1,t} dX_{1,t}$$

$$(22.63) \quad d\phi_{2,t} = m_{2,t}^* dt + s_{2,t} dX_{2,t}$$

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### Chapter 3

$$(3.3) \quad C(S, t) - P(S, t) = S - Ke^{-r(T-t)}$$

$$(3.9) \quad C(S + dS, t + dt) = C(S, t) + \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S} dS + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} dS^2 + \dots$$

$$(3.11) \quad C(S + dS, t + dt) = C(S, t) + \Theta dt + \Delta dS + \frac{1}{2} \Gamma dS^2$$

$$(3.16) \quad \frac{\partial C}{\partial t} + rS \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC$$

$$(3.17) \quad \frac{\partial C}{\partial t} + \frac{1}{2} \Gamma \Sigma^2 S^2 = 0$$

### Chapter 4

$$(4.1) \quad C(S, K, \tau, \sigma, r) = SN(d_1) - Ke^{-r\tau} N(d_2), \quad d_{1,2} = \frac{\ln\left(\frac{S}{K}\right) + \left(r \pm \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \quad N(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{1}{2}y^2} dy$$

$$(4.12) \quad \kappa_\pi = \int_0^\infty \rho(xS) S^2 f(x, \nu, \tau) dx$$

$$(4.32) \quad \frac{1}{T} \int_0^T \sigma_t^2 dt = \frac{2}{T} \left[ \int_0^T \frac{1}{S} dS - \ln\left(\frac{S_T}{S_0}\right) \right]$$

$$(4.41) \quad \pi(S_T, S_0, T, T) = \frac{2}{T} \left[ \left( \frac{S_T - S_0}{S_0} \right) - \ln\left(\frac{S_T}{S_0}\right) \right]$$

$$(4.43) \quad \sigma(K) = \sigma_F - b \frac{K - S_F}{S_F}$$

$$(4.44) \quad \sigma_K^2 = \sigma_F^2 (1 + 3Tb^2)$$

## Chapter 5

$$(5.1) \quad dS = \mu_S S dt + \sigma_S S dZ, \quad dB = Br dt$$

$$(5.2) \quad dC = \frac{\partial C}{\partial t} dt + \frac{\partial C}{\partial S} dS + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (\sigma_S S)^2 dt \\ = \left\{ \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S} \mu_S S + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (\sigma_S S)^2 \right\} dt + \frac{\partial C}{\partial S} \sigma_S S dZ \\ = \mu_C C dt + \sigma_C C dZ$$

$$(5.3) \quad \mu_C = \frac{1}{C} \left\{ \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S} \mu_S S + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} (\sigma_S S)^2 \right\}, \quad \sigma_C = \frac{S}{C} \frac{\partial C}{\partial S} \sigma_S = \frac{\partial \ln C}{\partial \ln S} \sigma_S$$

$$(5.10) \quad \frac{(\mu_C - r)}{\sigma_C} = \frac{(\mu_S - r)}{\sigma_S}$$

$$(5.12) \quad \frac{\partial C}{\partial t} + rS \frac{\partial C}{\partial S} + \frac{1}{2} \sigma_S^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC$$

$$(5.13) \quad C(S, K, t, T, \sigma, r) = e^{-r(T-t)} [S_F N(d_1) - KN(d_2)], \quad S_F = e^{r(T-t)} S$$

$$d_1 = \frac{\ln\left(\frac{S_F}{K}\right) + \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2 = \frac{\ln\left(\frac{S_F}{K}\right) - \left(\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$(5.19) \quad C_0 = C_T e^{-rT} - \int_0^T \Delta(S_x, x) [dS_x - S_x r dx] e^{-rx}$$

$$(5.20) \quad C_0 = C_T e^{-rT} - \int_0^T \Delta(S_x, x) \sigma S_x e^{-rx} dZ_x$$

$$(5.21) \quad E[C_0] = E[C_T] e^{-rT}$$

$$(5.22) \quad \pi(I, R) = V_I - \Delta_R S$$

$$(5.23) \quad \text{PV}[\text{P\&L}(I, R)] = V(S, \tau, \sigma_R) - V(S, \tau, \Sigma)$$

$$(5.25) \quad \Delta_R = e^{-D\tau} N(d_1), \quad d_1 = \frac{\ln\left(\frac{S_F}{K}\right) + \frac{1}{2} \sigma_R^2 \tau}{\sigma_R \sqrt{\tau}}, \quad S_F = S e^{(r-D)\tau}$$

$$(5.27) \quad d\text{P\&L}(I, R) = dV_I - rV_I dt - \Delta_R [dS - (r-D)S dt]$$

$$(5.28) \quad d\text{P\&L}(R, R) = 0 = dV_R - V_R r dt - \Delta_R [dS - (r-D)S dt]$$

$$(5.34) \quad \text{PV}[\text{P\&L}(I, R)] = e^{rt_0} [e^{-rT} \cdot 0 - e^{-rt_0} (V_{I,t} - V_{R,t})] = V_{R,t} - V_{I,t}$$

$$(5.38) \quad d\text{P\&L}(I, R) = \frac{1}{2} \Gamma_I S^2 (\sigma_R^2 - \Sigma^2) dt + (\Delta_I - \Delta_R) [(\mu - r + D)S dt + \sigma_R S dZ]$$

(page 100) The upper bound of the P&L is ...  $(V_{R,0} - V_{I,0})$

$$(5.41) \quad \text{PV}[\pi(I, R)]_L = (V_{R,0} - V_{I,0}) - 2K e^{-2r\tau} \left[ N\left(\frac{1}{2}(\sigma_R - \Sigma)\sqrt{\tau}\right) - \frac{1}{2} \right]$$

(This is a correction to the text formula)

$$(5.42) \quad d\text{P\&L}(I, I) = \frac{1}{2} \Gamma_I S^2 (\sigma_R^2 - \Sigma^2) dt$$

$$(5.43) \quad \text{PV}[\text{P\&L}(I, I)] = \frac{1}{2} \int_{t_0}^T e^{-r(t-t_0)} \Gamma_I S^2 (\sigma_R^2 - \Sigma^2) dt$$

$$(page 103, problem 5-4) \quad \text{PV}[\text{P\&L}(I, H)] = V_h - V_I + \frac{1}{2} \int_{t_0}^T e^{-r(t-t_0)} \Gamma_h S^2 (\sigma_R^2 - \sigma_h^2) dt$$

## Chapter 6

$$(6.2) \quad \pi = C - \frac{\partial C}{\partial S} S$$

$$(6.6) \quad HE \approx \sum_{i=1}^n \frac{1}{2} \Gamma_i \sigma_i^2 S_i^2 (Z_i^2 - 1) dt$$

$$(6.7) \quad \sigma_{HE}^2 \approx E \left[ \sum_{i=1}^n \frac{1}{2} (\Gamma_i S_i^2)^2 (\sigma_i^2 dt)^2 \right]$$

$$(6.12) \quad \sigma_{HE} \approx \sqrt{\frac{\pi}{4}} \frac{\sigma}{\sqrt{n}} \frac{\partial C}{\partial \sigma}$$

$$(6.14) \quad \sigma_{HE} \approx dC \approx \frac{\partial C}{\partial \sigma} d\sigma \approx \frac{\sigma}{\sqrt{n}} \frac{\partial C}{\partial \sigma}$$

$$(6.18) \quad \frac{\sigma_{HE}}{C} \approx \sqrt{\frac{\pi}{4n}} \approx \frac{0.89}{\sqrt{n}}$$

## Chapter 7

$$(7.14) \quad \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \frac{\partial C}{\partial t} - \sqrt{\frac{2}{\pi dt}} \left| \frac{\partial^2 C}{\partial S^2} \right| \sigma S^2 k = r \left( C - S \frac{\partial C}{\partial S} \right)$$

$$(7.18) \quad \check{\sigma}^2 = \sigma^2 + 2\sigma k \sqrt{\frac{2}{\pi dt}}$$

$$(7.19) \quad \check{\sigma} \approx \sigma \pm k \sqrt{\frac{2}{\pi dt}}$$

## Chapter 8

$$(8.3) \quad P[\ln(S_T) > \ln(K)] = P \left[ Z > \frac{-\ln\left(\frac{S_t}{K}\right) - \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \right] = N(d_2)$$

$$(8.6) \quad \Delta_{\text{ATM}} \approx \frac{1}{2} + \frac{d_1}{\sqrt{2\pi}} \approx \frac{1}{2} + \frac{\sigma\sqrt{\tau}}{2\sqrt{2\pi}}$$

$$(8.9) \quad \Delta \approx \Delta_{\text{ATM}} - \frac{1}{\sqrt{2\pi}} \frac{J}{\nu}$$

## Chapter 10

$$(10.3) \quad S = V - B, \quad \frac{dS}{S} = \frac{dV}{S} = \frac{V\sigma dZ}{S} = \sigma \frac{S+B}{S} dZ, \quad \sigma_S = \sigma \left( 1 + \frac{B}{S} \right)$$

$$(10.4) \quad \frac{dS}{S} = \mu(S, t) dt + \sigma S^{\beta-1} dZ$$

$$(10.5) \quad dS = \mu S dt + \sigma S dZ, \quad d\sigma = p\sigma dt + q\sigma dW, \quad E[dW dZ] = \rho dt$$

$$(10.10) \quad \text{Profit} = \frac{1}{2} \Gamma S^2 (\sigma^2 - \Sigma^2) dt = \frac{1}{2} \Gamma (dS)^2 - \frac{1}{2} \Gamma S^2 \Sigma^2 dt$$

$$(10.15) \quad D = -\frac{\partial C_{\text{BSM}}}{\partial K} - \frac{\partial C_{\text{BSM}}}{\partial \Sigma} \frac{\partial \Sigma}{\partial K}$$

## Chapter 11

$$(11.28) \quad x = \frac{\ln\left(\frac{S_T}{S_t}\right) - \left(r\tau - \frac{1}{2}\sigma^2\tau\right)}{\sigma\sqrt{\tau}}$$

$$(11.29) \quad N'(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$$

$$(11.34) \quad p(S_t, t, S_T, T) = \frac{e^{-\frac{x^2}{2}}}{\sigma S_T \sqrt{2\pi\tau}}$$

## Chapter 14

$$(14.4) \quad F = Se^{(r-b)dt}$$

$$(14.5) \quad F = qS_u + (1-q)S_d$$

$$(14.6) \quad q = \frac{F - S_d}{S_u - S_d}$$

$$(14.11) \quad S_u = Se^{\sigma(S,t)\sqrt{dt}}, \quad S_d = Se^{-\sigma(S,t)\sqrt{dt}}$$

$$(14.17) \quad \Sigma(S, K) \approx \sigma_0 + \frac{\beta}{2}(S + K)$$

$$(14.18) \quad \Sigma(S, K) \approx \sigma(S) + \frac{\beta}{2}(K - S)$$

## Chapter 17

$$(17.6) \quad d\pi_{\text{BSM}} = d\pi_{\text{loc}} - \varepsilon dS = \frac{1}{2}\Gamma_{\text{loc}}S^2 [\sigma_R^2 - \sigma_{\text{loc}}^2(S, t)] dt - \varepsilon dS$$

## Chapter 18

$$(18.6) \quad \Sigma_{\text{ATM}}(S) = \Sigma(S, S) = \Sigma_0 - \beta(S - S_0)$$

$$(18.7) \quad \Sigma(S, K) = \Sigma_0 - \beta(K - S)$$

$$(18.11) \quad \Sigma(S, K, t, T) = \Sigma_0(t, T) - \beta'(t, T)[0.5 - \Delta(S, K, t, T, \Sigma_{\text{ATM}}(S))]$$

$$(18.12) \quad \Sigma(S, K, \tau) = \Sigma_0 - \beta'(0.5 - \Delta(S, K, \tau, \Sigma_{\text{ATM}}))$$

## Chapter 19

$$(19.8) \quad \frac{dV}{V} = \alpha dt + \xi dW \quad \text{where } V = \sigma^2$$

$$(19.9) \quad dY = \alpha(m - Y)dt + \beta dW$$

$$(19.15) \quad Y_t = m + (Y_0 - m)e^{-\alpha t} + \beta \int_0^t e^{-\alpha(t-s)} dW_s$$

$$(19.21) \quad \text{Var}[Y_t] = \frac{\beta^2}{2\alpha} (1 - e^{-2\alpha t})$$

$$(19.24) \quad d\sigma = \alpha(m - \sigma)dt + \beta dW$$

$$(19.25) \quad dV = \alpha(m - V)dt + \beta dW$$

$$(19.26) \quad dV = \alpha(m - V)dt + \beta V dW$$

$$(19.27) \quad dV = \alpha(m - V)dt + \beta\sqrt{V}dW$$

**QFIQ-120-19: Chapter 6 and 7 of Pricing and Hedging Financial Derivatives, Marroni and Perdomo**

- (page 124)  $\Delta(\text{call}) = N(d_1)$
- (page 124)  $\Delta(\text{put}) = N(d_1) - 1$
- (page 127)  $\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$
- (page 128)  $d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$
- (page 128)  $N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$
- (page 133)  $\text{Vega} = S\sqrt{T-t}N'(d_1)$
- (page 137)  $\Theta(\text{call}) = -\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} - rKe^{-r(T-t)}N(d_2)$
- (page 137)  $d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$
- (page 137)  $d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$
- (page 137)  $\Theta(\text{put}) = -\frac{SN'(d_1)\sigma}{2\sqrt{T-t}} + rKe^{-r(T-t)}N(-d_2)$
- (page 138)  $\Theta \Delta t + \frac{1}{2}\Gamma \Delta S^2 \cong 0$
- (page 139)  $\frac{\Delta S}{S} \cong \frac{\sigma}{\sqrt{252}}$
- (page 143)  $\rho(\text{call}) = K(T-t)e^{-r(T-t)}N(d_2)$
- (page 143)  $\rho(\text{put}) = -K(T-t)e^{-r(T-t)}N(-d_2)$  (This is a correction to the text formula)
- (page 180)  $S(k) = S(k-1) * [\text{yield}(t) * dt + \text{vol}(t, S) * W * \text{sqrt}(dt)]$
- (page 181)  $\text{vol}^2 * (t-s) = \text{vol\_mkt}^2(t) * t - \text{vol\_mkt}^2(s) * s$

## QFIQ-128-20: Mitigating Interest Rate Risk in Variable Annuities: An Analysis of Hedging Effectiveness under Model Risk

$$(1) \quad dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_t^S$$

$$(2) \quad d\nu_t = \kappa(\theta - \nu_t)dt + \sigma_\nu \sqrt{\nu_t} dW_t^\nu$$

$$(page\ 505) \quad dr_t^{(j)} = a_j(b_j - r_t^{(j)})dt + \sigma_{t,j} \sqrt{r_t^{(j)}} dW_{t,j}^r$$

$$(page\ 506) \quad \Pi_{t+h} = (\Pi_t - \Delta_t S_t - n_t P_{t,t+T^B}) B_{t+h} / B_t + \Delta_t S_{t+h} + n_t P_{t+h,t+T^B}$$

$$(page\ 506) \quad dS_t = r_t S_t dt + \sigma_S S_t dZ_t^S$$

$$(page\ 506) \quad dr_t = (v(t) - ar_t)dt + \sigma_r dZ_t^r$$

$$(page\ 508) \quad dA_t = (\mu - \alpha)A_t dt - \omega_t dt + \sqrt{\nu_t} A_t dW_t^S$$

$$(page\ 508) \quad L_T = L_T^{(D)} + L_T^{(A)}$$

$$(page\ 508) \quad L_T^{(D)} = \int_0^T \left[ \max(G_t - A_t, 0) B_{t,T} - \int_0^t \alpha A_s B_{s,T} ds \right] {}_t p_x u_{x+t} dt$$

$$(page\ 509) \quad L_T^{(A)} = \left[ \max(G_T - A_T, 0) - \int_0^T \alpha A_t B_{t,T} dt \right] {}_T p_x$$

$$(3) \quad \Omega_t = \mathbb{E}^{\mathbb{Q}} \left[ \int_t^T \beta_{t,v} \max(A_v, G_v) {}_{v-t} p_{x+t} u_{x+v} dv + \beta_{t,T} \max(A_T, G_T) {}_{T-t} p_{x+t} | \mathcal{F}_t \right]$$

$$(page\ 510) \quad L_T = \frac{A_0}{T} \int_\tau^T B_{t,T} dt - \int_0^\tau \alpha A_t B_{t,T} dt$$

$$(5) \quad \Omega_t = \frac{A_0}{T} \int_t^T P_{t,v} dv + \mathbb{E}^{\mathbb{Q}}[\beta_{t,T} A_T | \mathcal{F}_t]$$

$$(page\ 512) \quad RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (HL_T^{(i)})^2}$$

$$(page\ 512) \quad CTE(1-p)\% = \frac{1}{N_p} \sum_{i=1}^{N_p} HL_T^{(i)}$$

$$(page\ 522) \quad P_{t,T} = \mathcal{A}(t, T) e^{-\mathcal{B}(t, T) r_t}$$

$$(page\ 522) \quad \mathcal{B}(t, T) = \frac{1}{a} [1 - e^{-a(T-t)}]$$

$$(page\ 522) \quad \rho_t^B = \frac{\partial P_{t,t+T^B}}{\partial r_t} = -\mathcal{B}(t, t+T^B) P_{t,t+T^B}$$

$$(page\ 523) \quad \Delta_t = \frac{\partial \mathcal{L}_t}{\partial A_t} \times \frac{\partial A_t}{\partial S_t} = \left[ \int_t^T \frac{\partial \Psi(t, v, A_t, G_v)}{\partial A_t} {}_{v-t} p_{x+t} u_{x+v} dv + \frac{\partial \Psi(t, T, A_t, G_T)}{\partial A_t} {}_{T-t} p_{x+t} - \left( 1 - \int_t^T e^{-\alpha(v-t)} {}_{v-t} p_{x+t} u_{x+v} dv - e^{-\alpha(T-t)} {}_{T-t} p_{x+t} \right) \right] \times \frac{A_t}{S_t}$$

(This is a correction to the text formula)

$$(page\ 523) \quad \rho_t = \int_t^T \frac{\partial \Psi(t, v, A_t, G_v)}{\partial r_t} {}_{v-t} p_{x+t} u_{x+v} dv + \frac{\partial \Psi(t, T, A_t, G_T)}{\partial r_t} {}_{T-t} p_{x+t}$$

(page 525)  $\Delta_t = (e^{-\alpha(T-t)}\Phi(z) - 1) \times \frac{A_t}{S_t}$

(page 525)  $\rho_t = -\frac{A_0}{T} \int_t^T \mathcal{B}(t, v) P_{t,v} dv + e^{-\alpha(T-t)} \frac{A_0}{T} \int_t^T e^{\alpha(v-t)} \mathcal{B}(t, v) P_{t,v} \Phi(z - m_v) dv$

(page 525)  $dv_t = \tilde{\kappa}(\tilde{\theta} - v_t)dt + \sigma_v \sqrt{v_t} d\tilde{W}_t^v$

(page 525) 1-year  $VIX_t = \sqrt{\mathbb{E}^{\mathbb{Q}^F} \left[ \int_t^{t+1} v_s ds | \mathcal{F}_t \right]} = \sqrt{A + Bv_t}$

(page 525)  $A = \tilde{\theta} \left( 1 - \frac{1 - e^{-\tilde{\kappa}}}{\tilde{\kappa}} \right), \quad B = \frac{1 - e^{-\tilde{\kappa}}}{\tilde{\kappa}}$

### QFIQ-129-21: Negative Interest Rates and Their Technical Consequences, Actuarial Association of Europe

(page 2)  $C_{DD}(T, K, F_t) = C_{B76}(T, K + \Theta, F_t + \Theta, \sigma_{impl}^{DD}(T, K + \Theta))$

1.0.1  $F_k(t) = \frac{1}{\tau_k} \left( \frac{P(t, T_{k-1})}{P(t, T_k)} - 1 \right)$

1.0.2  $dF_k(t) = \sigma_k(t) F_k(t) \sum_{j=\beta(t)}^k \frac{\tau_j \rho_{j,k} \sigma_j(t) F_j(t)}{1 + \tau_j F_j(t)} dt + \sigma_k(t) F_k(t) dZ_k^d(t)$

with  $\rho_{j,k} dt = d\langle Z_j, Z_k \rangle_t = dZ_j(t) \cdot dZ_k(t)$  and  $Z_k^d$  a M-dimensional (typically M=2) Brownian vector under  $\mathcal{Q}^d$

1.1.1  $dF_k(t) = (F_k(t) + \delta) \left( s(t)^2 \sum_{j=\beta(t)}^k \frac{\gamma_k(t) \gamma_j(t) \tau_j (F_j(t) + \delta)}{1 + \tau_j F_j(t)} dt + s(t) \gamma_k(t) dZ^d(t) \right)$

1.2.1  $d(F_k(t)) = (F_k(t) + \delta) V(t) \sum_{j=\beta(t)}^k \frac{\gamma_k(t) \gamma_j(t) \tau_j (F_j(t) + \delta)}{1 + \tau_j F_j(t)} dt + \sqrt{V(t)} \gamma_k(t) (F_k(t) + \delta) dZ^d(t)$

$dV(t) = \kappa(\theta - V(t))dt + \epsilon \sqrt{V(t)} dW(t)$  and  $d\langle W, Z^d \rangle_t = \rho dt$

2.1.1  $dr_t = a[f(t) - r_t]dt + \sigma dz$

2.2.1  $r(t) = x(t) + y(t) + \phi(t)$  with  $r(0) = r_0$ , where processes  $x$  and  $y$  satisfy:

$dx(t) = -ax(t)dt + \sigma dW_1(t)$  with  $x(0) = 0$  and  $dy(t) = -by(t)dt + \eta dW_2(t)$  with  $y(0) = 0$

### QFIQ-130-21: Interest Rate Models-Theory and Practice, 2nd ed., Brigo and Mercurio

(4.4)  $r(t) = x(t) + y(t) + \varphi(t)$ ,  $r(0) = r_0$ , where  $\{x(t) : t \geq 0\}$  and  $\{y(t) : t \geq 0\}$  satisfy

(4.5)  $dx(t) = -ax(t)dt + \sigma dW_1(t)$ ,  $x(0) = 0$  and  $dy(t) = -by(t)dt + \eta dW_2(t)$ ,  $y(0) = 0$

(page 144)  $r(t) = x(s)e^{-a(t-s)} + y(s)e^{-b(t-s)} + \sigma \int_s^t e^{-a(t-u)} dW_1(u) + \eta \int_s^t e^{-b(t-u)} dW_2(u) + \varphi(t)$

(4.6)  $E\{r(t) | \mathcal{F}_s\} = x(s)e^{-a(t-s)} + y(s)e^{-b(t-s)} + \varphi(t)$

$\text{Var}\{r(t) | \mathcal{F}_s\} = \frac{\sigma^2}{2a} [1 - e^{-2a(t-s)}] + \frac{\eta^2}{2b} [1 - e^{-2b(t-s)}] + 2\rho \frac{\sigma\eta}{a+b} [1 - e^{-(a+b)(t-s)}]$



$$(4.7) \quad r(t) = \sigma \int_0^t e^{-a(t-u)} dW_1(u) + \eta \int_0^t e^{-b(t-u)} dW_2(u) + \varphi(t)$$

$$(4.10) \quad V(t, T) = \frac{\sigma^2}{a^2} \left[ T - t + \frac{2}{a} e^{-a(T-t)} - \frac{1}{2a} e^{-2a(T-t)} - \frac{3}{2a} \right] \\ + \frac{\eta^2}{b^2} \left[ T - t + \frac{2}{b} e^{-b(T-t)} - \frac{1}{2b} e^{-2b(T-t)} - \frac{3}{2b} \right] \\ + 2\rho \frac{\sigma\eta}{ab} \left[ T - t + \frac{e^{-a(T-t)} - 1}{a} + \frac{e^{-b(T-t)} - 1}{b} - \frac{e^{-(a+b)(T-t)} - 1}{a+b} \right]$$

$$(4.11) \quad P(t, T) = \exp \left\{ - \int_t^T \varphi(u) du - \frac{1 - e^{-a(T-t)}}{a} x(t) - \frac{1 - e^{-b(T-t)}}{b} y(t) + \frac{1}{2} V(t, T) \right\}$$

$$(4.14) \quad P(t, T) = \frac{P^M(0, T)}{P^M(0, t)} \exp\{\mathcal{A}(t, T)\}$$

$$\mathcal{A}(t, T) := \frac{1}{2} [V(t, T) - V(0, T) + V(0, t)] - \frac{1 - e^{-a(T-t)}}{a} x(t) - \frac{1 - e^{-b(T-t)}}{b} y(t)$$

(page 162)  $r(t) = \tilde{\chi}(t) + \psi(t) + \varphi(t)$

$$d\tilde{\chi}(t) = -\bar{a}\tilde{\chi}(t)dt + \sigma_3 dZ_3(t)$$

$$d\psi(t) = -\bar{b}\psi(t)dt + \sigma_4 dZ_2(t)$$

$$\varphi(t) = r_0 e^{-\bar{a}t} + \int_0^t \theta(v) e^{-\bar{a}(t-v)} dv$$

### QFIQ-132-21: Investment Instruments with Volatility Target Mechanism, Albeverio, Steblovskaya, and Wallbaum

$$(2) \quad \mathcal{O}_0 = \frac{1}{B_T} \mathbb{E}^*(f(S))$$

$$(3) \quad V_t = \beta_k S_t + \gamma_k B_t$$

$$(5) \quad \tilde{V}_t = \beta_k \tilde{S}_t + \gamma_k$$

$$(7) \quad \hat{\mathcal{O}}_0 = \frac{1}{B_T} \mathbb{E}^*(f(V))$$

$$(8) \quad dS_t = S_t(rdt + \sigma dW_t)$$

$$(10) \quad P_{dp} = K + p_{dp} \cdot K \max \left\{ \frac{S_T}{S_0} - 1, 0 \right\}$$

$$(11) \quad RB = K(1 - e^{-rT})$$

$$(12) \quad g_{dp}(x) = \max \left\{ \frac{K}{S_0} x - K, 0 \right\}$$

$$(17) \quad p_{dp} = \frac{1 - e^{-rT}}{\mathcal{O}_0(f_{dp})}$$

$$(18) \quad P_{dp} = K + p_{dp} \cdot K \max \left\{ \frac{S_T}{S_0} - 1, 0 \right\}$$

$$(19) \quad P_{dpa} = \max \left\{ K; K \cdot \left( 1 + p_{dpa} \cdot \frac{1}{S_0} \left( \frac{1}{n} \sum_{i=0}^n S_{t_i} - S_0 \right) \right) \right\}$$

$$(20) \quad g_{dpa}(S) = \max \left\{ \frac{K}{S_0} \frac{1}{n} \sum_{i=0}^n S_{t_i} - K, 0 \right\}$$

$$(25) \quad p_{dpa} = \frac{1 - e^{-rT}}{\mathcal{O}_0(f_{dpa})}$$

**QFIQ-134-22: An Introduction to Computational Risk Management of Equity-Linked Insurance, Feng Chapter 1**

$$(page\ 15) \quad G_{(k+1)/n} = G_{k/n} \left( 1 + \frac{\rho}{n} \right), \quad \text{for } k = 0, 1, \dots$$

$$(1.15) \quad G_{(k+1)/n} = \max \{ G_{k/n}, F_{(k+1)/n} \}, \quad \text{for } k = 0, 1, \dots$$

$$(page\ 15) \quad G_{(k+1)/n} = \frac{G_{k/n}}{F_{k/n}} \max \{ F_{k/n}, F_{(k+1)/n} \}, \quad \text{for } k = 0, 1, \dots$$

$$(1.16) \quad \frac{G_{(k+1)/n} - G_{k/n}}{G_{k/n}} = \frac{(F_{(k+1)/n} - F_{k/n})_+}{F_{k/n}}$$

$$(page\ 16) \quad G_{(k+1)/n} = G_0 \prod_{j=0}^{k-1} \max \left\{ 1, \frac{F_{(j+1)/n}}{F_{j/n}} \right\}$$

$$(1.17) \quad G_{(k+1)/n} = \max \left\{ G_{k/n} \left( 1 + \frac{\rho}{n} \right), F_{(k+1)/n} \right\}, \quad \text{for } k = 0, \dots$$

$$(1.18) \quad G_{k/n} = \left( 1 + \frac{\rho}{n} \right)^k \max_{j=0, \dots, k} \left\{ \left( 1 + \frac{\rho}{n} \right)^{-j} F_{j/n} \right\}$$

$$(1.19) \quad G_t = \sup_{0 \leq s \leq t} \{ F_s \}$$

$$(1.23) \quad L_e^{(\infty)}(T_x) = e^{-rT} (G - F_T)_+ I(T_x > T) - \int_0^{T \wedge T_x} e^{-rs} m_e F_s ds$$

$$(1.24) \quad L_d^{(\infty)}(T_x) = e^{-rT_x} (G e^{\rho T_x} - F_{T_x})_+ I(T_x \leq T) - \int_0^{T \wedge T_x} e^{-rs} m_d F_s ds$$

$$(1.28) \quad L_w^{(n)} := \sum_{k=n\tau}^{(n\tau-1) \vee [nT]} e^{-rk/n} \frac{w}{n} - \sum_{k=1}^{(n\tau-1) \wedge [nT]} e^{-r(k-1)/n} F_{(k-1)/n} \frac{m_w}{n}$$

$$(page\ 24) \quad L_w^{(\infty)} := \int_{\tau}^{\tau \vee T} e^{-rt} w dt - \int_0^{t \wedge T} e^{-rt} m_w F_t dt, \quad \tau := \inf \{ t > 0 : F_t \leq 0 \}$$

$$(page\ 27) \quad L_{lw}^{(n)} := \sum_{k=n\tau}^{(n\tau-1) \vee [nT_x]} e^{-r(k+1)/n} G_{k/n} \frac{h}{n} - \sum_{k=0}^{(n\tau-1) \wedge [nT_x]} e^{-rk/n} G_{k/n} \frac{m_w}{n}$$

$$(page\ 27) \quad L_{lw}^{(\infty)} := \int_{\tau}^{\tau \vee T_x} e^{-rt} G_t h dt - \int_0^{t \wedge T_x} e^{-rt} G_t m_w dt$$

$$(1.39) \quad P \prod_{k=1}^T \max \left( \min \left( 1 + \alpha \frac{S_k - S_{k-1}}{S_{k-1}}, e^c \right), e^g \right)$$

$$(1.41) \quad P \prod_{k=1}^T \max \left( \min \left( \left( \frac{S_k}{S_{k-1}} \right)^\alpha, e^c \right), e^g \right)$$

$$(1.42) \quad \max \left( P \left( \frac{\max\{S_k : k = 1, \dots, T\}}{S_0} \right)^\alpha, G_T \right)$$

$$(1.43) \quad \max \left( P \left( \frac{\sup_{0 \leq t \leq T} \{S_t\}}{S_0} \right)^\alpha, G_T \right)$$

## Chapter 4

$$(4.52) \quad dF_t = (r - m)F_t dt + \sigma F_t dW_t, \quad 0 < t < T$$

$$(4.53) \quad B_e(t, F) = {}_T p_x \left[ G e^{-r(T-t)} \Phi \left( -d_2 \left( T-t, \frac{F}{G} \right) \right) - F e^{-m(T-t)} \Phi \left( -d_1 \left( T-t, \frac{F}{G} \right) \right) \right]$$

$$(4.54) \quad d_1(t, u) = \frac{\ln u + (r - m + \sigma^2/2)t}{\sigma \sqrt{t}}$$

$$(4.55) \quad d_2(t, u) = d_1(t, u) - \sigma \sqrt{t}$$

$$\text{(page 153)} \quad P_e(t, F) = m_e {}_t p_x F \int_0^{T-t} e^{-ms} {}_s p_{x+t} ds = m_e {}_t p_x F \bar{a}_{x+t:\overline{T-t}|m}$$

$$\text{(page 153)} \quad {}_T p_x \left[ e^{(\rho-r)T} \Phi \left( \frac{\rho - (r - m - \sigma^2/2)\sqrt{T}}{\sigma} \right) - e^{-mT} \Phi \left( \frac{\rho - (r - m + \sigma^2/2)\sqrt{T}}{\sigma} \right) \right] = m_e \bar{a}_{x:\overline{T}|m}$$

$$\text{(page 154)} \quad \tilde{\mathbb{E}} \left[ e^{-r(T_2-t)} (G_{T_1} - F_{T_2})_+ I(T_x > T_2) | \mathcal{F}_t \right] \\ = {}_{T_2} p_x e^{-r(T_2-t)} \left[ G_{T_1} \Phi(-d_2(T_2 - t, F_t/G_{T_1})) - F_t e^{(r-m)(T_2-t)} \Phi(-d_1(T_2 - t, F_t/G_{T_1})) \right] \\ \text{(This is a correction to the text formula)}$$

$$(4.60) \quad N_d(0, F_0) := B_d(0, F_0) - P_d(0, F_0) = \tilde{\mathbb{E}} \left[ \int_0^T e^{-rt} {}_t p_x \mu_{x+t} (G_t - F_t)_+ dt - \int_0^T e^{-rt} m_d {}_t p_x F_t dt \right]$$

$$(4.61) \quad B_w(t, F) = \tilde{\mathbb{E}} \left[ e^{-r(T-t)} F_{T-t} I(F_{T-t} > 0) | F_0 = F \right] + \frac{w}{r} (1 - e^{-r(T-t)})$$

$$(4.63) \quad N_w(t, F) = \tilde{\mathbb{E}} \left[ w \int_{\tau-t}^{(\tau \vee T)-t} e^{-rs} ds - m_w \int_{(\tau \wedge t)-t}^{(\tau \wedge T)-t} e^{-rs} \tilde{F}_s ds \middle| \tilde{F}_0 = F \right] \\ = \tilde{\mathbb{E}} \left[ w \int_{\tau}^{\tau \vee (T-t)} e^{-rs} ds - m_w \int_0^{\tau \wedge (T-t)} e^{-rs} F_s ds \middle| F_0 = F \right]$$

$$\text{(page 158)} \quad e^{-rT} F_T I(\tau > T) + \frac{w}{r} (1 - e^{-rT}) - F_0$$

$$\text{(page 158)} \quad w \int_{\tau \wedge T}^T e^{-rs} ds - m_w \int_0^{t \wedge T} e^{-rs} F_s ds$$

(page 159)  $F_0 - \mathbb{E} \left[ \int_0^{\tau \wedge T} e^{-rs} (mF_s + w) ds \right] = \mathbb{E}[e^{-r(\tau \wedge T)} F_{\tau \wedge T}] = \mathbb{E}[e^{-rT} F_T I(\tau > T)]$

(page 160)  $B_{lw}(F_0) := \tilde{\mathbb{E}} \left[ \int_0^{T_x} w e^{-rs} ds + e^{-rT_x} F_{T_x} I(F_{T_x} > 0) \right]$   
 $= \frac{w}{r} - \frac{\delta w}{r(\delta + r)} + \delta \tilde{\mathbb{E}} \left[ \int_0^\tau e^{-(\delta+r)t} F_t dT \right]$

(4.75)  $N_{lw}(F_0) = \frac{w}{r + \delta} \tilde{\mathbb{E}}[e^{-(\delta+r)\tau}] + m_w \tilde{\mathbb{E}} \left[ \int_0^\tau e^{-(\delta+r)u} F_u du \right]$

(page 162)  $B_{lw}(t, F) = \frac{w}{r} (1 - \tilde{q}_{x+t}(r)) + \tilde{\mathbb{E}} \left[ \int_0^\tau e^{-rs} F_s q_{x+t}(s) ds \right]$

(page 163)  $N_{lw}(t, F) = \frac{w}{r} \tilde{\mathbb{E}}[e^{-r\tau} \bar{Q}_{x+t}(\tau)] - \frac{w}{r} \tilde{\mathbb{E}} \left[ \int_\tau^\infty e^{-ru} q_{x+t}(u) du \right]$   
 $+ m_w \tilde{\mathbb{E}} \left[ \int_0^\tau e^{-ru} \bar{Q}_{x+t}(u) F_u du \right]$

(page 166) Investment Income:  $I[t] = A[t] \times \left( H[t] + U[t] - \frac{1}{2} L[t] \right)$

(page 167) Credited to Policyholder Account:  $J[t] = C[t] \times \left( Q[t] - \frac{1}{2} \times L[t] \right)$

(page 167) Risk Charges:  $K[t] = B[t] \times \left( Q[t] - \frac{1}{2} \times L[t] \right)$

(page 167) Mortality:  $L[t] = Q[t] \times F[t]$

(page 167) Lapses:  $M[t] = S[t] \times D[t]$

(page 167) Surrender Charge:  $N[t] = M[t] \times G[t]$

(page 167) Annuitization:  $O[t] = R[t] \times E[t]$

(page 167) Current Inforce (as % of initial):  $P[t] = P[t-1] \times (1 - D[t] - E[t] - F[t])$

(page 168) Policyholder Fund Value (BOY):  $Q[t] = T[t-1] + H[t]$

(page 168) Policyholder Fund Value before Lapses & Annuitizations (EOY):  $R[t] = Q[t] + J[t] - L[t]$

(page 168) Policyholder Fund Value before Lapses & after Annuitizations (EOY):  $S[t] = R[t] - O[t]$

(page 168) Policyholder Fund Value after Lapses & Annuitizations (EOY):  $T[t] = S[t] - M[t]$

(page 168) Statutory Reserve (BOY):  $U[t] = V[t-1]$

(page 168) Statutory Reserve (EOY):  $V[t] = T[t]$

(page 168) GMDB Benefit (5% - roll-up rate):  $W[t] = P[t-1] \times \max(Q[t], H[1] \times (1+5\%)^t)$

(page 168) GMDB Benefits:  $X[t] = (W[t] - U[t])_+ \times F[t]$

(page 168) Policy Fee Income (30 - annual policy fee:  $AG[t] = 30 \times AD[t-1]$

(page 168) Total Revenues:  $AH[t] = AE[t] + AF[t] + AG[t]$

(page 168) Premium-Based Administrative Expenses:  $AO[t] = AE[t] \times AJ[t]$

- (page 168) Per Policy Administrative Expenses (2% – inflation rate):  
 $AP[t] = AK[t] \times AM[t] \times (1 + 2\%)^{t-1}$
- (page 169) Commissions:  $AQ[t] = AE[t] \times AL[t]$
- (page 169) GMDB Cost (0.4% of account value – GMDDB cost):  $AR[t] = T[t] \times 0.4\%$
- (page 169) Total Expenses:  $AS[t] = AO[t] + AP[t] + AQ[t] + AR[t]$
- (page 169) Death Claims:  $AT[t] = L[t]$
- (page 169) Annuitization:  $AU[t] = O[t]$
- (page 169) Surrender Benefit:  $AV[t] = M[t] - N[t]$
- (page 169) Increase in Reserve:  $AW[t] = V[t] - V[t - 1]$
- (page 170) GMDDB Benefit:  $AX[t] = (W[t] - U[t])_+ \times F[t]$
- (page 170) Total Benefits:  $AY[t] = AT[t] + AU[t] + AV[t] + AW[t] + AX[t]$
- (page 170) Book Profit Before Tax:  $AZ[t] = AH[t] - AS[t] - AY[t]$
- (page 171) Taxes on Book Profit (37% – federal income tax rate):  $BF[t] = BE[t] \times 37\%$
- (page 171) Book Profits after Tax:  $BD[t] = BE[t] - BF[t]$
- (page 171) Target Surplus (BOY):  $BI[t] = BJ[t - 1]$
- (page 171) Target Surplus (EOY)(0.85% – target surplus rate):  $BI[t] = V[t] \times 0.85\%$
- (page 171) Increase in Target Surplus:  $BG[t] = BI[t] - BH[t]$
- (page 172) Interest on Target Surplus (5% – interest rate on surplus):  $BK[t] = BH[t] \times 5\%$
- (page 172) Taxes on Interest on Target Surplus:  $BL[t] = BK[t] \times 37\%$
- (page 172) After Tax Interest on Target Surplus:  $BJ[t] = BK[t] - BL[t]$
- (page 172) Distributable Earnings:  $BM[t] = BD[t] + BJ[t] - BG[t]$

$$(4.78) \quad \mathfrak{B} = \sum_{k=1}^{nT} \left(1 + \frac{r}{n}\right)^k P_{k/n}$$

$$(4.79) \quad \mathfrak{B} = \int_0^T e^{-rt} P_t dt = \int_0^T e^{-rt} k_t \mu_{x+t}^l {}_t p_x F_t dt + m \int_0^T e^{-rt} {}_t p_x F_t dt \\ - \int_0^T e^{-rt} E_t dt - \int_0^T e^{-rt} \mu_{x+t}^d {}_t p_x (G_t - F_t)_+ dt$$

## Chapter 6

$$(page 264) \quad B_e(t, F_t) = {}_T p_x \times \left[ G e^{-r(T-t)} \Phi \left( -d_2 \left( T-t, \frac{F_t}{G} \right) \right) - F_t e^{-m(T-t)} \Phi \left( -d_1 \left( T-t, \frac{F_t}{G} \right) \right) \right]$$

$$(6.16) \quad c(t, s) = - \left( \frac{\partial}{\partial t} + r s \frac{\partial}{\partial s} + \frac{1}{2} \sigma^2 s^2 \frac{\partial^2}{\partial s^2} - r \right) f(t, s) = m {}_t p_x F(t, s)$$

(page 269) 
$$\Delta_t = \frac{\partial}{\partial S} f(t, S_t) = -\frac{F_t}{S_t} \left[ {}_T p_x e^{-m(T-t)} \Phi \left( -d_1(T-t, \frac{F_t}{G}) \right) + m \int_t^T e^{-m(s-t)} {}_s p_x ds \right]$$

(page 275 Greeks)

$$\Delta_t = -{}_T p_x \frac{F}{S} e^{-m(T-t)} \Phi \left( -d_1(T-t, \frac{F}{G}) \right) - \frac{P_e}{S}$$

$$\Gamma_t = {}_T p_x \frac{F}{S^2} e^{-m(T-t)} \frac{\phi \left( d_1(T-t, \frac{F}{G}) \right)}{\sigma \sqrt{T-t}}$$

$$\Theta_t = {}_T p_x \left[ r G e^{-r(T-t)} \Phi \left( -d_2(T-t, \frac{F}{G}) \right) - m F e^{-m(T-t)} \Phi \left( -d_1(T-t, \frac{F}{G}) \right) - e^{-m(T-t)} \frac{\sigma F \phi \left( d_1(T-t, \frac{F}{G}) \right)}{2\sqrt{T-t}} \right] + m {}_t p_x F \quad (\text{This is a correction to the text formula})$$

$$\mathcal{V}_t = {}_T p_x F e^{-m(T-t)} \phi \left( d_1(T-t, \frac{F}{G}) \right) \sqrt{T-t}$$

### QFIQ-135-22: Structured Product Based Variable Annuities, Deng, et. al.

$$(1) \quad PV = I_0 e^{-r(t_n - t_0)} \mathbb{E} \left[ \prod_{i=1}^n (1 + R_i) \right] = I_0 e^{-r(t_n - t_0)} \prod_{i=1}^n \mathbb{E}[1 + R_i]$$

$$(2) \quad \mathbb{E}[1 + R_i] = 1 - \left( \frac{FV_{\text{Put}}(S_0, K_b, \tau, r, q, \sigma)}{S_0} \right) + \left( \frac{FV_{\text{Call}}(S_0, K = S_0, \tau, r, q, \sigma)}{S_0} \right) - \left( \frac{FV_{\text{Call}}(S_0, K_c, \tau, r, q, \sigma)}{S_0} \right)$$

$$(3) \quad K_b = S_0(1 - \text{Buffer})$$

$$(4) \quad K_c = S_0(1 + \text{Cap})$$

$$(5) \quad \mathbb{E}[1 + R_i] = 1 + \left( \frac{FV_{\text{Call}}(S_0, K = S_0, \tau, r, q, \sigma)}{S_0} \right) - \left( \frac{FV_{\text{Call}}(S_0, K_c, \tau, r, q, \sigma)}{S_0} \right)$$

$$(6) \quad \mathbb{E}[1 + R_i] = 1 + \left( \frac{FV_{\text{Binary Call}}(S_0, K = S_0, \tau, r, q, \sigma)}{S_0} \right)$$

$$(7) \quad FV_{\text{Call}}(S_0, K, \tau, r, q, \sigma) = FN(d_1) - KN(d_2)$$

$$(8) \quad FV_{\text{Put}}(S_0, K, \tau, r, q, \sigma) = KN(-d_2) - FN(-d_1)$$

$$(9) \quad FV_{\text{Binary Call}}(S_0, K, \tau, r, q, \sigma) = KN(d_2)$$

$$(10) \quad d_1 = \frac{\ln \left( \frac{F}{K} \right) + \frac{\sigma^2 \tau}{2}}{\sqrt{\sigma^2 \tau}} \quad d_2 = d_1 - \sqrt{\sigma^2 \tau} \quad F = e^{(r-q)\tau}$$