Advanced Portfolio Management Formula Package
February 2013

The exam committee felt that by providing many key formulas, candidates would be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorizing formulas. The formula package was developed sequentially by reviewing the syllabus material. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.** In general, formulas not in the package are either relatively fundamental or uncomplicated, or can be derived from formulas that are in the package.

Candidates should carefully observe the subtle differences in similar formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply to a specific situation in the exam question.

Candidates will note that the formula package provides minimal information about where the formula occurs in the syllabus, and does not provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent mastering the learning objectives and learning outcomes provided as part of the syllabus.

Fabozzi, Handbook of Fixed Income Securities

Treasury bills' rate:

$$Y_d = \frac{(F-P)}{F} \times \frac{360}{t}$$

where

Y_d = the rate on a discount basis,
F = the face value,
P = the price,
t = the number of days to maturity.

$$TIPS\ realized\ nominal\ yield = (1 + real\ yield) * (1 + inflation) - 1$$

$$Break\text{-}even\ \ inflation\ rate = \frac{1 + conventional\ \ nominal\ \ yield}{1 + TIPS\ \ real\ yield} - 1$$

Fixed income ETF premium/discount=(creation cost x *flow factor)*
+ execution risk adjustment

inverse floater: $K - L * (reference\ rate)$

Prepayment conventions:

$$\text{SMM} = 100 \times \frac{(\text{Scheduled Balance} - \text{Actual Balance})}{\text{Schedule Balance}}$$

$$\text{CPR} = 100 \times \left[1 - \left(1 - \frac{\text{SMM}}{100}\right)^{12}\right]$$

$$\text{PSA} = 100 \times \frac{\text{CPR}}{\text{minimum (age, 30)} \times 0.2}$$

Approximation of the relative change in value of the portfolio V:

$$\frac{dV(y)}{V(y)} \approx -MD[V(y)]dy + \frac{1}{2} RC[V(y)](dy)^2$$

where: MD = *modified duration*, and
RC = *convexity*

Duration Hedging:

$$\phi = -\frac{N_V \$dur\left[V\left(y\right)\right]}{N_H \$dur\left[H\left(y_H\right)\right]} = -\frac{N_V V\left(y\right) MD\left[V\left(y\right)\right]}{N_H H\left(y_H\right) MD\left[H\left(y_H\right)\right]}$$

Duration/Convexity Hedging:

$$\begin{cases} \phi_1 N_{H_1} H_1\left(y_1\right) MD\left[H_1\left(y_1\right)\right] + \phi_2 N_{H_2} H_2\left(y_2\right) MD\left[H_2\left(y_2\right)\right] = -N_V V\left(y\right) MD\left[V\left(y\right)\right] \\ \phi_1 N_{H_1} H_1\left(y_1\right) RC\left[H_1\left(y_1\right)\right] + \phi_2 N_{H_2} H_2\left(y_2\right) RC\left[H_2\left(y_2\right)\right] = -N_V V\left(y\right) RC\left[V\left(y\right)\right] \end{cases}$$

Nelson-Siegel / Svensson Yield-Curve Models:

$$R^C(0,\theta) = \beta_0 + \beta_1\left[\frac{1-\exp(-\theta/\tau_1)}{\theta/\tau_1}\right] + \beta_2\left[\frac{1-\exp(-\theta/\tau_1)}{\theta/\tau_1} - \exp(-\theta/\tau_1)\right] + \beta_3\left[\frac{1-\exp(-\theta/\tau_2)}{\theta/\tau_2} - \exp(-\theta/\tau_2)\right]$$

where

$R^C(0,\ \theta)$ = continuously compounded zero-coupon rate at time zero with maturity θ

β_0 = limit of $R^C(0,\ \theta)$ as θ goes to infinity (In practice, β_0 should be regarded as a long-term interest-rate.)

β_1 = limit of $R^C(0,\ \theta) - \beta_0$ as θ goes to 0 (In practice, β_1 should be regarded as the short- to long-term spread.)

β_2, β_3 = curvature parameters

τ_1 and τ_2 are scale parameters that measure the rate at which the short- and medium-term components decay to zero.

Babbel and Fabozzi , Investment Management for Insurers

$$D_S = (D_A - D_L)\frac{A}{S} + D_L$$

where D_S : duration of economic surplus

D_A : duration of assets

D_L : duration of liabilities

A : market value of assets

S : economic surplus = $A - L$ where L present value of liabilities

$$h = \frac{\Delta V - \Delta S}{\Delta F}$$

$$h = \frac{-\Delta S}{\Delta F} = \frac{-\beta_S}{\beta_F}$$

probability density of stock price changing from S_0 to S_f in time T assuming log normal distribution

$$dS_f P_T\left(S_f / S_n\right) = \frac{dS_f}{S_f\sqrt{2\pi\sigma^2 T}} \exp\left(-\frac{\left(\ln\left(\frac{S_f}{S_0}\right) - \mu T\right)^2}{2\sigma^2 T} \right)$$

where

S_0 : initial stock price S_f : value of stock on the expiry day of the option

C_0 : initial call price C_f : value of call on the expiry day of the option

W_0 : initial investment W_f : value of investment on the expiry day of the option

D : dividend received over the time period of the option

T : time to expiration

σ^2 : variance of log of stock price return ($\ln(S_f / S_0)$)

μ : mean per unit time of stock log price return

expected return from an investment in the combination of stock and option

$$\int dS_f P_T\left(\frac{S_f}{S_0}\right) \ln\left(\frac{W_f\left(S_f\right)}{W_0}\right)$$

for covered call position:

$$W_0 = S_0 - C_0$$

$$W_f = S_f - \max\left[0, S_f - E\right] + D$$

Litterman, Modern Investment Management

$$R_{L,t} - R_{f,t} = \beta(R_{B,t} - R_{f,t}) + \varepsilon_t$$

where $R_{L,t}$: total return on liability index at time t

$R_{f,t}$: risk-free rate of return

$R_{B,t}$: total return on a bond index

ε_t : noise term

$$SR_i = \frac{\mu_i - R_f}{\sigma_i}$$

$$RACS_t = \frac{E_t\left[S_{t+1} - S_t(1+R_f)\right]}{\sigma_t\left[S_{t+1}\right]}$$

$$RACS_t = \frac{E_t\left[A_t(1+R_{A,t+1}) - L_t(1+R_{L,t+1}) - (A_t - L_t)(1+R_f)\right]}{\sigma_t\left[A_t(1+R_{A,t+1}) - L_t(1+R_{L,t+1})\right]}$$

$$RACS_t = \frac{E_t\left[A_t(R_{A,t+1} - R_f)\right]}{\sigma_t\left[A_t(1+R_{A,t+1})\right]} = \frac{E_t\left[R_{A,t+1}\right] - R_f}{\sigma_t\left[R_{A,t+1}\right]}$$

$$E_t\left[F_{t+1}\right] = F_t E_t\left[\frac{1+R_{A,t+1}}{1+R_{L,t+a}}\right]\frac{1}{1-p} - \frac{p}{1-p}$$

$$E_0\left[F_t\right] = \left[\frac{1+E\left[R_x\right]}{1-p}\right]^t F_0 + p\frac{1-\left[\frac{1+E\left[R_x\right]}{1-p}\right]^t}{E\left[R_x\right]+p}$$

Maginn & Tuttle, Managing Investment Portfolios, A Dynamic Process

$$IR \approx IC\sqrt{Breadth}$$

where $IR =$ information ratio, $IC =$ information coefficient, $Breadth =$ investment discipline's breadth (# of independent active investment decision made each year)

$$\underset{by\,choice\,of\,managers}{maximize}\; U_A = r_A - \lambda_A \sigma_A^2$$

where $U_A =$ expected utility of active return of the manager mix

$r_A =$ expected return of the manager mix

$\lambda_A =$ the investor's trade-off between active risk and active return, measure risk aversion in active risk terms

$\sigma_A^2 =$ variance of the active return

$$portfolio\,active\,return = \sum_{i=1}^{n} h_{A_i} r_{A_i}$$

where h_{A_i} : weight assigned to the ith manager

r_{A_i} : active return of the ith manager

$$portfolio\,active\,risk = \sqrt{\sum_{i=1}^{n} h_{A_i}^2 \sigma_{A_i}^2}$$

where σ_{A_i} : active risk of the ith manager

$$manager's\ total\ active\ risk = \left[\left(manager's\ "true"\ active\ risk \right)^2 + \left(manager's\ "misfit"\ active\ risk \right)^2 \right]^{1/2}$$

$$total\ return\ on\ commodity\ index = collateral\ return + roll\ return + spot\ return$$

$$rate\ of\ return = \frac{\left[\left(ending\ value\ of\ portfolio \right) - \left(beginning\ value\ of\ portfolio \right) \right]}{\left(beginning\ value\ of\ portfolio \right)}$$

$$RR_{n,t} = \frac{\left(R_t + R_{t-1} + R_{t-2} + \ldots + R_{t-n} \right)}{n} \quad \text{where } RR_{n,t} = \text{rolling return}$$

$$downside\ deviation = \sqrt{\frac{\sum_{i=1}^{n} \left[\min\left(r_t - r^*, 0 \right) \right]^2}{n-1}}$$

where $r^* =$ specified return

$$sharpe\ ratio = \frac{(\text{annualized rate of return} - \text{annualized risk-free rate})}{\text{annualized standard deviation}}$$

$$gain-to-loss\ ratio = \left(\frac{number\ months\ with\ positive\ returns}{number\ months\ with\ negative\ returns} \right) * \left(\frac{average\ up-month\ return}{average\ down-month\ return} \right)$$

external cash flow at the beginning of the period

$$r_t = \frac{MV_1 - (MV_0 + CF)}{MV_0 + CF}$$

external cash flow at the end of period

$$r_t = \frac{(MV_1 - CF) - MV_0}{MV_0}$$

$$MV_1 = MV_0(1+R)^m + CF_1(1+R)^{m-L(1)} + \ldots + CF_n(1+R)^{m-L(n)}$$

where m : number of time units in the evaluation period, CF_i : ith cash flow $L(i)$: number of time units by which the ith cash flow is separated from the beginning of the evaluation period

$$R_p = a_p + \beta_p R_I + \varepsilon_p$$

$$r_V = \sum_{i=1}^{n} \left[w_{Vi} r_i \right] = \sum_{i=1}^{n} \left[(w_{pi} - w_{Bi}) r_i \right] = \sum_{i=1}^{n} w_{pi} r_i - \sum_{i=1}^{n} w_{Bi} r_i = r_p - r_B$$

where r_V : value-added return

r_P : portfolio return

r_B : benchmark return

$$r_{AC} = \sum_{i=1}^{A} w_i (r_{Ci} - r_f)$$

$$r_{IS} = \sum_{i=1}^{A} \sum_{j=1}^{M} w_i w_{ij} (r_{Bij} - r_{Ci})$$

$$r_{IM} = \sum_{i=1}^{A} \sum_{j=1}^{M} w_i w_{ij} (r_{Aij} - r_{Bij})$$

$$r_V = \sum_{i=1}^{n} \left[(w_{pi} - w_{Bi})(r_i - r_B) \right]$$

$$r_V = \sum_{j=1}^{S} (w_{pj} - w_{Bj})(r_{Bj} - r_B) + \sum_{j=1}^{S} (w_{pj} - w_{Bj})(r_{pj} - r_{Bj}) + \sum_{j=1}^{S} w_{Bj} (r_{pj} - r_{Bj})$$

where $\sum_{j=1}^{S} (W_{Pj} - W_{Bj})(r_{Bj} - r_B)$: pure sector allocation

$\sum_{j=1}^{S} (W_{Pj} - W_{Bj})(r_{Pj} - r_{Bj})$: allocation / selection interaction

$\sum_{j=1}^{S} W_{Bj} (r_{Pj} - r_{Bj})$: within-sector selection

$$R_{At} - r_{ft} = \alpha_A + \beta_A (R_{Mt} - r_{ft}) + \varepsilon_t$$

$$T_A = \frac{\overline{R}_A - \overline{r}_f}{\hat{\beta}_A}$$

$$S_A = \frac{\overline{R}_A - \overline{r}_f}{\hat{\sigma}_A}$$

$$M_A^2 = \overline{r}_f + \left[\frac{\overline{R}_A - \overline{r}_f}{\hat{\sigma}_A} \right] \hat{\sigma}_M$$

$$IR_A = \frac{\overline{R}_A - \overline{R}_B}{\hat{\sigma}_{A-B}}$$

where $\hat{\sigma}_{A-B}$: standard deviation of the difference between the return on the account and the return on the benchmark

V-C111-07

None

V-C119-07

None

V-C120-07

$$r = \frac{D}{P} + g$$

where r : rate of return

$\frac{D}{P}$: (expected) dividend yield

g : long-term growth rate

V-C122-07

None

V-C126-09 or FET-127-07 or 8V-114-00

None

V-C127-09 or FET-124-07 or 8V-323-05

$$L_0 R_{S(L)} = A_0 R_A - L_0 R_L$$

where L_0 : current liabilities

$R_{S(L)}$: liability-relative return of the surplus

A_0 : current asset

R_A : return on assets

R_L : return on liability

$$R_{S(L)} = \left(\frac{A_0}{L_0} R_A \right) - R_L$$

$$R_A = R_f + \beta_A r_Q + \alpha$$

where R_A : asset portfolio return

R_f : risk-free rate of return

r_Q : excess return of the total investable market (portfolio Q) over cash

$$\sigma_A^2 = \beta_A \sigma_Q^2 + \omega_A^2$$

where σ_A^2 : variance of the asset portfolio

σ_Q^2 : variance of the market risk premium on the relevant benchmark

ω_A^2 : variance of the alpha

$$\max(U_S) = R_S - \lambda \sigma_S^2$$

where U_S : surplus utility

λ : a constant representing the degree of risk aversion

$$\max(U_S) = \left(\frac{A_0}{L_0} - 1\right) R_F + \beta_S \mu_Q - \lambda_\beta \beta_S^2 \sigma_Q^2 + \left(\frac{A_0}{L_0}\alpha_A - \alpha_L\right) - \lambda_\omega \left[\left(\frac{A_0}{L_0}\right)^2 \omega_A^2 - 2\frac{A_0}{L_0}\omega_A \omega_L + \omega_L^2\right]$$

where μ_Q: the equilibrium or consensus, expected return of the total market across all asset classes

$\beta_S = (\frac{A}{L}\beta_A - \beta_L)$, surplus beta, the weighted relative betas of the assets and liabilities

ω : the standard deviation of the alphas, subscripted to indicate the assets and the liabilities, residual risk

$$P_{TIPS} = \frac{F}{(1+r)^T}$$

where P_{TIPS} : the price of TIPS bond

F : face value of bond

i : inflation rate

r : real interest rate

T : time

$$P_{EQUITY} = \sum_{t=0}^{\infty} \frac{Dvd_0 (1+g_r)^t}{(1+r)^t}$$

Where Dvd_0 : beginning dividend

g : growth rate of dividends

V-C135-08

None

V-C136-09 or FET-128-07 or 6-31-00

None

V-C138-09 or FET-126-07 or 8V-120-03

None

V-C140-09 or FET-115-07

$$E[D] = \sum_n (DthBen - actuarialreserve)_t * {}_t p_x * q_{x+t} * v_t$$

$$E[D^2] = \sum_n [(DthBen - actuarialreserve)_t * v^t]^2 *_t p_x * q_{x+t}$$

$$Var[D] = E[D^2] - (E[D])^2$$

V-C143-09

None

V-C144-09

None

V-C146-09

None

V-C148-09

investor's utility function $U(W) = \left[\frac{1}{(1-A)}\right] W^{(1-A)}$

where $A =$ coefficient of relative risk aversion

$W =$ investor's wealth

arithmetic equity premium $EP \approx A(\sigma^2)$

where $\sigma =$ standard deviation of return on investor's portfolio

V-C150-09

$$S_p = \frac{\overline{R}_p - \overline{R}_f}{\sigma_p}$$

where $S_p =$ Sharpe ratio for a portfolio

$\overline{R}_p =$ mean return on the portfolio

$\overline{R}_f =$ mean return on the U.S. T-bill (proxy for risk-free rate of interest)

$\sigma_p =$ sample standard deviation of returns

approximate Sharpe ratio for multi-period investment horizon

$$S_n = \frac{(1+R_1)^n - (1+R_f)^n}{\left\{\left[\sigma_1^2 + (1+R_1)^2\right]^n - (1+R_1)^{2n}\right\}^{1/2}}$$

where R_1 *and* σ_1 are one period expected return and standard deviation

$R_n = (1+R_1)^n - 1$ n-period expected return

$\sigma_n = \left\{\left[\sigma_1^2 + (1+R_1)^2\right]^n - (1+R_1)^{2n}\right\}^{1/2}$ n-period standard deviation

$HPR_n = \prod_{i=1}^{n}(1+R_i)$ n-year holding period return

V-C154-09

None

V-C164-09

$$x_S + x_L = 1$$

$$x_S D_S + x_L D_L = D_B$$

$$\overline{r} = \sum_{s=1}^{4} p_i r_i$$

$$\sigma^2 = \sum_{s=1}^{4} p_i (r_i - \overline{r})^2$$

$$p_i^{PERFECT} = 1/n_W \quad \text{if } i \text{ is correct decision}$$

$$0 \quad \text{otherwise}$$

where n_W are correct decisions among n choices, $n_L = n - n_W$ are incorrect decisions

$$p_i(s) = (1-s)\,p_i^{RANDOM} + sp_i^{PERFECT} = \frac{(n_W + sn_L)}{n_W(n_W + n_L)} \quad \text{if } i \text{ is correct decision}$$

$$\frac{(1-s)}{(n_W + n_L)} \quad \text{otherwise}$$

$$r = \sum_j w_j r_j$$

where w_j percentage market capitalization of the index in cell j

r_j strategy outperformance of the index within cell j

$$\bar{r} = \sum_j w_j \bar{r}_j$$

$$\sigma^2 = \sum_j w_j^2 \sigma_j^2$$

$$r = \frac{1}{n}\sum_{i=1}^{n} r_i$$

where r_i outperformance due to decision i

r overall portfolio outperformance

$$\mu_{strategy} = \mu_{decision} \qquad \sigma_{strategy} = \sigma_{decision}/\sqrt{n}$$

$$R_{S,b} = bR_S + (1-b)R_B = R_B + b(R_S - R_B)$$

where R_B benchmark performance R_S strategy performance

b portion of portfolio assets is committed to strategy

$$\mu_{S,b} = E(R_{S,b} - R_B) = E(b(R_S - R_B)) = bE(R_S - R_B) = b\mu_s$$

$$\sigma_{S,b}^2 = Var(R_{S,b} - R_B) = Var(b(R_S - R_B)) = b^2 Var(R_S - R_B) = b^2\sigma_S^2$$

strategy information ratio $IR_S = \dfrac{\mu_S}{\sigma_S}$

$$IR_{S,b} = \frac{\mu_{S,b}}{\sigma_{S,b}} = \frac{b\mu_S}{b\sigma_S} = \frac{\mu_S}{\sigma_S} = IR_S$$

$$E(y) = E(E(y|x))$$

$$Var(y) = Var(E(y|x)) + E(Var(y|x))$$

V-C165-09
None

V-C168-09

$$Total\ return = income\ return + price\ return + currency\ return$$

$$duration\ return = roll\ down + shift + twist + shape\ return$$

V-C169-09

total return $TR = (1+TR_t)(1+TR_{t+1})...(1+TR_n) - 1$

$$TR_f = \sum_{f=1}^{F} RF_{f,t}$$

where $RF_{f,t}$ = fth attribution effect obtained at time t

V-C171-09
None

V-C172-09
None

V-C174-09
None

V-C179-10
None

V-C180-10

$$\Pr ob(H) + \Pr ob(H^C) = 1$$

$$\sum_{k=1}^{\infty} 2^{-k} \log(2^k) = 2\log 2 \approx 4$$

V-C182-10
None

V- C183-10

Equation 1: CDS Spread as a Function of Default Probability (PD) and Recovery Rate (R)
$S = PD \times (1 - R)$

Equation 2: CDS Pricing Equation – From upfront plus running to full running, using the CDS risky annuity (RA) and accrued interest (AI)

$$Full\ Running = \frac{Upfront - AI}{RA} + Fixed\ Coupon$$

Equation 3: Par Asset Swap Spread Calculation

$$\text{Asset swap spread} = \frac{PV\left[\text{Coupon} + \text{Principal}\right] - \text{Bond Price}}{\text{Risk free annuity}}$$

Equation 4: Basis Trade Profit on Default

$$\begin{aligned}&\text{CDS Notional x (100} - \text{Recovery} - \text{CDS Upfront} - \text{CDS Coupons Paid} - \text{CDS Funding Costs Paid)}\\&+ \text{Bond Notional x (Recovery} + \text{Bond Coupons Received} - \text{Bond Price} - \text{Bond Funding Costs Paid)}\end{aligned}$$

Note: Bond Price refers to the dirty bond price.

Equation 5: Basis Trade Profit on Maturity

$$\begin{aligned}&\text{Bond Notional x (100} + \text{Bond Coupons Received} - \text{Bond Price} - \text{Bond Funding Costs Paid)}\\&- \text{CDS Notional x (CDS Upfront} + \text{CDS Coupons Paid} + \text{CDS Funding Costs Paid)}\end{aligned}$$

Note: Bond Price refers to the dirty bond price.

Equation 6: Basis Trade Profit on Default splitting the trade cash flows into running and one-off payments

$$\begin{aligned}&\underline{\text{From one - off payments:}}\\&\text{CDS Notional } x \text{ (100} - \text{Recovery} - \text{CDS Upfront)} + \text{Bond Notional } x \text{ (Recovery} - \text{Bond Price)}\\&\underline{\text{From running payments:}}\\&\text{Bond Notional } x \text{ (Bond Coupons Received} - \text{Bond Funding Costs Paid)}\\&- \text{CDS Notional } x \text{ (CDS Coupons Paid} + \text{CDS Funding Costs Paid)}\end{aligned}$$

Equation 7: CDS Notional in a “Capital-at-Risk” Basis Trade

$$\text{CDS Notional} = \frac{\text{Bond Price} - \text{Recovery}}{100 - \text{Recovery} - \text{CDS Upfront}}\, x \text{ Bond Notional}$$

Equation 8: Equal Notional Basis Trade Profit on Default or Maturity (Ignoring risk-free discounting and funding costs)

$$(100 - \text{Bond Price} + \text{Bond Coupons Received} - \text{CDS Upfront} - \text{CDS Coupons Paid})$$

Note: Bond Price refers to the dirty bond price.

V-C184-11

None

V-C185-11

$$ABO = \frac{mGW_0}{(1+n)^T}\left[\frac{1}{n} - \frac{1}{n(1+n)^Z}\right]$$

where:

m is the multiplier
G is the number of years already worked
W_0 **is the current annual wage**
n is the annual discount rate
T is the expected number of years until retirement
Z is the remaining expected lifetime of the retiree

$$PBO = \frac{mGW_0(1+w)^T}{(1+n)^T}\left[\frac{1}{n} - \frac{1}{n(1+n)^Z}\right]$$

where:

m is the multiplier
G is the number of years already worked
W_0 **is the current annual wage**
n is the annual discount rate
T is the expected number of years until retirement
Z is the remaining expected lifetime of the retiree
W is the assumed annual growth rate in wages over the T years until retirement

$$NP = \frac{MV\ (\text{Target Duration} - \text{Portfolio Duration})}{\text{Swap Duration}}$$

Where NP is the notional principal
MV is the market value

V-C186-11

None

V-C187-11

None

V-C188-11

None

V-C189-11

$$\text{Actual} - \text{Projected Price Change} = \Delta P - \Delta \hat{P}$$

$$\cong P\left[-D_s\Delta s - D_v\Delta v - D_c\Delta c + \frac{1}{2}C_y\Delta y^2 - \sum D_{y_j}(\Delta y_j - \Delta y)\right]$$

V-C190-11

1.2.1 Proposition *Under the assumption that the severity and the default event D are uncorrelated, the unexpected loss of a loan is given by*

$$UL = EAD \times \sqrt{\mathbb{V}[SEV] \times DP + LGD^2 \times DP(1-DP)}\ .$$

$$\text{EL}_{PF} = \sum_{i=1}^{m} \text{EL}_i = \sum_{i=1}^{m} \text{EAD}_i \times \text{LGD}_i \times \text{DP}_i\ . \qquad (1.\ 6)$$

1.2.3 Proposition *For a portfolio with constant severities we have*

$$UL_{PF}^2 = \sum_{i,j=1}^{m} EAD_i \times EAD_j \times LGD_i \times LGD_j \times$$
$$\times \sqrt{DP_i(1-DP_i)DP_j(1-DP_j)}\,\rho_{ij}$$

where $\rho_{ij} = Corr[L_i, L_j] = Corr[\mathbf{1}_{D_i}, \mathbf{1}_{D_j}]$ *denotes the* default correlation *between counterparties* i *and* j.

V-C191-11

$$\text{DV01} \equiv -\frac{\Delta P}{10,000 \times \Delta y} \qquad (5.1)$$

$$D \equiv -\frac{1}{P}\frac{\Delta P}{\Delta y} \qquad (5.10)$$

$$C = \frac{1}{P}\frac{d^2 P}{dy^2} \qquad (5.14)$$

V-C192-11

None

CIA Educational Note: Liquidity Risk Measurement

None

Byrne & Brooks, "Behavioral Finance: Theory and Evidence"

None

Chapter 3 of Active Credit Portfolio Management in Practice, Structure Model

$$cpd_t = 1 - (1 - pd)^t$$

$$D = e^{-r_f T} D^* - CDS$$

$$E + e^{-r_f T} D^* = CDS + A$$

$$DD = \frac{\ln\left(\frac{A}{X}\right) + \left(\mu_A - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A \sqrt{T}}$$

$$dA = \mu_A A dt + \sigma_A A\, dz$$

$$\frac{1}{2}\frac{\partial^2 E}{\partial A^2}\sigma_A^2 A^2 + \frac{\partial E}{\partial A} rA + \frac{\partial E}{\partial t} - rE = 0$$

$$\sigma_E = \frac{\partial E}{\partial A}\ \frac{A}{E}\sigma_A$$

$$PD = \Pr(A \le X) = \Phi(-DD)$$

$$E = A_0\Phi\left(d_1\right) - Xe^{-rt}\Phi\left(d_2\right)$$

$$d_1 = \frac{\ln A_0 + \left(r + \frac{1}{2}\sigma_A^2\right)T - \ln X}{\sigma_A\sqrt{T}}$$

$$d_2 = \frac{\ln A_0 + \left(r - \frac{1}{2}\sigma_A^2\right)T - \ln X}{\sigma_A\sqrt{T}} = d_1 - \sigma_A\sqrt{T}$$

$$cpd_T^Q = \Phi\left(\Phi^{-1}(cpd_T) + R\lambda\sqrt{T}\right)$$

$$D = e^{-rT}\left(1 - cpd_T^Q\ L\right)$$

$$\Pr(At \le K) = cpd_t$$

$$= 1 - [\Phi\left(\frac{\ln\left(\frac{A}{K}\right) + \left(\mu_A - a - \frac{1}{2}\sigma_A^2\right)t}{\sigma_A\sqrt{t}}\right)$$

$$-\left(\frac{A}{K}\right)^{\frac{2\left(\mu_A - a - \frac{1}{2}\sigma_A^2 - \gamma\right)}{\sigma_A^2}}\Phi\left(\frac{\ln\left(\frac{K}{A}\right) + \left(\mu_A - a - \frac{1}{2}\sigma_A^2\right)t}{\sigma_A\sqrt{t}}\right)$$

$$D = X\frac{e^{b\mu}}{e^{b\tilde{\mu}}}\left[\Phi\left(\frac{b - \tilde{\mu}T}{\sqrt{T}}\right) + e^{2\tilde{\mu}b}\Phi\left(\frac{b + \tilde{\mu}T}{\sqrt{T}}\right)\right]$$

where:

$$b = \frac{\ln\left(\frac{X}{A}\right) - \gamma T}{\sigma_A}; \quad \mu = \frac{r - a - \frac{1}{2}\sigma_A^2 - \gamma}{\sigma_A}; \quad \tilde{\mu} = \sqrt{\mu^2 + 2\alpha}$$

$$D^*(A) = \frac{c}{r} + \lambda\left(D(A) - \frac{c}{r}\right) \text{ where } \lambda = \left(\frac{X - \frac{c}{r}}{D(X) - \frac{c}{r}}\right)$$

$$B(T) = e^{a(T)+b(T)X(T)}$$

$$a(t) = \frac{2\kappa\mu_h}{\sigma_h^2}\ln\left(\frac{2\gamma e^{\frac{(\gamma+\kappa)T}{2}}}{2\gamma + (\gamma+\kappa)(e^{\gamma T} - 1)}\right);$$

$$\gamma = \sqrt{\kappa^2 + 2\sigma_h^2}; \quad b(t) = \frac{-2\left(e^{\gamma T} - 1\right)}{2\gamma + (\gamma+\kappa)(e^{\gamma T} - 1)}$$

$$D = \frac{C}{r}\left(1 - \left(\frac{A}{X}\right)^{-\gamma}\right) + (1-L)X\left(\frac{A}{S}\right)^{-\gamma}$$

$$\text{where } \gamma = \alpha + \xi; \quad \alpha = \frac{r - p - \frac{1}{2}\sigma_A^2}{\sigma_A^2};$$

$$\xi = \frac{\left(\left(r - p - \frac{1}{2}\sigma_A^2\right)^2 + 2\sigma_A^2 r\right)^{\frac{1}{2}}}{\sigma_A^2}$$

$$A_{LT} = A + \frac{\tau C}{r}\left[1 - \left(\frac{A}{X}\right)^{-\tau}\right] - LX\left(\frac{A}{X}\right)^{-\tau}$$

$$E = A_{LT} - D$$

$$cpd_T = \Phi\left(\frac{-\beta - \left(\mu_A - p - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right) + e^{\frac{2\beta\left(\mu_A - p - \frac{1}{2}\sigma_A^2\right)}{\sigma_A^2}}\Phi\left(\frac{-\beta + \left(\mu_A - p - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}\right)$$

$$R = \begin{cases} \left(\dfrac{1 - \dfrac{\mu}{\sqrt{\mu^2 + \sigma^2}}}{1 + \dfrac{\mu}{\sqrt{\mu^2 + \sigma^2}}} \right)^{\frac{L}{\sqrt{\mu^2 + \sigma^2}}} & \text{if } \dfrac{\mu}{\sqrt{\mu^2 + \sigma^2}} > 0 \\ 1 & \text{otherwise} \end{cases}$$

$$\pi(s-t, x-\underline{A}) = 1 - \left[\Phi\left(\frac{x - \underline{A} + m(s-t)}{\sqrt{s-t}} \right) - e^{-2m(x-\underline{A})} \Phi\left(\frac{-(x-\underline{A}) + m(s-t)}{\sqrt{s-t}} \right) \right]$$