SOCIETY OF ACTUARIES

EXAM MLC ACTUARIAL MODELS–LIFE CONTINGENCIES

EXAM MLC SAMPLE QUESTIONS AND SOLUTIONS

MLC-09-07 **PRINTED IN U.S.A.**

1 You are given:

(i) Z is the present-value random variable for an insurance on the lives of (x) and (y), where:

$$Z = \begin{cases} v^{T(y)} & T(x) \le T(y) \\ 0 & T(x) > T(y) \end{cases}$$

(ii) (x) is subject to a constant force of mortality, 0.07.

(iii) (y) is subject to a constant force of mortality, 0.09.

(iv) (x) and (y) are independent lives.

(v) $\delta = 0.06$

Calculate E[Z].

(A) 0.191

(B) 0.318

(C) 0.409

(D) 0.600

(E) 0.727

2 You are given:

(i) $T(x)$ and $T(y)$ are independent.

(ii) The survival function for (x) follows de Moivre's law with $\omega = 95$.

(iii) The survival function for (y) is based on a constant force of mortality, $\mu_{y+t} = \mu$ for $t \geq 0$.

(iv) $n < 95 - x$

Determine the probability that (x) dies within n years and predeceases (y).

(A) $\dfrac{e^{-\mu n}}{95 - x}$

(B) $\dfrac{ne^{-\mu n}}{95 - x}$

(C) $\dfrac{1 - e^{-\mu n}}{\mu(95 - x)}$

(D) $\dfrac{1 - e^{-\mu n}}{95 - x}$

(E) $1 - e^{-\mu n} + \dfrac{e^{-\mu n}}{95 - x}$

3. You are given:

(i) $T(30)$ and $T(40)$ are independent.

(ii) Deaths of (30) and (40) are uniformly distributed over each year of age.

(iii) $q_{30} = 0.4$

(iv) $q_{40} = 0.6$

Calculate ${}_{0.25}q^{\,2}_{30.5:40.5}$.

(A) 0.0134

(B) 0.0166

(C) 0.0221

(D) 0.0275

(E) 0.0300

4 You are given the following extract from a select-and-ultimate mortality table:

$[x]$	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
60	80,625	79,954	78,839	62
61	79,137	78,402	77,252	63
62	77,575	76,770	75,578	64

Calculate $1000\,{}_{0.7}q_{[60]+0.8}$, using the hyperbolic assumption for mortality at fractional ages.

(A) 8.6

(B) 8.7

(C) 8.8

(D) 8.9

(E) 9.0

5. You are given:

(i) (x) and (y) are independent lives.

(ii) $\mu_{x+t} = 5t$ for $t \geq 0$ is the force of mortality for (x).

(iii) $\mu_{y+t} = 1t$ for $t \geq 0$ is the force of mortality for (y).

Calculate $q_{\overset{1}{x}y}$.

(A) 0.16

(B) 0.24

(C) 0.39

(D) 0.79

(E) 0.83

6. You are given:

(i) Mortality follows de Moivre's law with $\omega = 110$.

(ii) $T(80)$ and $T(85)$ are independent.

(iii) G is the probability that (80) dies after (85) and before 5 years from now.

(iv) H is the probability that the first death occurs after 5 and before 10 years from now.

Calculate $G + H$.

(A) 0.25

(B) 0.28

(C) 0.33

(D) 0.38

(E) 0.41

7 You are given:

(i) $\mu_x = \sqrt{\dfrac{1}{80 - x}}, \qquad 0 \le x < 80$

(ii) F is the exact value of $s(10.5)$.

(iii) G is the value of $s(10.5)$ using the Balducci assumption.

Calculate $F - G$.

(A) -0.0183

(B) -0.0005

(C) 0

(D) 0.0006

(E) 0.0172

8. Z is the present-value random variable for an insurance on the lives of Bill and John. This insurance provides the following benefits:

(1) 500 at the moment of Bill's death if John is alive at that time; and

(2) 1000 at the moment of John's death if Bill is dead at that time.

You are given:

(i) Bill's survival function follows de Moivre's law with $\omega = 85$.

(ii) John's survival function follows de Moivre's law with $\omega = 84$.

(iii) Bill and John are both age 80.

(iv) Bill and John are independent lives.

(v) $i = 0$.

Calculate E[Z].

(A) 600

(B) 650

(C) 700

(D) 750

(E) 800

9 You are given:

(i) (x) is subject to a uniform distribution of deaths over each year of age.

(ii) (y) is subject to a constant force of mortality of 0.25.

(iii) $q_{\overset{1}{x}y} = 0.125$

(iv) $T(x)$ and $T(y)$ are independent.

Calculate q_x.

(A) 0.130

(B) 0.141

(C) 0.167

(D) 0.214

(E) 0.250

10-14. Use the following information for questions 10 through 14

You are given:

(i) (30) and (50) are independent lives, each subject to a constant force of mortality, $\mu = 0.05$

(ii) $\delta = 0.03$

10 Calculate ${}_{10}q_{\overline{30:50}}$.

(A) 0.155

(B) 0.368

(C) 0.424

(D) 0.632

(E) 0.845

11 Calculate $\mathring{e}_{\overline{30:50}}$.

(A) 10

(B) 20

(C) 30

(D) 40

(E) 50

12. Calculate $\bar{A}^{\,1}_{30:50}$.

(A) 0.23

(B) 0.38

(C) 0.51

(D) 0.64

(E) 0.77

10-14. (Repeated for convenience) Use the following information for questions 10 through 14.

You are given:

(i) (30) and (50) are independent lives, each subject to a constant force of mortality, $\mu = 0.05$

(ii) $\delta = 0.03$

13 Calculate $Var[T(30:50)]$.

(A) 50

(B) 100

(C) 150

(D) 200

(E) 400

14 Calculate $Cov[T(30:50), T(\overline{30:50})]$.

(A) 10

(B) 25

(C) 50

(D) 100

(E) 200

15-18 Use the following information for questions 15 through 18

For a special fully discrete whole life insurance on (x), you are given:

(i) Deaths are distributed according to the Balducci assumption over each year of age.

(ii)

k	Net annual premium at beginning of year k	Death benefit at end of year k	Interest rate used during year k	q_{x+k-1}	${}_kV$
2	---	---	---	---	84
3	18	240	0.07	---	96
4	24	360	0.06	0.101	---

15. Calculate q_{x+2}.

(A) 0.046

(B) 0.051

(C) 0.055

(D) 0.084

(E) 0.091

16 Calculate ${}_4V$.

(A) 101

(B) 102

(C) 103

(D) 104

(E) 105

15-18. (Repeated for convenience) Use the following information for questions 15 through 18

For a special fully discrete whole life insurance on (x), you are given:

(i) Deaths are distributed according to the Balducci assumption over each year of age.

(ii)

k	Net annual premium at beginning of year k	Death benefit at end of year k	Interest rate used during year k	q_{x+k-1}	${}_kV$
2	---	---	---	---	84
3	18	240	0.07	---	96
4	24	360	0.06	0.101	---

17 Calculate ${}_{0.5}q_{x+3.5}$

(A) 0.046

(B) 0.048

(C) 0.051

(D) 0.053

(E) 0.056

18. Calculate ${}_{3.5}V$.

(A) 99

(B) 103

(C) 106

(D) 108

(E) 111

19-23. Use the following information for questions 19 through 23.

A 30-year term insurance on Janet age 30 and Andre age 40 provides the following benefits:

- A death benefit of 140,000 if Janet dies before Andre and within 30 years.
- A death benefit of 180,000 if Andre dies before Janet and within 30 years.

You are given:

(i) Mortality follows de Moivre's law with $\omega = 100$

(ii) $i = 0$.

(iii) The death benefit is payable at the moment of the first death.

(iv) Premiums, $\bar{P}$, are paid continuously while both are alive, for a maximum of 20 years.

19. Calculate the probability that at least one of Janet and Andre will die within 10 years.

(A) $\frac{1}{42}$

(B) $\frac{1}{12}$

(C) $\frac{1}{7}$

(D) $\frac{2}{7}$

(E) $\frac{13}{42}$

20. Calculate ${}_{10}q^{2}_{30:40}$.

(A) 0.012

(B) 0.024

(C) 0 042

(D) 0.131

(E) 0.155

19-23 (Repeated for convenience) Use the following information for questions 19 through 23.

A 30-year term insurance on Janet age 30 and Andre age 40 provides the following benefits:

- A death benefit of 140,000 if Janet dies before Andre and within 30 years.
- A death benefit of 180,000 if Andre dies before Janet and within 30 years.

You are given:

(i) Mortality follows de Moivre's law with $\omega = 100$.

(ii) $i = 0$

(iii) The death benefit is payable at the moment of the first death

(iv) Premiums, $\bar{P}$, are paid continuously while both are alive, for a maximum of 20 years.

21 Calculate the probability that the second death occurs between times $t = 10$ and $t = 20$.

(A) 0.071

(B) 0.095

(C) 0.293

(D) 0.333

(E) 0.357

22 Calculate the present value at issue of the death benefits.

(A) 81,000

(B) 110,000

(C) 116,000

(D) 136,000

(E) 150,000

19-23. (Repeated for convenience) Use the following information for questions 19 through 23

A 30-year term insurance on Janet age 30 and Andre age 40 provides the following benefits:

- A death benefit of 140,000 if Janet dies before Andre and within 30 years.
- A death benefit of 180,000 if Andre dies before Janet and within 30 years

You are given:

(i) Mortality follows de Moivre's law with $\omega = 100$

(ii) $i = 0$.

(iii) The death benefit is payable at the moment of the first death.

(iv) Premiums, $\bar{P}$, are paid continuously while both are alive, for a maximum of 20 years

23. Calculate the present value at issue of premiums in terms of $\bar{P}$.

(A) $11.2\bar{P}$

(B) $14.4\bar{P}$

(C) $16.9\bar{P}$

(D) $18.2\bar{P}$

(E) $19.3\bar{P}$

1 The insurance is payable on the death of (y), if (x) predeceases (y).

$$E[Z] = \bar{A}_{x\overset{2}{y}} = \int_0^\infty v^t \, {}_tq_x \, {}_tp_y \, \mu_{y+t} \, dt$$

$$= \int_0^\infty e^{-.06t}(1-e^{-.07t})(e^{-.09t})(.09)\, dt$$

$$= .09 \int_0^\infty (e^{-.15t} - e^{-.22t})\, dt$$

$$= .09\left(\frac{1}{.15} - \frac{1}{.22}\right)$$

$$= .191 \quad \text{(A)}$$

2 $\quad {}_tp_x = \frac{95-x-t}{95-x} \qquad \mu_{x+t} = \frac{1}{95-x-t} \qquad {}_tp_y = e^{-\mu t}$

$$\int_0^n {}_tp_{xy}\,\mu_{x+t}\,dt = \int_0^n \frac{e^{-\mu t}}{95-x}\,dt = \frac{1-e^{-\mu n}}{\mu(95-x)} \qquad \text{(C)}$$

3 $\quad {}_{0.25}q^{2}_{\overline{30.5}:\overline{40.5}} = \int_0^{0.25} {}_tp_{30.5}\, \mu_{30.5+t}\; {}_tp_{40.5}\, dt$

$$= \int_0^{0.25} \frac{0.6t}{1-(0.5)(0.6)} \times \frac{0.4}{1-(0.5)(0.4)}\, dt$$

$$= \int_0^{0.25} \frac{0.24t}{0.56}\, dt$$

$$= \left(\frac{0.24}{0.56}\right) \frac{t^2}{2} \Big|_0^{0.25}$$

$$= 0.0134 \qquad \text{(A)}$$

4. Under hyperbolic (Balducci),

$$\frac{1}{l_{[60]+0.8}} = 0.2\left(\frac{1}{l_{[60]}}\right) + 0.8\left(\frac{1}{l_{[60]+1}}\right)$$

$$= \frac{0.2}{80{,}625} + \frac{0.8}{79{,}954}$$

$$= 0.0000124864$$

$$l_{[60]+0.8} = 80{,}087$$

$$\frac{1}{l_{[60]+1.5}} = 0.5\left(\frac{1}{l_{[60]+1}}\right) + 0.5\left(\frac{1}{l_{[60]+2}}\right)$$

$$= \frac{0.5}{79{,}954} + \frac{0.5}{78{,}839}$$

$$= 0.0000125956$$

$$l_{[60]+1.5} = 79{,}393$$

$$1000\,{}_{0.7}q_{[60]+0.8} = 1000\left(1 - {}_{0.7}p_{[60]+0.8}\right)$$

$$= 1000\left(1 - \frac{l_{[60]+1.5}}{l_{[60]+0.8}}\right)$$

$$= 1000\left(1 - \frac{79{,}393}{80{,}087}\right)$$

$$= 8.67 \quad \text{(B)}$$

5. Solution

$$q^1_{xy} = \int_0^1 {}_tp_y \, {}_tp_x \, \mu_{x+t} \, dt$$

$${}_tp_y = e^{-\int_0^t s\,ds} = e^{-\frac{s^2}{2}\Big|_0^t} = e^{-t^2/2}$$

$${}_tp_x = e^{-\int_0^t 5s\,ds} = e^{-\frac{5s^2}{2}\Big|_0^t} = e^{-\frac{5t^2}{2}}$$

$$q^1_{xy} = \int_0^1 e^{-t^2/2} e^{-\frac{5t^2}{2}} \cdot 5t \, dt$$

$$= \int_0^1 e^{-3t^2} \cdot 5t \, dt$$

$$= \frac{5}{6} \int_0^1 6te^{-3t^2} \, dt$$

$$= -\frac{5}{6} e^{-3t^2} \Big|_0^1$$

$$\frac{5}{6}\left[1 - e^{-3}\right]$$

$$= 0.7918$$

(D)

6. Solution:

$$G = \int_0^5 \frac{t}{25} \cdot \frac{30-t}{30} \, \frac{1}{30-t} \, dt = \frac{t^2}{2 \cdot 25 \cdot 30} \Big|_0^5 = \frac{1}{60}$$

$$H = \frac{110-80-5}{110-80} \cdot \frac{110-85-5}{110-85} - \frac{110-80-10}{110-80} \cdot \frac{110-85-10}{110-85}$$

$$= \frac{25}{30} \cdot \frac{20}{25} - \frac{20}{30} \cdot \frac{15}{25} = \frac{2}{3} - \frac{2}{5} = \frac{4}{15}$$

$$G + H = \frac{1}{60} + \frac{16}{60} = \frac{17}{60} = .28 \quad \text{(B)}$$

7. $s(x) = e^{-\int_0^x \mu_t\, dt}$

$$= e^{-\int_0^x (80-t)^{-1/2}\, dt}$$

$$= e^{2(80-t)^{1/2}\Big|_0^x}$$

$$= e^{2[(80-x)^{1/2} - 80^{1/2}]}$$

$$F = s(10.5) = e^{2[69.5^{1/2} - 80^{1/2}]}$$

$$= e^{-2(0.60761)} = 0.29665$$

Similarly, $s(10) = e^{2[70^{1/2} - 80^{1/2}]}$

$$= e^{-2(0.57767)} = 0.31495$$

$$s(11) = e^{2[69^{1/2} - 80^{1/2}]}$$

$$= e^{-2(0.63765)} = 0.27935$$

With the hyperbolic / Balducci assumption:

$$\frac{1}{s(10.5)} = 0.5\left(\frac{1}{s(10)}\right) + 0.5\left(\frac{1}{s(11)}\right)$$

$$= \frac{0.5}{0.31495} + \frac{0.5}{0.27935} = 3.37742$$

$$G = s(10.5) = \frac{1}{3.37742} = 0.29608$$

$$F - G = 0.29665 - 0.29608 = 0.00057$$

(D)

8 $$E[Z] = 500\int_0^4 v^\tau \, {}_\tau p_{80}^{B} \cdot {}_\tau p_{80}^{J} \cdot \mu_{80+\tau}^{B} \cdot d\tau + 1000 \cdot \int_0^4 v^\tau \, {}_\tau p_{80}^{J} \, \mu_{80+\tau}^{J} \, {}_\tau q_{80}^{B} \cdot d\tau$$

$$= 500\int_0^4 \frac{(5-\tau)}{5} \cdot \frac{(4-\tau)}{4} \cdot \frac{1}{(5-\tau)} \cdot d\tau + 1000\int_0^4 \frac{4-\tau}{4} \cdot \frac{1}{4-\tau} \cdot \frac{\tau}{5} \cdot d\tau$$

$$= \frac{500}{20}\int_0^4 (4-\tau) \cdot d\tau + \frac{1000}{20}\int_0^4 \tau \cdot d\tau$$

$$= 25\,(16-8) + 50 \cdot 8 = 600 \quad \text{(A)}$$

Solution:

9 $$q^1_{xy} = \int_0^1 {}_tp_{xy}\,\mu_{x+t}\,dt$$

$$= \int_0^1 {}_tp_x\,{}_tp_y\,\mu_{x+t}\,dt \quad \text{due to independence}$$

$$= q_x \int_0^1 {}_tp_y\,dt$$

$$= q_x \int_0^1 e^{-0.25t}\,dt$$

$$= q_x \left(-\frac{e^{-0.25t}}{0.25} \right) \Big|_0^1$$

$$= q_x \left(\frac{1-e^{-0.25}}{0.25} \right) = 0.8848\,q_x = 0.125$$

$$q_x = 0.141$$

(B)

Solution:

10. $${}_{10}p_{\overline{30:50}} = {}_{10}p_{30} + {}_{10}p_{50} - {}_{10}p_{30:50}$$

$$= e^{-10(.05)} + e^{-10(.05)} - e^{-20(.05)}$$

$$= .845$$

$${}_{10}q_{\overline{30:50}} = 1 - .845 = .155 \quad \text{(A)}$$

Solution:

11 $$\mathring{e}_{xy} = \int_0^\infty {}_tp_{xy}\,dt = \int_0^\infty e^{-.1t}\,dt$$

$$= \left.\frac{e^{-.1t}}{-.1}\right|_0^\infty$$

$$= 10$$

$$\mathring{e}_x = \mathring{e}_y = \int_0^\infty {}_tp_{x}\,dt = \int_0^\infty e^{-.05t}\,dt$$

$$= 20$$

$$\mathring{e}_{\overline{xy}} = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy} = 30 \quad \text{(C)}$$

Solution:

12 $$\bar{A}^{\,1}_{30:50} = \int_0^\infty e^{-\delta t} e^{-2\mu t}\mu\,dt = \frac{\mu}{2\mu+\delta} = \frac{.05}{.13} = .38 \quad \text{(B)}$$

Solution:

13 $Var[T(\overline{30:50})] = 2\int_0^\infty t e^{-.1t} dt - (\mathring{e}_{xy})^2$

$= 2\left(\frac{1}{2\mu}\right)^2 - \left(\frac{1}{2\mu}\right)^2$

$= \frac{1}{4\mu^2}$

$= 100$ (B)

Solution:

14 $Cov = (\mathring{e}_x - \mathring{e}_{xy})(\mathring{e}_y - \mathring{e}_{xy})$

$= (20-10)(20-10)$

$= 100$ (D)

15. $\left({}_2V + \pi\right)(1+i) - q_{x+2}\left(\text{Benefit} - {}_3V\right) = {}_3V$

$$(84 + 18)(1.07) - q_{x+2}(240 - 96) = 96$$

$$q_{x+2} = (109.14 - 96)/144$$

$$= 0.091 \quad \text{(E)}$$

16. $${}_4V = \frac{\left({}_3V + \pi\right)(1+i) - \left(q_{x+3}\right)(\text{Benefit})}{p_{x+3}}$$

$$= \frac{(96 + 24)(1.06) - (0.101)(360)}{1 - 0.101}$$

$$= (127.2 - 36.36)/0.899$$

$$= 101.05 \quad \text{(A)}$$

17. Under Balducci / hyperbolic

$${}_{0.5}q_{x+3.5} = (0.5)\, q_{x+3.5} = (0.5)(0.101)$$

$$= 0.0505 \quad \text{(C)}$$

18. $${}_{3.5}V = v^{1/2}\left({}_{0.5}p_{x+3.5}\right){}_4V + v^{1/2}\left({}_{0.5}q_{x+3.5}\right)(\text{Benefit})$$

$$= \left(\frac{1}{1.06^{1/2}}\right)(1 - 0.0505)(101.05) + \left(\frac{1}{1.06^{1/2}}\right)(0.0505)(360)$$

$$= 110.85 \quad \text{(E)}$$

Solution

19. $1 - {}_{10}p_{30:40} = 1 - {}_{10}p_{30} \cdot {}_{10}p_{40}$

$= 1 - \left(1 - \frac{10}{70}\right)\left(1 - \frac{10}{60}\right) = 1 - \frac{6}{7} \cdot \frac{5}{6} = \frac{2}{7}$ (D)

Solution

20

$$ {}_{10}q^{2}_{30:40} = \int_0^{10} (1 - {}_tp_{40})\, {}_tp_{30}\, \mu_{30+t}\, dt $$
$$ = \int_0^{10} \frac{1}{70} \frac{t}{60} dt $$
$$ = \frac{1}{70} \frac{1}{60} \frac{t^2}{2} \Big|_0^{10} = \frac{1}{7 \cdot 6 \cdot 2} = 0.012 $$ (A)

Solution

21.

$$ {}_{10|10}q_{\overline{30\,40}} = {}_{10|10}q_{30} + {}_{10|10}q_{40} - {}_{10|10}q_{30\,40} $$
$$ = {}_{10}p_{30}\; {}_{10}q_{40} + {}_{10}p_{40}\; {}_{10}q_{50} - {}_{10}p_{30:40} \cdot {}_{10}q_{40:50} $$
$$ = \left(1 - \frac{10}{70}\right)\left(\frac{10}{60}\right) + \left(1 - \frac{1}{6}\right)\left(\frac{1}{5}\right) - \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{6}\right)\left[1 - \left(1 - \frac{1}{6}\right)\left(1 - \frac{1}{5}\right)\right] $$
$$ = \frac{1}{7} + \frac{1}{6} - \frac{5}{7} \cdot \frac{1}{3} = \frac{6 + 7 - 10}{42} = \frac{3}{42} = \frac{1}{14} $$ (A)

$= 0.071$

Solution

22.

$$140{,}000 \int_0^{30} {}_tp_{30}\, \mu_{30+t}\, {}_tp_{40}\, dt + 180{,}000 \int_0^{30} {}_tp_{40}\, \mu_{40+t}\, {}_tp_{30}\, dt$$

$$000 \times \left\{ 140 \cdot \frac{1}{70} \int_0^{30} \left(1 - \frac{t}{60}\right) dt + 180 \cdot \frac{1}{60} \int_0^{30} \left(1 - \frac{t}{70}\right) dt \right\}$$

$$= 1000 \left[2 \left(t - \frac{t^2}{120}\right)\Big|_0^{30} + 3 \left(t - \frac{t^2}{140}\right)\Big|_0^{30} \right]$$

$$= 1000 \left[2 \left(30 - \frac{30}{4}\right) + 3 \left(30 - \frac{90}{14}\right)\right] = 115{,}714$$

(C)

23. Solution:

$$PVFP = \bar{P} \int_0^{20} v^t\, {}_tp_{30}\, {}_tp_{40}\, dt$$

$$= \bar{P} \int_0^{20} 1 \cdot \left(1 - \frac{t}{70}\right)\left(1 - \frac{t}{60}\right) dt$$

$$= \bar{P} \int_0^{20} \left(1 - \frac{t}{70} - \frac{t}{60} + \frac{t^2}{4200}\right) dt$$

$$= \bar{P} \left(20 - \frac{400}{140} - \frac{400}{120} + \frac{8000}{3(4200)}\right)$$

$$= 14.4\, \bar{P}$$

(B)