Quantitative Finance and Investments Advanced Formula Sheet

Fall 2014/Spring 2015

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas, candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula sheet was developed sequentially by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

Changes from Spring 2014 are:

- Addition of formulas from Chapters 69 and 71 of Fabozzi
- Addition of a formula from Chapter 6 of Bluhm
- Deletion of Chapters 3 and 8 of Tsay

Interest Rate Models - Theory and Practice, Brigo and Mercurio

Chapter 3

Table 3.1 Summary of instantaneous short rate models

Model	Dynamics	$r > 0$	$r \sim$	AB	AO
V	$dr_t = k[\theta - r_t]dt + \sigma dW_t$	N	$\mathcal{N}$	Y	Y
CIR	$dr_t = k[\theta - r_t]dt + \sigma\sqrt{r_t}dW_t$	Y	NCχ^2	Y	Y
D	$dr_t = ar_t dt + \sigma r_t dW_t$	Y	L$\mathcal{N}$	Y	N
EV	$dr_t = r_t[\eta - a\ln r_t]dt + \sigma r_t dW_t$	Y	L$\mathcal{N}$	N	N
HW	$dr_t = k[\theta_t - r_t]dt + \sigma dW_t$	N	$\mathcal{N}$	Y	Y
BK	$dr_t = r_t[\eta_t - a\ln r_t]dt + \sigma r_t dW_t$	Y	L$\mathcal{N}$	N	N
MM	$dr_t = r_t\left[\eta_t - \left(\lambda - \frac{\gamma}{1+\gamma t}\right)\ln r_t\right]dt + \sigma r_t dW_t$	Y	L$\mathcal{N}$	N	N
CIR++	$r_t = x_t + \varphi_t, \quad dx_t = k[\theta - x_t]dt + \sigma\sqrt{x_t}dW_t$	Y*	SNCχ^2	Y	Y
EEV	$r_t = x_t + \varphi_t, \quad dx_t = x_t[\eta - a\ln x_t]dt + \sigma x_t dW_t$	Y*	SL$\mathcal{N}$	N	N

*rates are positive under suitable conditions for the deterministic function φ.

(3.5) $dr(t) = k[\theta - r(t)]dt + \sigma dW(t), \qquad r(0) = r_0$

(3.6) $r(t) = r(s)e^{-k(t-s)} + \theta\left(1 - e^{-k(t-s)}\right) + \sigma\int_s^t e^{-k(t-u)}dW(u)$

(3.7) $E\{r(t)|\mathcal{F}_s\} = r(s)e^{-k(t-s)} + \theta\left(1 - e^{-k(t-s)}\right)$

$$\mathrm{Var}\{r(t)|\mathcal{F}_s\} = \frac{\sigma^2}{2k}\left[1 - e^{-2k(t-s)}\right]$$

(3.8) $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

(3.9) $dr(t) = [k\theta - B(t,T)\sigma^2 - kr(t)]dt + \sigma dW^T(t)$

(3.11) $dr(t) = [k\theta - (k + \lambda\sigma)r(t)]dt + \sigma dW^0(t), \qquad r(0) = r_0$

(3.12) $dr(t) = [b - ar(t)]dt + \sigma dW^0(t)$

(3.13) $r(t) = r(s)e^{-a(t-s)} + \frac{b}{a}\left(1 - e^{-a(t-s)}\right) + \sigma\int_s^t e^{-a(t-u)}dW^0(u)$

(3.14) $\hat{\alpha} = \dfrac{n\sum_{i=1}^n r_i r_{i-1} - \sum_{i=1}^n r_i \sum_{i=1}^n r_{i-1}}{n\sum_{i=1}^n r_{i-1}^2 - \left(\sum_{i=1}^n r_{i-1}\right)^2}$

(3.15) $\hat{\beta} = \dfrac{\sum_{i=1}^n [r_i - \hat{\alpha}r_{i-1}]}{n(1-\hat{\alpha})}$

(3.16) $\widehat{V^2} = \dfrac{1}{n}\sum_{i=1}^n \left[r_i - \hat{\alpha}r_{i-1} - \hat{\beta}(1-\hat{\alpha})\right]^2$

(3.19) $E\{r(t)|\mathcal{F}_s\} = r(s)e^{a(t-s)}$ and $\mathrm{Var}\{r(t)|\mathcal{F}_s\} = r^2(s)e^{2a(t-s)}\left(e^{\sigma^2(t-s)} - 1\right)$

(3.20) $P(t,T) = \dfrac{\bar{r}^p}{\pi^2}\int_0^\infty \sin(2\sqrt{\bar{r}}\sinh y)\int_0^\infty f(z)\sin(yz)dzdy + \dfrac{2}{\Gamma(2p)}\bar{r}^p K_{2p}(2\sqrt{\bar{r}})$

(3.21) $dr(t) = k(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t), \qquad r(0) = r_0$

(3.22) $dr(t) = [k\theta - (k + \lambda\sigma)r(t)]dt + \sigma\sqrt{r(t)}dW^0(t), \qquad r(0) = r_0$

(3.23) $E\{r(t)|\mathcal{F}_s\} = r(s)e^{-k(t-s)} + \theta\left(1 - e^{-k(t-s)}\right)$

$$\mathrm{Var}\{r(t)|\mathcal{F}_s\} = r(s)\frac{\sigma^2}{k}\left(e^{-k(t-s)} - e^{-2k(t-s)}\right) + \theta\frac{\sigma^2}{2k}\left(1 - e^{-k(t-s)}\right)^2$$

(3.24) $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

(3.25) $A(t,T) = \left[\dfrac{2h\exp\{(k+h)(T-t)/2\}}{2h + (k+h)(\exp\{(T-t)h\} - 1)}\right]^{2k\theta/\sigma^2}$

$$B(t,T) = \frac{2(\exp\{(T-t)h\} - 1)}{2h + (k+h)(\exp\{(T-t)h\} - 1)}, \qquad h = \sqrt{k^2 + 2\sigma^2}$$

(3.27) $dr(t) = [k\theta - (k + B(t,T)\sigma^2)r(t)]dt + \sigma\sqrt{r(t)}dW^T(t)$

(3.28) $p^T_{r(t)|r(s)}(x) = p_{\chi^2(v,\delta(t,s))/q(t,s)}(x) = q(t,s)p_{\chi^2(v,\delta(t,s))}(q(t,s)x)$

$$q(t,s) = 2[\rho(t-s) + \psi + B(t,T)] \text{ and } \delta(t,s) = \frac{4\rho(t-s)^2 r(s)e^{h(t-s)}}{q(t,s)}$$

Page 68 $R(t,T) = \alpha(t,T) + \beta(t,T)r(t), \qquad P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

(3.29) $\sigma_f(t,T) = \dfrac{\partial B(t,T)}{\partial T}\sigma(t,r(t))$

Page 69 $dr(t) = b(t,r(t))dt + \sigma(t,r(t))dW(t)$

$$b(t,x) = \lambda(t)x + \eta(t), \qquad \sigma^2(t,x) = \gamma(t)x + \delta(t)$$

$$\frac{\partial}{\partial t}B(t,T) + \lambda(t)B(t,T) - \frac{1}{2}\gamma(t)B(t,T)^2 + 1 = 0, \qquad B(T,T) = 0$$

$$\frac{\partial}{\partial t}[\ln A(t,T)] - \eta(t)B(t,T) + \frac{1}{2}\delta(t)B(t,T)^2 = 0, \qquad A(T,T) = 1$$

Page 69/70 Vasicek $\lambda(t) = -k, \quad \eta(t) = k\theta, \quad \gamma(t) = 0, \quad \delta(t) = \sigma^2$

Page 70 CIR $\lambda(t) = -k, \quad \eta(t) = k\theta, \quad \gamma(t) = \sigma^2, \quad \delta(t) = 0$

$$b(x) = \lambda x + \eta, \qquad \sigma^2(x) = \gamma x + \delta$$

Page 71 $\displaystyle\lim_{t\to\infty} E\{r(t)|\mathcal{F}_s\} = \exp\left(\frac{\theta}{a} + \frac{\sigma^2}{4a}\right)$

(3.31) $\displaystyle\lim_{t\to\infty} \mathrm{Var}\{r(t)|\mathcal{F}_s\} = \exp\left(\frac{2\theta}{a} + \frac{\sigma^2}{2a}\right)\left[\exp\left(\frac{\sigma^2}{2a}\right) - 1\right]$

(3.32) $dr(t) = [\vartheta(t) - a(t)r(t)]dt + \sigma(t)dW(t)$

(3.33) $dr(t) = [\vartheta(t) - ar(t)]dt + \sigma dW(t)$

(3.34) $\vartheta(t) = \dfrac{\partial f^M(0,t)}{\partial T} + af^M(0,t) + \dfrac{\sigma^2}{2a}(1 - e^{-2at})$

(3.35) $r(t) = r(s)e^{-a(t-s)} + \int_s^t e^{-a(t-u)}\vartheta(u)du + \sigma\int_s^t e^{-a(t-u)}dW(u)$

$$= r(s)e^{-a(t-s)} + \alpha(t) - \alpha(s)e^{-a(t-s)} + \sigma\int_s^t e^{-a(t-u)}dW(u)$$

(3.36) where $\alpha(t) = f^M(0,t) + \dfrac{\sigma^2}{2a^2}(1 - e^{-at})^2$

(3.37) $E\{r(t)|\mathcal{F}_s\} = r(s)e^{-a(t-s)} + \alpha(t) - \alpha(s)e^{-a(t-s)}$

$$\mathrm{Var}\{r(t)|\mathcal{F}_s\} = \frac{\sigma^2}{2a}\left[1 - e^{-2a(t-s)}\right]$$

(3.38) $dx(t) = -ax(t)dt + \sigma dW(t), \qquad x(0) = 0$

Page 74 $x(t) = x(s)e^{-a(t-s)} + \sigma \int_s^t e^{-a(t-u)}dW(u)$

(3.47) $E\{x(t_{i+1})|x(t_i) = x_{i,j}\} = x_{i,j}e^{-a\Delta t_i} =: M_{i,j}$

$$\mathrm{Var}\{x(t_{i+1})|x(t_i) = x_{i,j}\} = \frac{\sigma^2}{2a}\left[1 - e^{-2a\Delta t_i}\right] =: V_i^2$$

(3.48) $\Delta x_i = V_{i-1}\sqrt{3} = \sigma\sqrt{\frac{3}{2a}[1 - e^{-2a\Delta t_{i-1}}}$

(3.49) $k = \mathrm{round}\left(\frac{M_{i,j}}{\Delta x_{i+1}}\right)$

(3.50) $p_u = \frac{1}{6} + \frac{\eta_{j,k}^2}{6V_i^2} + \frac{\eta_{j,k}}{2\sqrt{3}V_i}, \; p_m = \frac{2}{3} - \frac{\eta_{j,k}^2}{3V_i^2}, \; p_d = \frac{1}{6} + \frac{\eta_{j,k}^2}{6V_i^2} - \frac{\eta_{j,k}}{2\sqrt{3}V_i}$

(3.64) $dx_t^\alpha = \mu(x_t^\alpha;\alpha)dt + \sigma(x_t^\alpha;\alpha)dW_t^x$

(3.65) $P^x(t,T) = \Pi^x(t,T,x_t^\alpha;\alpha)$

(3.66) $r_t = x_t + \varphi(t;\alpha), \qquad t \geq 0$

(3.67) $P(t,T) = \exp\left[-\int_t^T \varphi(s;\alpha)ds\right]\Pi^x(t,T,r_t - \varphi(t;\alpha);\alpha)$

(3.68) $\varphi(t;\alpha) = \varphi^*(t;\alpha) := f^M(o,t) - f^x(0,t;\alpha)$

(3.69) $\exp\left[-\int_t^T \varphi(s;\alpha)ds\right] = \Phi^*(t,T,x_0;\alpha) := \frac{P^M(0,T)}{\Pi^x(0,T,x_0;\alpha)}\frac{\Pi^x(0,t,x_0;\alpha)}{P^M(0,t)}$

(3.70) $\Pi(t,T,r_t;\alpha) = \Phi^*(t,T,x_0;\alpha)\Pi^*(t,T,r_t - \varphi^*(t;\alpha);\alpha)$

(3.71) $V^x(t,T,\tau,K) = \Psi^x(t,T,\tau,K,x_t^\alpha;\alpha)$

(3.74) $dr_t = \left[k\theta + k\varphi(t;\alpha) + \frac{d\varphi(t;\alpha)}{dt} - kr_t\right]dt + \sigma dW_t$

Page 100 $\varphi^{VAS}(t;\alpha) = f^M(0,t) + (e^{-kt} - 1)\frac{k^2\theta - \sigma^2/2}{k^2} - \frac{\sigma^2}{2k^2}e^{-kt}(1 - e^{-kt}) - x_0e^{-kt}$

Page 101 $P(t,T) = \frac{P^M(0,T)A(0,t)\exp\{-B(0,t)x_0\}}{P^M(0,t)A(0,T)\exp\{-B(0,T)x_0\}}$

$$\times A(t,T)\exp\{-B(t,T)[r_t - \varphi^{VAS}(t;\alpha)]\}$$

(3.76) $dx(t) = k(\theta - x(t))dt + \sigma\sqrt{x(t)}dW(t), \qquad x(0) = x_0, \qquad r(t) = x(t) + \varphi(t)$

(3.77) $\varphi^{CIR}(t;\alpha) = f^M(0,t) - f^{CIR}(0,t;\alpha)$

$$f^{CIR}(0,t;\alpha) = \frac{2k\theta(\exp\{th\} - 1)}{2h + (k+h)(\exp\{th\} - 1)} + x_0\frac{4h^2\exp\{th\}}{[2h + (k+h)(\exp\{th\} - 1)]^2}$$

$$h = \sqrt{k^2 + 2\sigma^2}$$

Chapter 4

(4.4) $\quad r_t = x(t) + y(t) + \varphi(t), \qquad r(0) = r_0$

(4.5) $\quad dx(t) = -ax(t)dt + \sigma dW_1(t), \qquad x(0) = 0$

$$dy(t) = -by(t)dt + \eta dW_2(t), \qquad y(0) = 0$$

(4.6) $\quad E\{r(t)|\mathcal{F}_s\} = x(s)e^{-a(t-s)} + y(s)e^{-b(t-s)} + \varphi(t)$

$$\text{Var}\{r(t)|\mathcal{F}_s\} = \frac{\sigma^2}{2a}\left[1 - e^{-2a(t-s)}\right] + \frac{\eta^2}{2b}\left[1 - e^{-2b(t-s)}\right] + 2\rho\frac{\sigma\eta}{a+b}\left[1 - e^{-(a+b)(t-s)}\right]$$

(4.7) $\quad r(t) = \sigma\int_0^t e^{-a(t-u)}dW_1(u) + \eta\int_0^t e^{-b(t-u)}dW_2(u) + \varphi(t)$

(4.8) $\quad dx(t) = -ax(t)dt + \sigma d\widetilde{W}_1(t) \qquad dy(t) = -by(t)dt + \eta\rho d\widetilde{W}_1(t) + \eta\sqrt{1-\rho^2}d\widetilde{W}_2(t)$

where $dW_1(t) = dd\widetilde{W}_1(t)$ and $dW_2(t) = \rho d\widetilde{W}_1(t) + \sqrt{1-\rho^2}d\widetilde{W}_2(t)$

(4.9) $\quad M(t,T) = \dfrac{1 - e^{-a(T-t)}}{a}x(t) + \dfrac{1 - e^{-b(T-t)}}{b}y(t)$

(4.10) $$V(t,T) = \frac{\sigma^2}{a^2}\left[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right]$$
$$+\frac{\eta^2}{b^2}\left[T - t + \frac{2}{b}e^{-b(T-t)} - \frac{1}{2b}e^{-2b(T-t)} - \frac{3}{2b}\right]$$
$$+2\rho\frac{\sigma\eta}{ab}\left[T - t + \frac{e^{-a(T-t)} - 1}{a} + \frac{e^{-b(T-t)} - 1}{b} - \frac{e^{-(a+b)(T-t)} - 1}{a+b}\right]$$

(4.11) $$P(t,T) = \exp\left\{-\int_t^T \varphi(u)du - \frac{1 - e^{-a(T-t)}}{a}x(t) - \frac{1 - e^{-b(T-t)}}{b}y(t) + \frac{1}{2}V(t,T)\right\}$$

(4.12) $$\varphi(t) = f^M(0,T) + \frac{\sigma^2}{2a^2}\left(1 - e^{-aT}\right)^2 + \frac{\eta^2}{2b^2}\left(1 - e^{-bT}\right)^2 + \rho\frac{\sigma\eta}{ab}(1 - e^{-aT})(1 - e^{-bT})$$

(4.13) $$\exp\left\{-\int_t^T \varphi(u)du\right\} = \frac{P^M(0,T)}{P^M(0,t)}\exp\left\{-\frac{1}{2}[V(0,T) - V(0,t)]\right\}$$

(4.14) $$P(t,T) = \frac{P^M(0,T)}{P^M(0,t)}\exp\{\mathcal{A}(t,T)\}$$

$$\mathcal{A}(t,T) := \frac{1}{2}[V(t,T) - V(0,T) + V(0,t)] - \frac{1 - e^{-a(T-t)}}{a}x(t) - \frac{1 - e^{-b(T-t)}}{b}y(t)$$

(4.15) $\quad P(t,T) = A(t,T)\exp\{-B(a,t,T)x(t) - B(b,t,T)y(t)\}$

(4.16) $\quad \sigma_f(t,T) = \sqrt{\sigma^2 e^{-2a(T-t)} + \eta^2 e^{-2b(T-t)} + 2\rho\sigma\eta e^{-(a+b)(T-t)}}$

Page 152
$$\frac{Cov(df(t,T_1),df(t,T_2))}{dt}$$
$$= \sigma^2 \frac{\partial B}{\partial T}(a,t,T_1)\frac{\partial B}{\partial T}(a,t,T_2) + \eta^2 \frac{\partial B}{\partial T}(b,t,T_1)\frac{\partial B}{\partial T}(b,t,T_2)$$
$$+\rho\sigma\eta \left[\frac{\partial B}{\partial T}(a,t,T_1)\frac{\partial B}{\partial T}(b,t,T_2) + \frac{\partial B}{\partial T}(a,t,T_2)\frac{\partial B}{\partial T}(b,t,T_1)\right]$$
$$= \sigma^2 e^{-a(T_1+T_2-2t)} + \eta^2 e^{-b(T_1+T_2-2t)}$$
$$+\rho\sigma\eta \left[e^{-aT_1-bT_2+(a+b)t} + e^{-aT_2-bT_1+(a+b)t}\right]$$
$$Corr(df(t,T_1),df(t,T_2)) = \frac{\sigma^2 e^{-a(T_1+T_2-2t)} + \eta^2 e^{-b(T_1+T_2-2t)}}{\sigma_f(t,T_1)\sigma_f(t,T_2)}$$
$$+\frac{\rho\sigma\eta \left[e^{-aT_1-bT_2+(a+b)t} + e^{-aT_2-bT_1+(a+b)t}\right]}{\sigma_f(t,T_1)\sigma_f(t,T_2)}$$

Page 153
$$f(t,T_1T_2) = \frac{\ln P(t,T_1) - \ln P(t,T_2)}{T_2 - T_1}$$
$$df(t,T_1,T_2) = ...dt + \frac{B(a,t,T_2) - B(a,t,T_1)}{T_2 - T_1}\sigma dW_1(t)$$
$$+\frac{B(b,t,T_2) - B(b,t,T_1)}{T_2 - T_1}\eta dW_2(t)$$
$$\sigma_f(t,T_1,T_2) = \sqrt{\sigma^2\beta(a,t,T_1,T_2)^2 + \eta^2\beta(b,t,T_1,T_2)^2 + 2\rho\sigma\eta\beta(a,t,T_1,T_2)\beta(b,t,T_1,T_2)}$$

where
$$\beta(z,t,T_1,T_2) = \frac{B(z,t,T_2) - B(z,t,T_1)}{T_2 - T_1}$$
$$\frac{Cov(df(t,T_1,T_2),df(t,T_3,T_4))}{dt}$$
$$\sigma^2 \frac{B(a,t,T_2) - B(a,t,T_1)}{T_2 - T_1}\frac{B(a,t,T_4) - B(a,t,T_3)}{T_4 - T_3}$$
$$+\eta^2 \frac{B(b,t,T_2) - B(b,t,T_1)}{T_2 - T_1}\frac{B(b,t,T_4) - B(b,t,T_3)}{T_4 - T_3}$$
$$+\rho\sigma\eta \left[\frac{B(a,t,T_2) - B(a,t,T_1)}{T_2 - T_1}\frac{B(b,t,T_4) - B(b,t,T_3)}{T_4 - T_3}\right.$$
$$\left. + \frac{B(a,t,T_4) - B(a,t,T_3)}{T_4 - T_3}\frac{B(b,t,T_2) - B(b,t,T_1)}{T_2 - T_1}\right]$$

Page 160
$$\sigma_3 = \sqrt{\sigma_1^2 + \frac{\sigma_2^2}{(\bar{a}-\bar{b})^2} + 2\bar{\rho}\frac{\sigma_1\sigma_2}{\bar{b}-\bar{a}}}$$
$$dZ_3(t) = \frac{\sigma_1 dZ_1(t) - \frac{\sigma_2}{\bar{a}-\bar{b}}dZ_2(t)}{\sigma_3}, \qquad \sigma_4 = \frac{\sigma_2}{\bar{a}-\bar{b}}$$

Page 161
$$a = \bar{a}, \quad b = \bar{b}, \quad \sigma = \sigma_3, \quad \eta = \sigma_4, \quad \rho = \frac{\sigma_1\bar{\rho} - \sigma_4}{\sigma_3}$$

$$\varphi(t) = r_0 e^{-\bar{a}t} + \int_0^t \theta(v) e^{-\bar{a}(t-v)} dv$$

$$\bar{a} = a, \quad \bar{b} = b, \quad \sigma_1 = \sqrt{\sigma^2 + \eta^2 + 2\rho\sigma\eta}, \quad \sigma_2 = \eta(a-b)$$

$$\bar{\rho} = \frac{\sigma\rho + \eta}{\sqrt{\sigma^2 + \eta^2 + 2\rho\sigma\eta}}, \quad \theta(t) = \frac{d\varphi(t)}{dt} + a\varphi(t)$$

Managing Credit Risk: The Great Challenge for Global Financial Markets, Caouette, et. al.

Chapter 20

(20.2) $R_p = \sum_{i=1}^{N} X_i EAR$

(20.3) $V_p = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij}$

(20.5) $UAL_p = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij} 1$

Page 403 $$CVaR(CL) = EAD \bullet LGD \bullet \left(\Phi \left(\frac{\sqrt{\rho}\Phi^{-1}(CL) + \Phi^{-1}(PD)}{\sqrt{1-\rho}} \right) - PD \right) \times \frac{1 + (M - 2.5) \bullet b(PD)}{1 - 1.5b(PD)}$$

Liquidity Risk Measurement and Management: A Practioner's Guide to Global Best Practices, Matz and Neu

Chapter 2

Page 33 $\log V(t) = \alpha + \beta t + \sigma \varepsilon_t$

Page 33 $\log V_q(t) = \alpha + \beta t - \sigma \Phi^{-1}(q) \sqrt{t}$

Bond-CDS Basis Handbook: Measuring, Trading and Analysing Basis Trades, Elizalde, Doctor, and Saltuk

Page 13, Equation 1 $S = PD \times (1 - R)$

Page 15, Equation 2 $FR = \frac{U - AI}{RA} + FC$

Page 18, Equation 3 $SS = \frac{PV[c+p] - BP}{RFA}$

Page 25, Equation 4 $BTP1 = CN \times (100 - R - U - CP - FC) + BN \times (R + CR - BP - FC)$

Page 25, Equation 5 $BTP2 = BN \times (100 + CR - BP - FC) - CN \times (U + CP + FC)$

Page 43, Equation 7 $CN = \frac{BP - R}{100 - R - U} \times BN$

A Survey of Behavioral Finance, Barberis and Thaler

(1) $(x,p:y,q)=\pi(p)v(x)+\pi(q)v(y)$

(2) $\sum_i \pi_i v(x_i)$ where $v = \begin{array}{ll} x^\alpha & \text{if } x\geq 0 \\ -\lambda(-x)^\alpha & \text{if } x<0 \end{array}$ and $\pi_i = w(P_i) - w(P_i^*)$,

$$w(P)=\frac{P^\gamma}{(P^\gamma+(1-P)^\gamma)^{1/\gamma}}$$

(3) $\frac{D_{t+1}}{D_t}=e^{g_D+\sigma_D\varepsilon_{t+1}}$

(4) $\frac{C_{t+1}}{C_t}=e^{g_C+\sigma_C\eta_{t+1}}$

(5) $\begin{pmatrix}\varepsilon_t\\ \eta_t\end{pmatrix}\tilde{\ }N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1 & w\\ w & 1\end{pmatrix}\right)$, i.i.d.over time

(6) $E_0\sum_{t=0}^{\infty}\rho^t\frac{C_t^{1-\gamma}}{1-\gamma}$

(7) $1=\rho E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1}\right]$

(8) $R_{t+1}=\frac{D_{t+1}+P_{t+1}}{P_t}=\frac{1+P_{t+1}/D_{t+1}}{P_t/D_t}\frac{D_{t+1}}{D_t}$

(9) $r_{t+1}=\Delta d_{t+1}+\text{const.}\equiv d_{t+1}-d_t+\text{const.}$

(10) $E_\pi v[(1-w)R_{f,t+1}+wR_{t+1}-1]$

(11) $E_0\sum_{t=0}^{\infty}\left[\rho^t\frac{C_t^{1-\gamma}}{1-\gamma}+b_0\overline{C}_t^{-\gamma}\hat{v}(X_{t+1})\right]$

(13) $R_{t+1}=\frac{P_{t+1}+D_{t+1}}{P_t}$

(14) $p_t-d_t=E_t\sum_{j=0}^{\infty}\rho^t\Delta d_{t+1+j}-E_t\sum_{j=0}^{\infty}\rho^t r_{t+1+j}+E_t\lim_{j\to\infty}\rho^j(p_{t+j}-d_{t+j})+\text{const.}$

(15) $E_0\sum_{t=0}^{\infty}\left[\rho^t\frac{C_t^{1-\gamma}}{1-\gamma}+b_0\overline{C}_t^{-\gamma}\tilde{v}(X_{t+1},z_t)\right]$

(16) $\overline{r}_i-r_f=\beta_{i.1}(\overline{F}_1-r_f)+\ldots+\beta_{i,K}(\overline{F}_K-r_f)$

(17) $r_{i,t}-r_{f,t}=\alpha_i+\beta_{i,1}(F_{1,t}-r_{f,t})+\ldots+\beta_{i,K}(F_{K,t}-r_{f,t})+\varepsilon_{i,t}$

(18) $R_f=\frac{1}{\rho}e^{\gamma g_C+0.5\gamma^2\sigma_C^2}$

(19) $1=\rho\frac{1+f}{f}e^{g_D-\gamma G_C+0.5(\sigma_D^2+\gamma^2\sigma_C^2-2\gamma\sigma_C\sigma_D w)}$

(20) $R_{t+1}=\frac{D_{t+1}+P_{t+1}}{P_t}=\frac{1+P_{t+1}/D_{t+1}}{P_t/D_t}\frac{D_{t+1}}{D_t}=\frac{1+f}{f}e^{g_D+\sigma_D\varepsilon_{t+1}}$

CAIA Level II: Advanced Core Topics in Alternative Investments, Black, Chambers, Kazemi

Chapter 16

(16.1) $P_t^{\text{reported}} = \alpha + \beta_0 P_t^{\text{true}} + \beta_1 P_{t-1}^{\text{true}} + \beta_2 P_{t-2}^{\text{true}} + \cdots$

(16.2) $P_t^{\text{reported}} = \alpha P_t^{\text{true}} + \alpha(1-\alpha) P_{t-1}^{\text{true}} + \alpha(1-\alpha)^2 P_{t-2}^{\text{true}} + \cdots$

(16.3) $P_t^{\text{true}} = (1/\alpha) \times P_t^{\text{reported}} - [(1-\alpha)/\alpha] \times P_{t-1}^{\text{reported}}$

(16.4) $P_t^{\text{true}} = P_{t-1}^{\text{reported}} + [(1/\alpha) \times (P_t^{\text{reported}} - P_{t-1}^{\text{reported}})]$

(16.5) $R_{t,\text{reported}} \approx \beta_0 R_{t,\text{true}} + \beta_1 R_{t-1,\text{true}} + \beta_2 R_{t-2,\text{true}} + \cdots$

(16.6) $P_t^{\text{reported}} = (1-\rho) P_t^{\text{true}} + \rho P_{t-1}^{\text{reported}}$

(16.7) $\Delta P_t^{\text{reported}} = (1-\rho) \Delta P_t^{\text{true}} + \rho \Delta P_{t-1}^{\text{reported}}$

(16.8) $R_{t,\text{reported}} \approx (1-\rho) R_{t,\text{true}} + \rho R_{t-1,\text{reported}}$

(16.9) $R_{t,\text{true}} = (R_{t,\text{reported}} - \rho R_{t-1,\text{reported}})/(1-\rho)$

(16.10) $\hat{\rho} = \text{corr}(R_{t,\text{reported}} - R_{t-1,\text{reported}})$

(16.11) $\rho_{i,j} = \sigma_{ij}/(\sigma_i \sigma_j)$

(16.12) $R_t^{\text{reported}} = \alpha + \beta_1 R_{t-1}^{\text{reported}} + \beta_2 R_{t-2}^{\text{reported}} + \cdots + \beta_k R_{t-k}^{\text{reported}} + \varepsilon_t$

Chapter 21

Page 262 $Y = S \times I \times E \times H$ where Y = yield, S = total solar radiation over the area per period, I = fraction of solar radiation captured by the crop canopy, E = photosynthetic efficiency of the crop (total plant dry matter per unit of solar radiation), H = harvest index (fraction of total dry matter that is harvestable)

Managing Investment Portfolio: A Dynamic Process, Maginn, Tuttle, Pinto, McLeavey

Chapter 8

Page 523 $TRCI = CR + RR + SR$

Page 553 $RR_{n,t} = (R_t + R_{t-1} + R_{t-2} + \ldots + R_{t-n})/n$

Page 554 $DD = \sqrt{\dfrac{\sum_{i=1}^{n} [\min(r_i - r^*, 0)]^2}{n-1}}$

Page 555 $SR = \dfrac{ARR - rf}{SD}$

Page 556 $SR = \dfrac{ARR - rf}{DD}$

The Secular and Cyclic Determinants of Capitalization Rates: The Role of Property Fundamentals, Macroeconic Factors, and "Structural Changes," Chervachidze, Costello, Wheaton

(1) $$Log(C_{j,t}) = a_0 + a_1 \log(C_{j,t-1}) + a_2 \log(C_{j,t-4}) + a_3 \log(RRI_{j,t}) + a_4 RTB_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j$$

(1.1) $$RRI_{j,t-s} = RR_{j,t}/MRR_j$$

(2) $$Log(C_{j,t}) = a_0 + a_1 \log(C_{j,t-1}) + a_2 \log(C_{j,t-4}) + a_3 \log(RRI_{j,t-s}) + a_4 RTB_t + a_5 SPREAD_t + a_6 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j$$

(2.1) $$DEBTFLOW_t = TNBL_t/GDP_t$$

(3) $$Log(C_{j,t}) = a_0 + a_1 \log(C_{j,t-1}) + a_2 \log(C_{j,t-4}) + a_3 \log(RRI_{j,t-s}) + a_4 RTB_t + a_5 SPREAD_t + a_6 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t$$

(4) $$Log(C_{j,t}) = a_0 + a_1 yearq + a_2 \log(C_{j,t-1}) + a_3 \log(C_{j,t-4}) + a_4 \log(RRI_{j,t-s}) + a_5 RTB_t + a_6 SPREAD_t + a_7 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j$$

Analysis of Financial Time Series, Tsay

Chapter 9

(9.1) $$r_{it} = \alpha_i + \beta_{i1} f_{1t} + \cdots + \beta_{im} f_{mt} + \epsilon_{it},\ \ t = 1, \ldots, T,\ \ i = 1, \ldots, k$$

(9.2) $$\mathbf{r}_t = \alpha + \beta \mathbf{f}_t + \epsilon_t,\ \ t = 1, \ldots, T$$

(9.3) $$\mathbf{R}_i = \alpha_i \mathbf{1}_T + \mathbf{F} \beta_i' + \mathbf{E}_i$$

(9.4) $$\mathbf{R} = \mathbf{G} \xi' + \mathbf{E}$$

(9.5) $$r_{it} = \alpha_i + \beta_i r_{mt} + \epsilon_{it},\ \ i = 1, \ldots, k\ \ t = 1, \ldots, T$$

(9.11) $$\mathrm{Var}(y_i) = \mathbf{w}_i' \mathbf{\Sigma}_r \mathbf{w}_i,\ \ i = 1, \ldots, k$$

(9.12) $$\mathrm{Cov}(y_i, y_j) = \mathbf{w}_i' \mathbf{\Sigma}_r \mathbf{w}_j,\ \ i, j = 1, \ldots, k$$

(9.13) $$\sum_{i=1}^{k} \mathrm{Var}(r_i) = \mathrm{tr}(\mathbf{\Sigma}_r) = \sum_{i=1}^{k} \lambda_i = \sum_{i=1}^{k} \mathrm{Var}(y_i)$$

(9.14) $$\hat{\mathbf{\Sigma}}_r \equiv [\hat{\sigma}_{ij,r}] = \frac{1}{T-1} \sum_{t=1}^{T} (\mathbf{r}_t - \bar{\mathbf{r}})(\mathbf{r}_t - \bar{\mathbf{r}})',\ \ \bar{\mathbf{r}} = \frac{1}{T} \sum_{t=1}^{T} \mathbf{r}_t$$

(9.15) $$\hat{\rho}_r = \hat{\mathbf{S}}^{-1} \hat{\mathbf{\Sigma}}_r \hat{\mathbf{S}}^{-1}$$

(9.16) $$\mathbf{r}_t - \mu = \beta \mathbf{f}_t + \epsilon_t$$

(9.17) $$\mathbf{\Sigma} r = \mathrm{Cov}(\mathbf{r}_t) = E[(\mathbf{r}_t - \mu)(\mathbf{r}_t - \mu)'] = E[(\beta \mathbf{f}_t + \epsilon_t)(\beta \mathbf{f}_t + \epsilon_t)'] = \beta \beta' + \mathbf{D}$$

(9.18) $$\mathrm{Cov}(\mathbf{r}_t, \mathbf{f}_t) = E[(\mathbf{r}_t - \mu)\mathbf{f}_t'] = \beta E(\mathbf{f}_t \mathbf{f}_t') + E(\epsilon_t \mathbf{f}_t') = \beta$$

(9.19) $$\hat{\beta} \equiv [\hat{\beta}_{ij}] = \left[\sqrt{\hat{\lambda}_1} \hat{\mathbf{e}}_1 | \sqrt{\hat{\lambda}_2} \hat{\mathbf{e}}_2 | \cdots | \sqrt{\hat{\lambda}_m} \hat{\mathbf{e}}_m \right]$$

(9.20) $$\mathrm{LR}(m) = -\left[T - 1 - \frac{1}{6}(2k+5) - \frac{2}{3} m \right] \left(\ln |\hat{\mathbf{\Sigma}}_r| - \ln |\hat{\beta} \hat{\beta}' + \hat{\mathbf{D}}| \right)$$

Handbook of Fixed Income Securities, Fabozzi

Chapter 69

(69 − 4) Asset Allocation $\sum_s (w_s^P - w_s^B) \cdot R_s^B$

(69 − 5) Security Selection $\sum_s w_s^P \cdot (R_s^P - R_s^B)$

(69 − 12) $\alpha_k^P f_k^P - \alpha_k^B f_k^B = \sum_s \alpha_{k,s}^P f_{k,s}^P - \sum_s \alpha_{k,s}^B f_{k,s}^B$

Chapter 70

(70 − 1) Asset Allocation $w^P \cdot \sum_s \left(\frac{w_s^P}{w^P} - \frac{w_s^B}{w^B} \right) \cdot (TR_s^B - TR^B)$

(70 − 2) Sector Management $\sum_s w_s^P \cdot (TR_s^P - TR_s^B)$

(70 − 3) Top-Level Exposure $(w^P - w^B) \cdot TR^B$

(70 − 4) Asset Allocation $w^P \cdot \sum_s \left(\frac{w_s^P}{w^P} - \frac{w_s^B}{w^B} \right) \cdot (ER_s^B - ER^B)$

(70 − 5) Sector Management $\sum_s w_s^P \cdot (ER_s^P - ER_s^B)$

(70 − 6) Top-Level Exposure $(w^P - w^B) \cdot ER^B$

Chapter 71

Page 1737

$R^P - R^B = \sum_t \beta_t (R_t^P - R_t^B)$

$$A = \frac{(R^P - R^B)/T}{(1 + R^P)^{1/T} - (1 + R^B)^{1/T}}$$

$$C = \frac{R^P - R^B - A \sum_{t=1}^{T} (R_t^P - R_t^B)}{\sum_{t=1}^{T} (R_t^P - R_t^B)^2}$$

$\beta_t = A + C(R_t^P - R_t^B)$

Introduction to Credit Risk Modeling, 2nd ed., Bluhm, Overbeck, Wagner

Chapter 6

Page 237

$M_n = M_1^n$