A Stochastic Approach To Long Term Disability Valuation

Peter Douglas, FSA, FCIA

University of Regina

Introduction

This paper discusses stochastic treatments for insured Long term Disability (LTD) benefits under a group insurance policy. It builds on earlier work by the author, including a paper delivered by the author at a previous Canadian Institute of Actuaries Stochastic Modeling Symposium.

The initial sections provide background on Canadian insured LTD benefit design and experience, as the nature of the LTD risk is a fundamental element in the argument in favor of a stochastic treatment. A proposed model for stochastic treatment is developed and compared to the current deterministic models commonly used.

The Analysis section includes illustrative results are provided based on a sample of actual disability claims experience. The key findings from this analysis are contained in the Conclusions and Recommendations section.

Description of the LTD Benefit

Long Term Disability (LTD) insurance provides a monthly income to replace lost wages to injured or sick employees who are unable to work. The benefit amount is based on the worker's pre-disability and a typical plan is designed to replace 80-90% of pre-disability income. The benefit is usually offset (or reduced on a dollar for dollar basis) for any benefits the disabled employer receives from government sponsored programs like Canada or Quebec Pension Plans (CPP or QPP), Employment Insurance (EI) or Worker's Compensation (WCB).

To qualify for LTD benefits, the disabled must remain unable to work for a minimum elimination period of 3-12 months. (Income replacement during this time is provided by sick time, EI or short term disability programs.) After the end of the elimination period, LTD Benefits are paid until the disabled attains age 65. However, the disabled must continue to meet the definition of disability and benefits will cease when the disabled is judged able to return to work, or dies.

A key feature of LTD plan design is the definition of disability. During an initial period (of 18-36 months depending on plan design) the individual is consider disabled as long as he/she is unable to do their own occupation. This is referred to as the "Own Occ" definition of disability and the period to which it applies is referred to as "Own Occ" period. After the expiration of the Own Occ period, the definition becomes stricter and the individual must be unable to do any occupation for which they are qualified (the

“Any Occ” definition) to continue qualifying as disabled. To illustrate the difference, a surgeon who suffers a crippling hand injury would be unable to perform surgery and thus meet the “Own Occ’ definition. However, the surgeon may well be able to teach at a University or act in an administrative role and therefore would not meet the “Any Occ’ definition.

Modeling LTD

Past studies of LTD experience consider termination rates, where terminations include deaths and recoveries, and show consistent patterns. Many disabilities are short term in nature and in most cases benefits cease before the end of two years. There is typically a spike in termination rates at the expiration of the “Own Occ” period since many claimants are considered able to return to work under the application of the stricter definition of disability. Conversely, any disabilities that last beyond this point are much more serious and typically continue until age 65 unless the claimant dies before that time. In addition, by the end of two of more years claimants have acclimatized to their changed circumstances and consider themselves disabled and unlikely to return to work. In fact the majority of claim terminations after 2 years of disability are from death and not from recovery.

LTD experience studies produce termination tables showing expected termination rates, representing the probability that an individual disabled at Age X and remaining disabled at age $X+t_0$, will cease to be disabled prior to age $X+t_1$ where $t_1 > t_0$. The actuarial notion is ${}_{t_1-t_o}q_{[X]+t_0} = 1 - {}_{t_1-t_o}p_{[X]+t_0}$ where ${}_{t_1-t_o}\,p_{[X]+t_0}$ which is probability that an individual disabled at age X continues to disabled at age X+t_1 given that they were disabled at age X+t_o. In practice most insurers do not treat deaths and recoveries as separate contingencies. It should also be noted that the 1987 GLTD (the standard base LTD morbidity valuation table used in Canada) does not distinguish between terminations due to death and recovery.

Raw rates from the CIA inter-company LTD study show the clear pattern of high recovery rates initially, which tail off rapidly except for a spike corresponding to the expiration of the Own Occ period. Figure 1 below shows the probability distribution of the number of months of disability, based on the terminations rates for a Male aged 40 based on the most recent Canadian Institute of Actuaries (CIA) LTD experience study (based on 1988-1994 data, see the appendices for additional details). Figure 1A also shows the same distribution but focuses on the first 10 years to better illustrate the left tail.

Figure 1

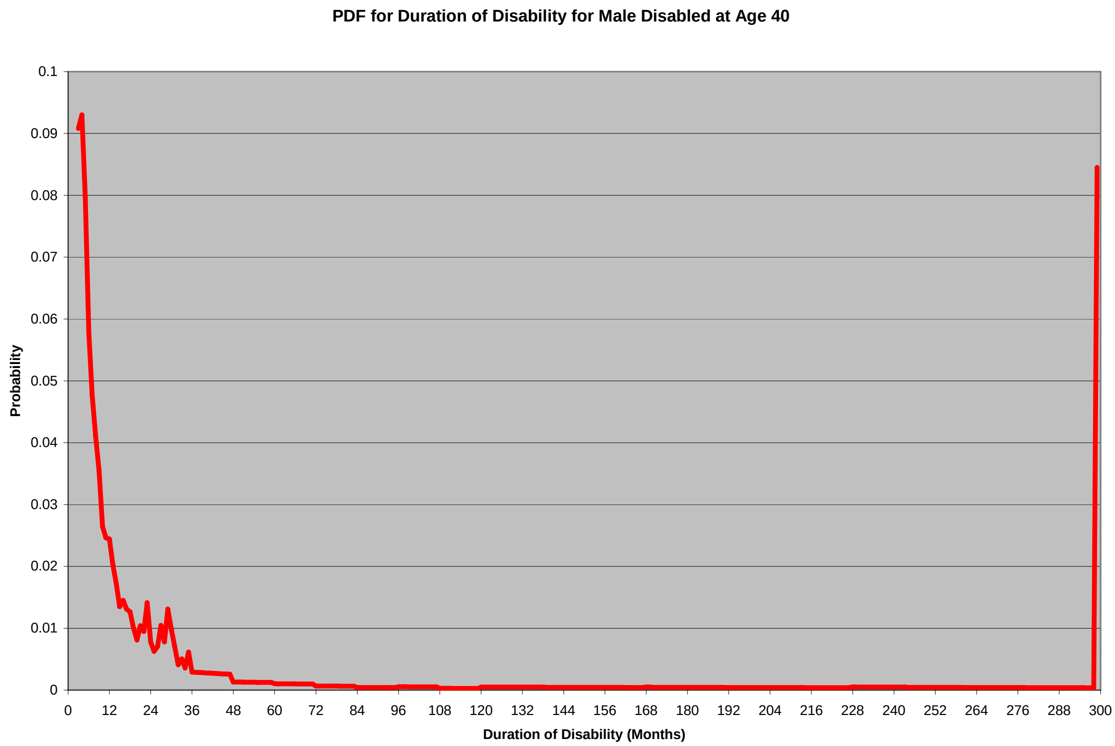
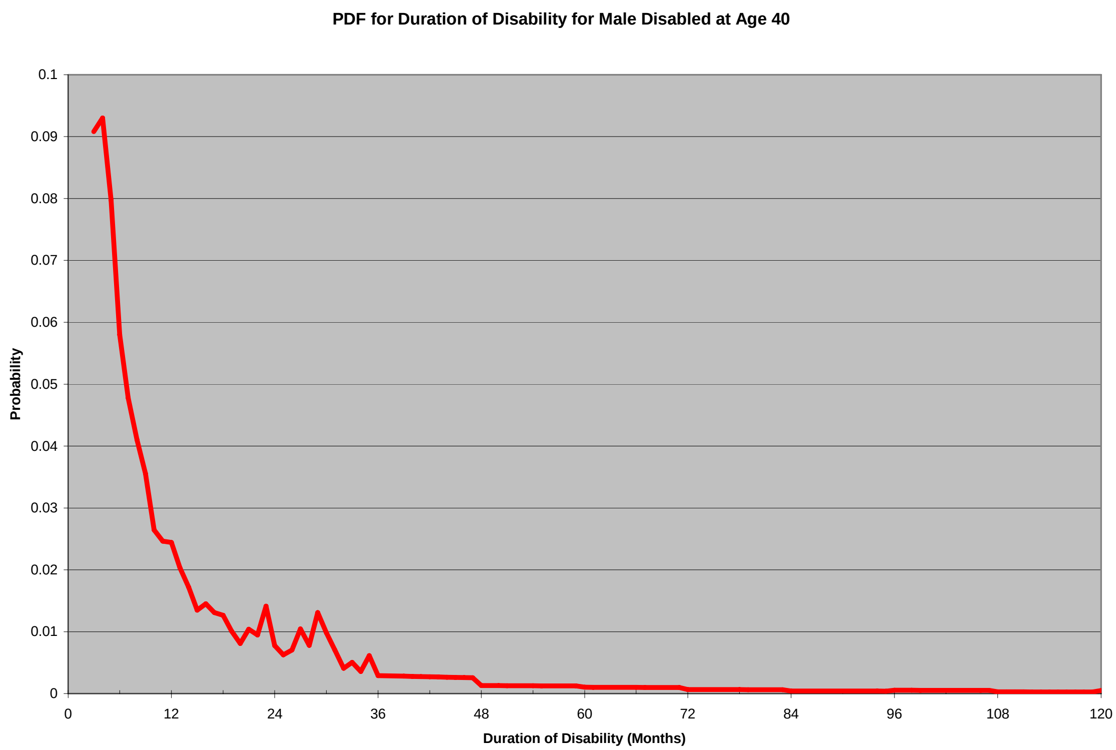

PDF for Duration of Disability for Male Disabled at Age 40
Probability
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0 12 24 36 48 60 72 84 96 108 120 132 144 156 168 180 192 204 216 228 240 252 264 276 288 300
Duration of Disability (Months)

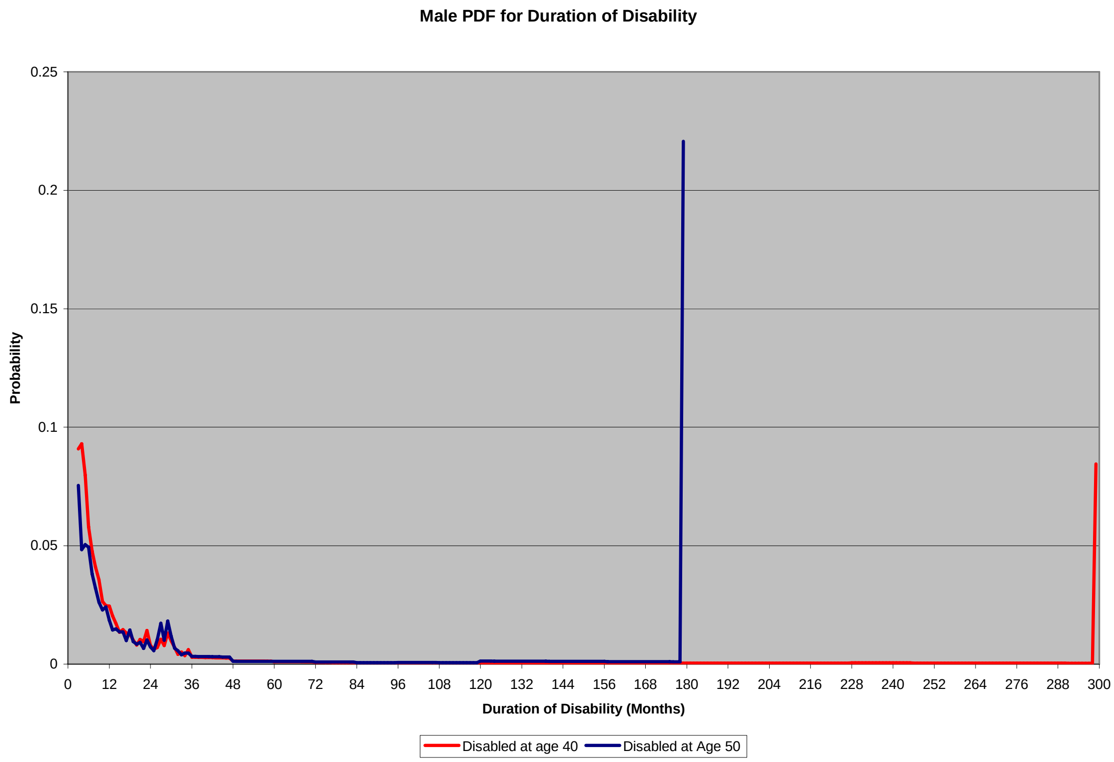
Figure 1A


PDF for Duration of Disability for Male Disabled at Age 40
Probability
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0 12 24 36 48 60 72 84 96 108 120
Duration of Disability (Months)

These figures show a marked bimodal distribution with most of the probability split between very short durations (less than 2 years) and a spike at the maximum duration (i.e. until age 65 or 300 months for a 40 year old). Note that only 35.7% of individuals remain disabled after 24 months, but 9.8% of individuals remain disabled for 25 years. The spike in termination rates between 18 and 36 months results from the application of the stricter Any Occ definition of disability. Note there is a double spike effect, which is due to differences in the application of the Own Occ definition between insurers (and indeed between policies within the same insurer).

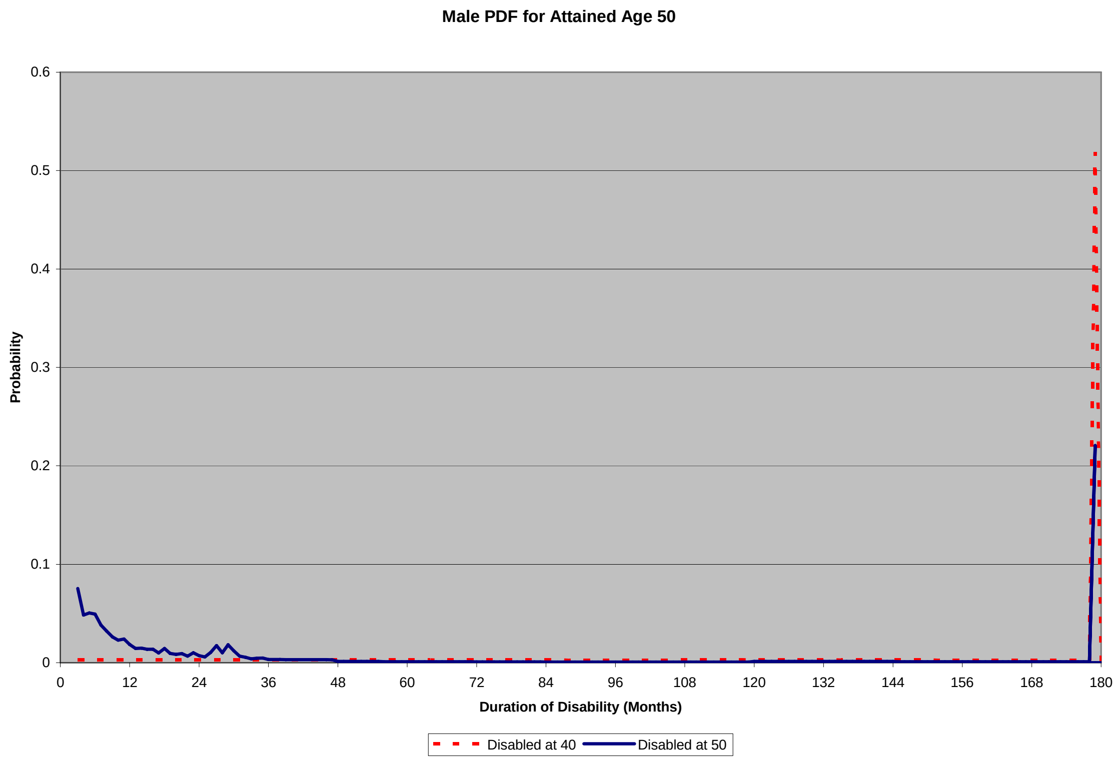
The same basic histogram shape appears for males and females and at other ages, although termination rates will differ at specific points in time for males versus females. Similarly the age at disability will change not only the recovery rates but also the maximum duration (and thus the location of the probability spike for benefit payments to age 65). However, while these may change the relative shape and size of the mass and location and size of the spike, the bimodal shape remains. Figure 2 compares the distributions for Males aged 40 and 50. The impact of higher age at disability can be seen in the dramatic increase in the proportion of claimants who remain disabled until age 65 (under 10% for males disabled at age 40 and over 20% for those 10 years older).

Figure 2



The preceding figures show the distribution of results for individuals who have just started their LTD benefits (i.e. at the end of a 3-6 month elimination period). Claimants who have been disabled for some time have a conditional probability distribution which effectively shifts probability from the left tail to the right, as shown below. For any individuals who have been disabled for more than 3 years, the distribution has very low probabilities for most duration with a very large spike at age 65. Figure 2a compares the probability distribution for a Male aged 50 and recently disabled to Male disabled at age 40 after 10 years of disability (therefore currently aged 50). For claimants disabled at age 50, just over 20% will remain disabled until age 65. However, a claimant who is disabled at age 40 and remains disabled at age 50 has over a 50% likelihood of remaining disabled until age 65.

Figure 2a



Traditionally, LTD valuation has used a deterministic life annuity function of the form

$a_{[X]+d:\overline{65-x-d|}} = \sum_{k=0}^{Age65} v^k \, {}_k p_{[X]+d}$ where the index is taken in monthly increments and v represents the present value of $1 paid in one month. This formula is the expected value of the present value of the LTD benefits and can also be expressed in the form

$a_{[X]+d:\overline{65-x-d|}} = \sum_{k=0}^{Age65} a_{\overline{K|}} * P[T = k \mid T > d]$, where T is the time at which the payments cease and $a_{\overline{K|}}$ is the present value of a term certain annuity making payments of $1 at the end of every month for K months.

The pricing of LTD uses a similar deterministic formula under which the annual cost per $1 monthly benefit $= q_x^{(i)} \times a_{x:\overline{65-x|}}$. Here $q_x^{(i)}$ is the incidence rate, or probability that an employee currently aged *x* who is actively at work becomes disabled under the plan within the next year and $a_{x:\overline{65-x|}}$ is the expected value for a monthly life annuity that starts payments at age *x* and ceases on the attainment of age 65. In addition, the pricing formula must also make assumptions for the expected offset for CPP benefits in developing the expected monthly benefit.

With respect to incidence rates, there is no publically available study of Canadian incidence rates (although at least one private intercompany study exists). The 1987 GLTD table includes a set of incidence rates, these based on US data that is over 20 years old and should not be considered relevant in the current Canadian market place. Tracking incidence experience is a major challenge for many smaller insurers, as administration practices often do not provide sufficient information on exposure.

Suitability for Stochastic Treatment

There is a strong case to be made that a deterministic approach does not adequately measure the risk for an LTD benefit, especially for disabilities in the Own Occ period. I believe risk that these can be better modeled using a stochastic approach.

To begin with let us consider the circumstances where a stochastic model is preferred to a deterministic model. In general an argument can be made that a stochastic approach should be applied in any actuarial application where additional information on the nature of the risk is desirable. This would certainly apply to LTD liabilities given the variability inherent in the LTD benefit and the difficulty of developing an algebraic model for the aggregate loss distribution. Furthermore, for many insurers the LTD liabilities represent both a significant proportion of their total liabilities and a leading source of variability in financial results.

In particular, a stochastic model is often suggested if any one of the following conditions exist:

1) the loss distribution has a long or heavy right tail;
2) there is a trigger point or cliff at which the shape of the random variable changes significantly; or
3) there are inter-dependencies between underlying risks.

The first two conditions are clearly shown in the loss distribution for individual disabled lives (see Figures 1 and 2). The long right tail can be seen in the probability spike at age 65. Note that the spike arises from the censoring of the payments at age 65 and that the right tail would be even longer if benefits were paid for life. The end of the Own Occ period represents a trigger point in that any claimants who remain disabled past this point are likely to receive benefits to age 65. As may be expected, the impacts of both the long tail and the trigger point are mitigated when a large sample size is modeled (i.e. with a block of several hundred claimants). However, the presence of these features gives the aggregate loss distribution a relatively high variance.

Dependencies Between Risks

There are dependencies of risks between LTD claimants, which can be seen by additional examination of the LTD insurance environment. For an insurance company, termination experience on the entire block of LTD claimants is often affected external or internal events and forces, including the following:

1) The National Employment Climate

Recovery rates are inversely correlated to unemployment rates. When unemployment is high, there is less incentive to return to work and may be that no job is available even if the claimant wants to return to work. Conversely when trained workers are scarce, an employer may be more willing to accommodate a disabled worker.

2) Change in the Administration of CPP and QPP

In the past, both CPP and QPP have changed their claims adjudication, and have become both tougher and more lax on occasion. These changes can have an important impact on LTD recovery experience since the presence of a CPP disability benefit will limit an insurer's ability to terminate an existing disability claim.

3) Legal Rulings or Regulation Changes

In the past, these have forced insurers to provide or extend benefits to certain types of claimants (i.e. for disability related to pregnancy or mental and nervous conditions). Future rulings or legislation may lead to further changes reducing the ability of an insurer to terminate existing LTD claims. In addition, with retirement ages extending beyond age 65 there may be a movement to extend LTD benefits past age 65.

4) Medical Breakthroughs

These may be thought to have an advantageous impact, in that they would generally lessen the severity of disability. However, in practice while such breakthroughs may extend the lifetime on an individual they may not improve their condition enough to allow them to return to work permanently. Therefore the impact of such breakthroughs may be to reduce termination rates by reducing mortality rates.

5) Stability in Key Claims Adjudication Areas

Successful claims management requires a stable well-trained claims adjudication staff. Recovery experience deteriorates when an insurer suffers internal reorganizations, periods of high staff turnover or changes in claims procedures.

It should be noted that although company management have some control over the stability of key claims personnel, these forces are otherwise both outside of management's control and difficult to predict in advance.

Dependencies of Risks Within a Single Group Policy or Smaller Block of Claims

In addition to the external forces that affect an insurance company's entire portfolio, subsets of this portfolio are subject to additional local external forces. Changes in the economic cycle rarely affect all sectors equally, as there are variations depending on industry and geographic location. These are particularly felt in smaller or more remote employment markets (i.e. outside of the Quebec- Windsor corridor) and in resource-based industries. In the past, downturns in mining, forestry, agriculture and fishing based economies have been sharp and had a strong impact on regional economies. In addition, rural or remote locations often lack adequate medical and rehabilitation facilities to easily support the return to work of disabled workers. There are also regional differences in the administration of CPP benefits and even in the cultural acceptance of disability.

Most group policies cover the employees of a single employer. For these cases, one of the most important influences affecting the cost of an LTD program is the attitude of the employer to returning disabled workers back into the workforce. If the employer supports rehabilitative training, gradual return to work schedules and modified work loads, then the recovery rates can be higher than expected. If the employer is unwilling to make these accommodations, then recovery rates suffer. In the worst cases, management may view the LTD plan as an easy way of handling “problem” employees.

In addition to inter-dependencies of risks, the financial underwriting arrangements for larger group insurance policies also magnify the risk to an insurance company. Under most insurance arrangements, experience can be pooled among an entire portfolio of insured risks. For example, Individual Life Insurance policies are evaluated on a portfolio basis so that the gain or loss on the portfolio is based on the premiums and claims experience of all similar policies. However, many large Group policies use Retroactive Experience Rated or “Refund” accounting policies. Under these arrangements, experience on each policy is evaluated separately from the rest of the portfolio. Worse still, a large proportion of any gains on the policy are refunded back to the policyholder while losses are retained by the insurer.

To show the impact of this accounting method, consider two policies: Policy A, which has a gain of $1 million, and Policy B, with a loss of $0.75 million. If these policies are underwritten on a pooled (or non-refund) basis, the insurance company would offset the gains and losses and show a net gain of $0.25 million. With refund underwriting, the insurer would show a net loss of close to $0.75 million, as they would refund most of the gain on policy A and be liable for all of the loss on Policy B. Refund underwriting makes each policy a “heads you win, tails we’re even” game, and leverages the risks from these policies for the insurer. This again creates a cliff in the overall portfolio experience.

Because the underwriting arrangements limit the insurer's ability to offset gains and losses, the experience on the LTD portfolio cannot be considered as a whole and must be evaluated as a series of smaller subsets. While the entire block of Pooled business could be treated as one aggregate risk portfolio, each Refund policy should be treated as a separate risk.

The Proposed Valuation Model

The model considers the valuation of an existing block of ongoing LTD claimants. It calculates present value of future benefits for an open block of LTD claims, calculated at a valuation date. It is an individual risk model, with each existing LTD claimant representing a single risk. The outcomes for an individual claimant are the present value of future benefits and are discrete distributions, since there are a finite non-negative number of outcomes (the number of payments made for each claimant). The cost random variable for the i[th] disabled life risk can be expressed in the form $Y_i = B_i * a_{\overline{T|}}$ where B_i is the current monthly benefit amount, $a_{\overline{k|}}$ is the present value of a term certain annuity of duration *k* months and $T = 0,1,2,3...$ is a discrete random variable representing the number of months that the claimant receives benefits before these are discontinued due to recovery, death, or the attainment of age 65.

Under the traditional discrete reserving method the liability for each individual risk is taken as the expected value of the risk, and the liability for the portfolio as the sum of the expected values of the individual risks. Note that this assumes that the risks are independent, which is a questionable assumption for LTD risks for the reasons noted above. The notation is $E[S] = \sum_{i=1}^{n} E[Y_i] = \sum_{i=1}^{n} B_i * E\left(a_{\overline{T}|}\right) = \sum_{i=1}^{n} B_i * a_{[x_i]+d|\overline{\varpi_i}|}$ where $a_{[x]_i+d|\overline{\varpi_i}|}$ is the life annuity function for an individual disabled at age x_i and currently aged $x_i + d$ and ϖ_i is the time at which the claimant attains age 65. The life annuity uses termination rates for LTD experience, which differ by age, gender and duration since disability. Typically insurers use a published table based on Inter-company experience, which may be modified for their own in-company experience. In some cases, the termination table may be further modified for particular groups of claimants based on experience. (For ease of notation, all annuity functions are based on monthly payment periods and use an effective interest per month $\frac{i^{(12)}}{12} = (1+i)^{1/12} - 1$ using standard notation where i is the effective annual interest rate.)

It would be theoretically possible to develop the distribution of the random variable *S*, the total present value of benefits for the existing claimants. However, this would be an extremely complex and time consuming process due to the large range of possible results. Similarly, the distribution of *S* could be approximated with a fitted continuous distribution, but this may not provide an accurate estimate of the shape of the distribution of S, particularly at the right tail.

However, it is quite possible to determine the full distribution of each random variable Y_i, the present value of benefits for the i^{th} risk (or claimant). There are a finite number of outcomes (for instance for a claimant aged 45, there can be a most 240 months of payments) and simple life contingencies functions can be used to develop the full distribution of Y_i. The most useful functions are the conditional probabilities $f_{X+d}(T) = f_X(T + d \mid T >= d)$ and $S_{X+d}(T) = S_X(T + d \mid T >= d)$ where T is the number of payments made after the valuation date, X is the age at disability and d is the duration from disability (therefore $X+d$ is the attained age). Using these functions, the distribution of Y_i can be calculated based on the valuation termination table and the claimant's gender, duration from disability and current age. These distributions become the heart of the proposed stochastic model

Under the proposed stochastic model, Monte Carlo techniques are used to simulate the outcome on each random variable Y_i for a large number of trials with $\hat{Y}_{ij}$ being the simulated outcome for the i^{th} claimant on the j^{th} trial. For each trial the outcome $\hat{S}_j = \sum_{j=1}^{n} \hat{Y}_{ij}$ is the sum of the individual outcomes of Y_i. The empirical distribution of the trial results for $\hat{S}$ is used to estimate the distribution of S. In addition, statistical quantities such as $E\lfloor\hat{S}\rfloor$, $Var\lfloor\hat{S}\rfloor$, VaR_α and CTE_α can be used to approximate the related quantities for S. The same model output can be sued to approximate the distribution of future cash flows (i.e. projected benefit payments). Simulation examples are presented in the Appendices.

<u>Simulation Technique</u>

A stochastic model was created to simulate the distribution of the present value of benefits for an existing block of claimants. The basic model used the following methods and assumptions:

i) The model was created using the @Risk Professional 5.0 software package developed by Palisades Corporation. This software is a Monte Carlo simulation package that is based on Microsoft Excel and is used in many insurance and financial applications. For this paper, @Risk was used to generate random variables from the uniform distribution over the interval (0,1) and provided the charts and tables. For illustration purposes 10,000 trials were used.

ii) For illustration purposes, life contingencies were taken from the Canadian Institute of Actuaries study *Canadian Group Long Term Disability Termination Experience 1988-1994*. The raw termination rates were used without modification. No differentiation was made between terminations from death and those from recovery.

iii) Present values were based on an annual effective interest rate of 5% per annum. The interest rate remained constant at all times. The actual LTD valuation interest rates used today vary between insurers, and these may decrease as the projection period increases. A change in interest rate would change the scale of results but not the underlying shape of the distribution.

iv) For simplicity, benefits were assumed to remain at the current levels. For some LTD plans, benefits may be increased annually based on inflation indices, or less commonly based on a fixed percentage increase. Inflation indexing may be subject to a maximum percentage increase in any given year. In addition, no assumption regarding future CPP awards were made. Therefore no contingency was allowed for the possibility that insured benefits would decrease in the future due to CPP offsets. Many LTD insurers make assumptions regarding future CPP awards for claimants who currently receive no CPP benefits. The model could be modified to simulate inflation future indexing or CPP awards, but at the cost of additional complexity.

v) Again for simplicity, disabled individuals were assumed to share risk characteristics regardless of the cause of the disability. Studies of LTD claimants show that experience differs between causes. In particular, two types of disability (Musculoskeletal and Mental/nervous disabilities) show different recovery patterns from other disabilities. The model could be expanded to account for these differences but would require additional information on a claim by claim level. In addition, the actuary would need to develop separate termination tables for each group as inter-company termination studies do not typically differentiate between causes of disability.

vi) For illustration, data was obtained from 500 claims for an existing block of business. This data was further modified to test the impact of various factors such as age, duration, benefit amount, and number of lives on results. The data is described in more detail below.

Modifications for Pricing Models

The model described above can easily be expanded to consider the pricing of LTD benefits. This would require additional assumptions including a set of incidence rates (which typically differ by age and gender) and assumptions regarding CPP awards and benefit amounts. As noted above, accurate estimation of future incidence rates remains a major challenge in the Canadian Group LTD marketplace.

LTD pricing should also consider the risk characteristics for the group of insured lives. These include geographic, industry and employer specific characteristics and may affect incidence rates, termination rates or most likely both contingencies.

Modifications for dependencies of risks

While the model described above provides additional insights over the deterministic approach, it still assumes that all risks are independent. The basic model does not consider the impact of external forces on the portfolio on the whole. Two approaches used to account for these forces, as described below.

Each of these modifications requires an additional random variable that accounts for overall variation in termination rates. It is easy to show that this variation exists (for instance preliminary results comparing Canadian experience by year over the period 1988-1997 to the GLTD table shows actual to expected ratios that range from 113% to 143% on an annual basis. However, separating the impact of external forces from the impact of changes in demographics and policy types is difficult. Therefore modeling of the fluctuations due to external factors will be highly subjective. Therefore the actuary is advised to focus on the potential impact of future events rather than replication of past events.

1) Constant Modification to Monthly recovery rates

For this modification, the distribution of the random variable Y_i (present value of benefits for the ith claimant) is affected by an additional random variable, which modifies the base termination tabular rates. Let the new random variable be m_j representing the external forces affecting the j^{th} trial, where m_j has mean 0 and variance σ_m^2. The new random variable is used to create a new distribution for the random variables Y_{ij} based on the modified termination rates $q'_{[x]+d} = q_{[x]+d} \cdot m_j$. Therefore the modified survival function becomes $S'([x]+d) = \prod_{k=1}^{d}(1 - m_j \cdot_t \; q_{[x]+d})$. Note that within each trial the termination rates are modified equally for all claimants to account for an event affecting the experience of entire portfolio or group of claimants. A value of m_j greater than zero indicates higher termination rates, an improvement in expected experience. Conversely a value of m_j less than zero indicates lower termination rates and thus a worsening of expected experience.

Under this proposed modification, termination rates are modified by the same factor at all durations of disability. This simplifies the impact of external forces, which may be of limited duration and can be cyclical in nature. The model could be further modified to allow for such impacts by simulating a different modifying random variable for various projections periods. However, it can be seen that this would make the model much more complex. The single modifying random variable does simulate the impact of changes in the LTD environment. Furthermore it can be argued that external events in the near future would have lasting impact on the current block disabled lives.

This modification contains a great deal of subjectivity regarding the distribution of m_j. It is easy to argue that this should have mean 0, but the shape and variance are highly debatable. Nor is there credible experience which can be used to base any estimates. For illustration purposes, a normal random variable with mean 0 and standard deviation 0.1 was used. Despite these limitations, this technique can be used to explore the potential impact of fluctuations due to external forces and represent additional variance in experience.

2) Exponential Modification to Survival functions

This second technique employs a modification which is similar to the Cox Proportional Hazard Rate model. The distribution of the random variable Y_i is again modified by another random variable, but in this case the factor is applied as an exponent to the base survival function. The modified distribution is found based on $S'([x]+d+t)=S([x]+d+t)^{m'_j}$.

A value of m'_j greater than one will reduce the survival function and represents an improvement in expected experience. Conversely a value of m'_j less will increase the survival probabilities thus represents a worsening of expected experience.

Under this modification, the impact of the random variable varies according to the underlying termination rates. The random variable m'_j has a greater impact when the termination rates are high (i.e. the early months of disability) and a relatively small impact when the termination rates are low (i.e. after 2-3 years of disability). This is a better representation of the impact of external forces on existing claimants than the constant modification discussed above. In addition, this modification is easier to accommodate into an excel model than the constant modification to termination rates, and thus shortens run times.

As with the previous modification, a new random variable m'_j is simulated for each trial and applied equally to each risk (claimant). The variable m'_j should have mean 1 and variance $\sigma^2_{m'}$ but the nature of this random variable is subjective. For illustrative purposes a lognormal random variable with parameters having mean 1 and standard deviation 0.1 was used.

Data

The sample data used in this paper was obtained from a major actuarial consulting firm with a significant disability practice. The sample was taken from an existing client group with a mature block of disabled lives. For reasons of confidentiality, I have no further information on the group or the individual disabled lives. However, this sample was selected as being representative of a typical block of disabled lives and therefore is appropriate for the purposes of this paper.

A sample of 500 disabled lives was obtained from an existing group of claimants. Data records for each individual include gender, date of birth, date of disability and current benefit amount. Several records were removed; representing claimants with a zero dollar monthly benefit and those who had attained age 65 leaving a block of 488 active claims. These represent a mature block with durations of up to 26 years. Sample statistics are shown in the tables A1-A4 in the Appendix.

This sample was modified to illustrate a number of scenarios as shown in Table 1 below. These modifications were selected to provide representative samples of differing sizes and durations. In addition, modifications were made to investigate the impact of changes in demographics such as age, gender and benefit amount.

Table 1 Data Sets

Scenario	Description	Number of Claims	Average Attained Age	Average Duration (Months)	Average Monthly Benefit
1	Base Model	488	51.3	77.1	$1,706
1A	Subset With Durations < 2 Years	97	47.2	14.4	$2,044
1B	Subset With Durations < 5 Years	234	49.3	28.7	$1,858
1C	Subset With Durations >=2 Years	391	52.4	92.6	$1,622
2	Modified Durations < 2 Years	488	51.3	13.0	$1,706
2a	Modified Durations >=2 Years	488	51.3	82.0	$1,706

Analysis

Basic Model With Full Data

For the base data set (the full unmodified set of 488 claims), a series of 10,000 trials was run. The summary statistics are in Table 2 while Figure 3 shows the histogram of results.

Table 2 Base Model Summary Statistics

Statistics		Percentile		Ratio to Mean
Minimum	$ 54,180,455	**50%**	$ 60,918,227	100.0%
Maximum	$ 67,423,964	**75%**	$ 62,151,413	102.0%
Mean	$ 60,922,683	**80%**	$ 62,443,907	102.5%
Std Dev	$ 1,805,768	**85%**	$ 62,809,934	103.1%
Median	$ 60,918,227	**90%**	$ 63,236,550	103.8%
Mode	$ 61,256,273	**95%**	$ 63,886,737	104.9%

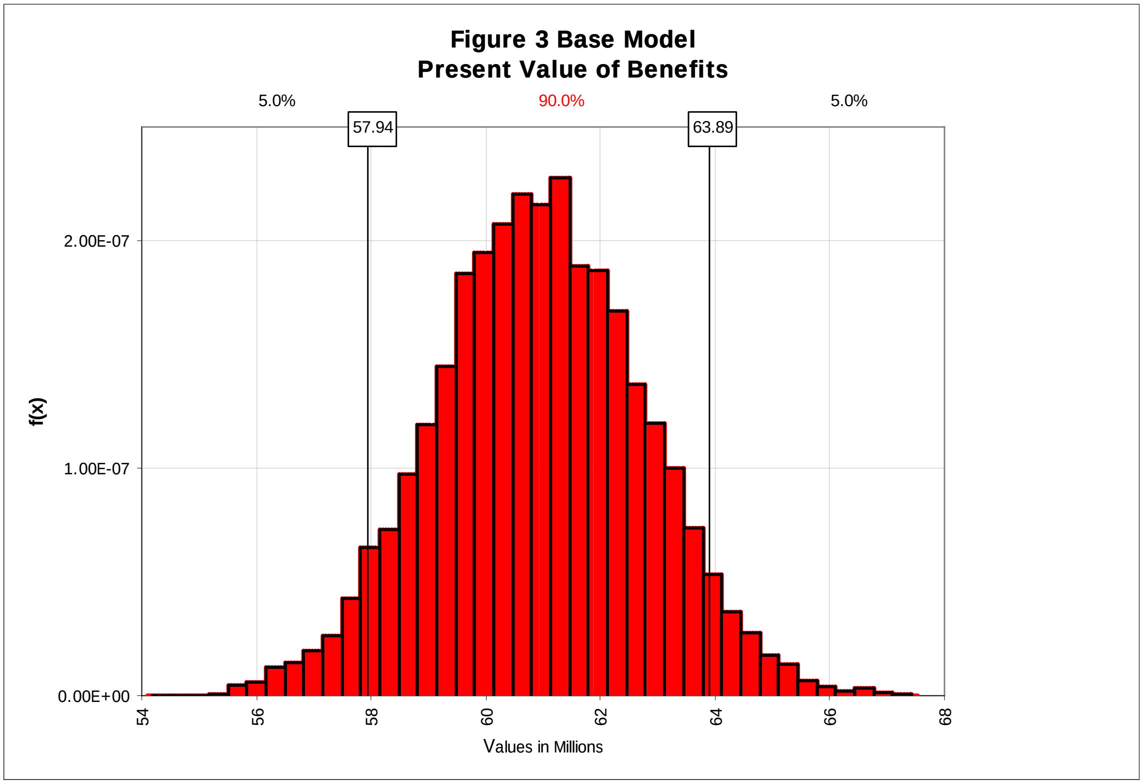
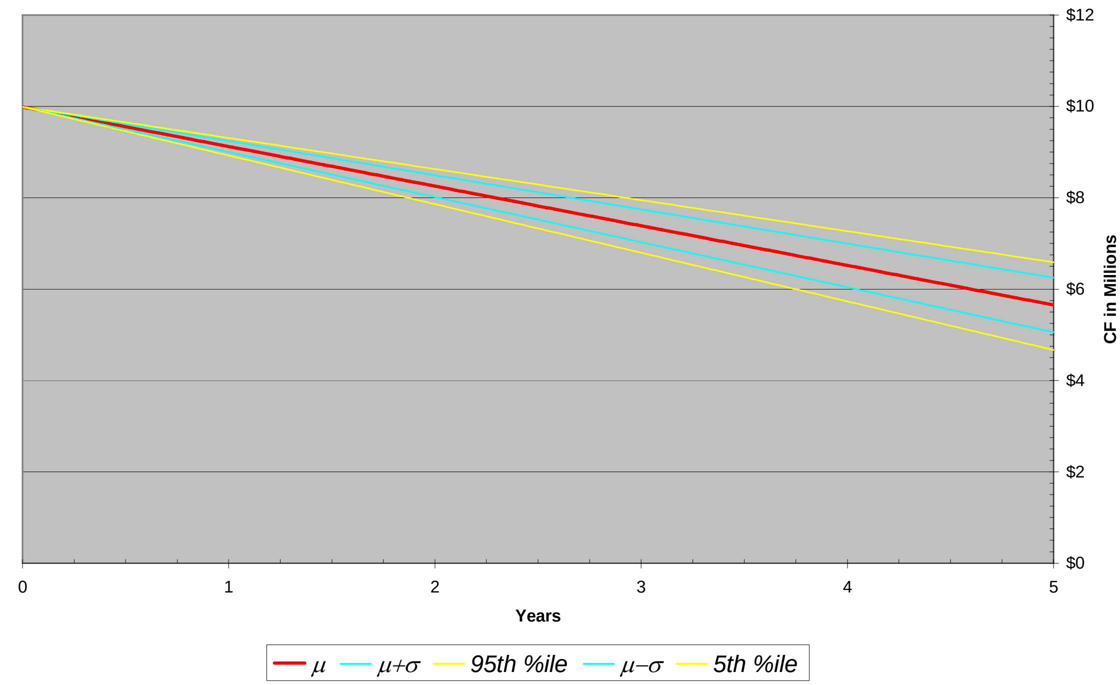


Figure 3 shows the results of the Central Limit Theorem. Although the individual claim random variables are anything but normally distributed (as shown in Figures 1-2 above), the aggregate distribution for the sum of 488 claims is approaching a normal bell curve. However, there is still a significant right tail to the simulated results. Note that 20% of trials are at least 2.5% greater than the mean result and that nearly 5% are more than 5% greater than the mean. With group insurance priced with typical profit margins in the 2-3% of premium range, these tails have significant meaning for the insurer. To put these in perspective, the 80th percentile is equivalent to the use of a Provision for Adverse Deviation (Pfad) of 7.0% (i.e. the expected present value calculated using 93.0% of the tabular termination rates) and the 95th percentile is equivalent to a Pfad of 12.9% (using 87.1% of the tabular rates).

Furthermore, the symmetry of the distribution does not exist if the block of claims is administered under a Refund accounting method. This accounting would effectively left censor the distribution, as the benefit of favourable experience would be ceded to the policyholder in the form of a experience rating refund or claims fluctuation reserve. Using Figure 3, consider the case where a case where the claims fluctuation reserve is set at the maximum of 0 and 95% of expected costs less actual costs. Therefore the minimum cost to the insurer is 95% of expected costs, which in this case is $57.9 million (approximately the 5th percentile of results). Therefore, the insurer remains liable for the full right of possible results but loses any benefit of the left tail.

The results of these trials have been restated in Figure 3A to show projected cash flows over the next five years. This shows the mean values, the mean plus standard deviation, the mean minus one standard deviation and the 5th and 95th percentiles. The cash flows decrease over time, but the variance of results increases significantly.

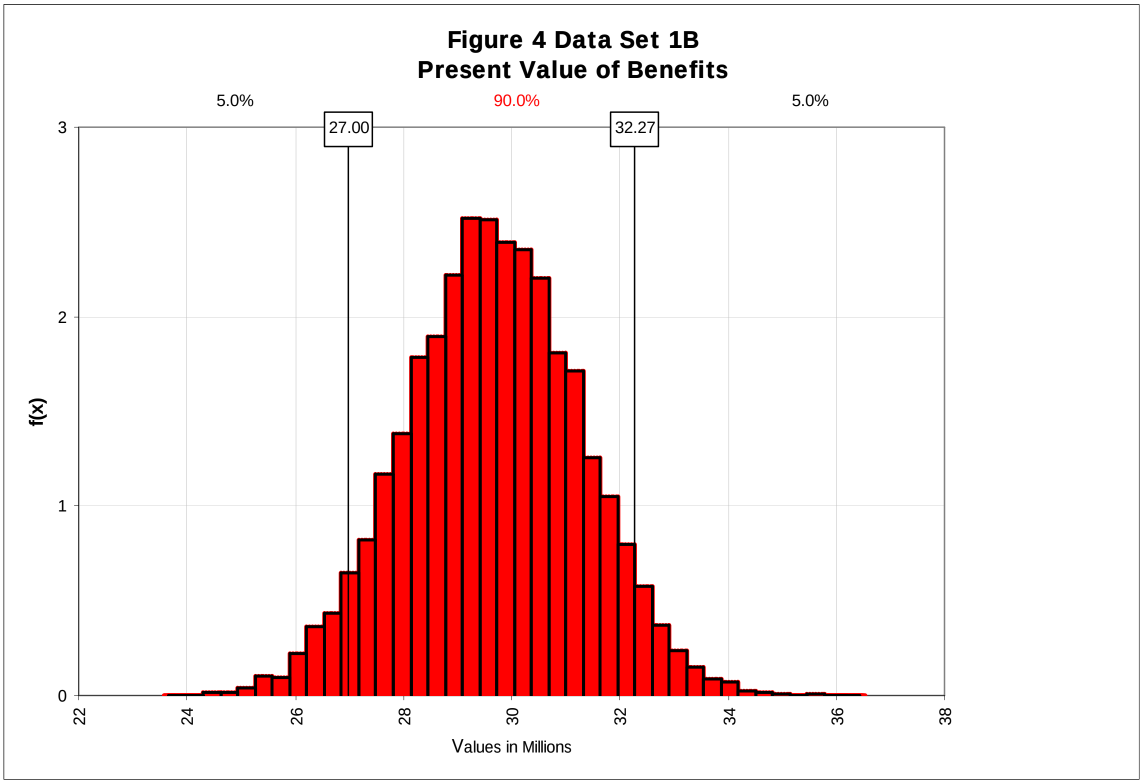
Figure 3A Basic Model Projected Annual Cash Flow



Basic Model With Smaller Subsets of Data

With the smaller subset of 234 claims within five years of disability, the sample statistics and histogram are shown below in Table 3 and Figure 4 (again based on 10,000 trials). Figure 4 also shows an approximate normal shape but the distribution has been flattened right tail and has lost full symmetry. Compared to the full data set, not only has the sample size has been halved but the all claims are of shorter duration disabilities and therefore have the bimodal distribution illustrated above in Figure 1. In this case, the 80% percentile is approximately 4.5% greater than the mean and the 95% percentile is nearly 9% greater than the mean. These correspond to Pfads of 7.6% and 14.2% (i.e. using 92.4% and 85.8% of tabular termination rates) respectively.

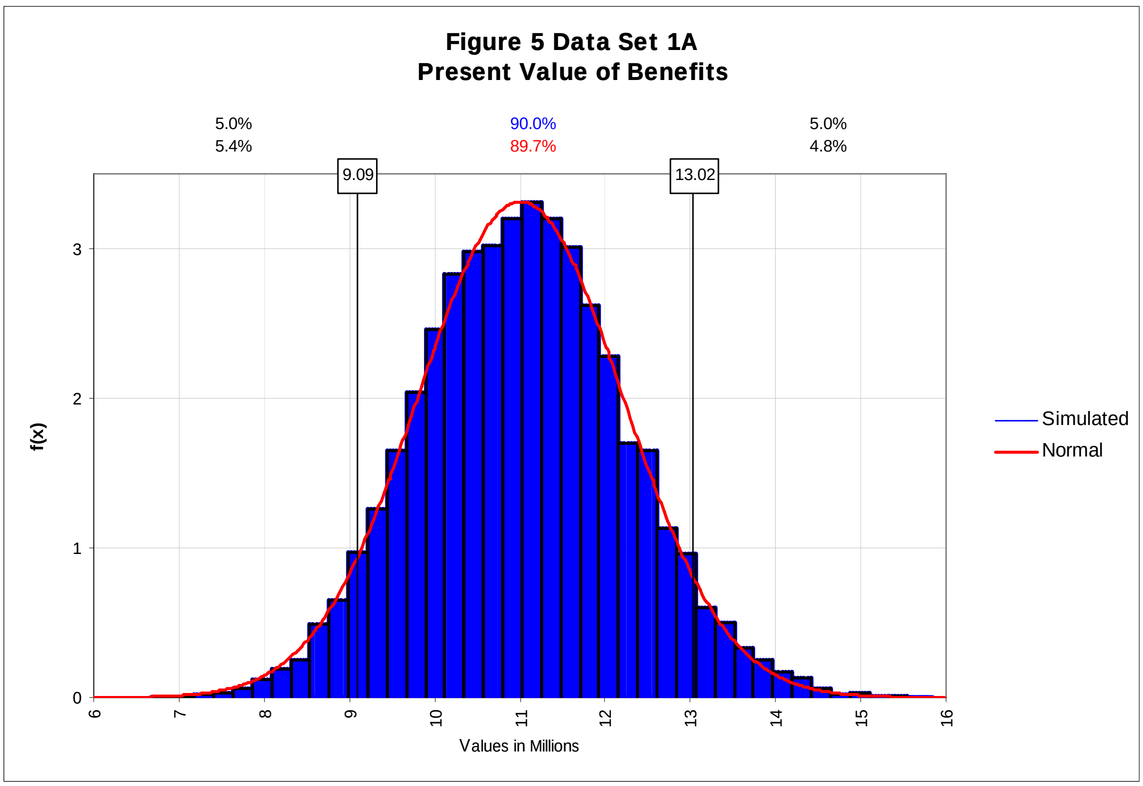
Table 3 Data Set 1B Summary Statistics				
Statistics		**Percentile**		**Ratio to Mean**
Minimum	$ 23,669,616	**50%**	$ 29,659,108	100.01%
Maximum	$ 36,418,844	**75%**	$ 30,731,930	103.63%
Mean	$ 29,656,836	**80%**	$ 31,015,377	104.58%
Std Dev	$ 1,596,005	**85%**	$ 31,312,696	105.58%
Median	$ 29,659,108	**90%**	$ 31,710,659	106.93%
Mode	$ 29,362,875	**95%**	$ 32,269,200	108.81%



With the smallest subset of 97 claims within two years of disability, the sample statistics and histogram are shown below in Table 4 and Figure 5. The histogram still shows an approximately normal shape, which contrasts greatly with the histogram of individual claim durations shown above. (To further illustrate this, a normal distribution has been fit to the trial results and overlaid on the histogram.) However, both the histogram and the statistics show a much longer tails than the larger sample set. In this case the 80%

percentile is approximately 9% greater than the mean and the 95% percentile is over 18% greater than the mean. These correspond to Pfads of 8.3% and 18.4% (i.e. using 91.7% and 81.6% of tabular termination rates) respectively.

Table 4 Data Set 1A (Durations <=2 Years) Summary Statistics				
Statistics		Percentile		Ratio to Mean
Minimum	$ 6,727,662	**50%**	$ 11,015,611	99.90%
Maximum	$ 15,776,995	**75%**	$ 11,827,622	107.26%
Mean	$ 11,026,731	**80%**	$ 12,025,382	109.06%
Std Dev	$ 1,203,539	**85%**	$ 12,275,109	111.32%
Median	$ 11,015,611	**90%**	$ 12,576,507	114.05%
Mode	$ 11,210,940	**95%**	$ 13,024,946	118.12%

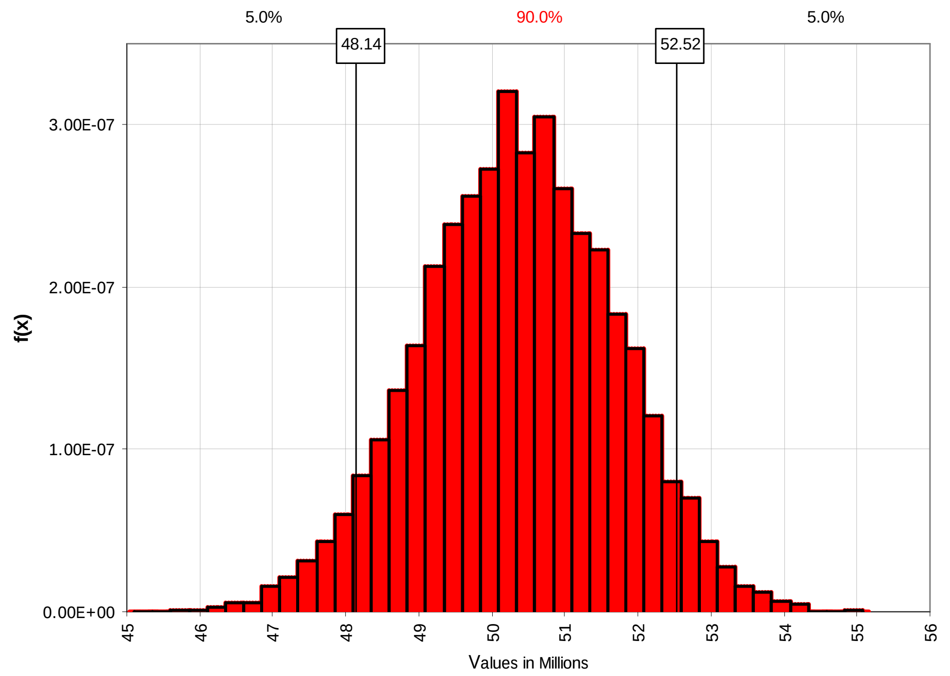


These two subsets of the full data differ from the full sample in two important aspects. First they are obviously considerably smaller (with approximately 20% and 50% of the full 488 claims). In addition, since all claims have shorter durations since disability, a larger proportion of the individual risks have the bimodal distribution shown above in Figures 1-2. With the full data set, the longer duration claims have a smaller variance, due to the high likelihood of continuing on disability to age 65.

Analysis was also performed to consider the claims in the Any Occ period (i.e. mature claims disabled for two years or more) separately. Results for the subset of 391 claims disabled two years or more are shown in Figure 5 and Table 6. The upper percentiles for this sample are similar to the results for the full block of 488 claims, when expressed as a percentage of the mean. However, the histogram is not symmetrical and the normal approximation does not appear appropriate for these results. In fact the left tail appears longer than the right (a result that is consistent with the distribution for mature claims disabled for more than 2 years). Given that a high proportion of these claimants are expected to remain on claim until age 65, deviations from the expected are likely to lead to shorter times on claim and thus smaller costs. This is in complete contrast to the claimants within the Own Occ period, shown in the last two examples. For these subsets, most claimants are expected to terminate prior to the end of 3 years, and therefore deviations from the expected are more likely to result in longer times on claim and high benefit costs.

Table 5 Data Set 1C (Durations >=2 Years) Summary Statistics				
Statistics		Percentile		Ratio to Mean
Minimum	$ 45,363,169	**50%**	$ 50,372,693	100.0%
Maximum	$ 55,600,063	**75%**	$ 51,263,079	101.8%
Mean	$ 50,357,880	**80%**	$ 51,467,135	102.2%
Std Dev	$ 1,325,976	**85%**	$ 51,718,768	102.7%
Median	$ 50,372,693	**90%**	$ 52,056,872	103.4%
Mode	$ 50,550,922	**95%**	$ 52,523,907	104.3%

Figure 6 Data Set 1C
Present Value of Benefits



<u>Basic Model With Modified Data Sets</u>

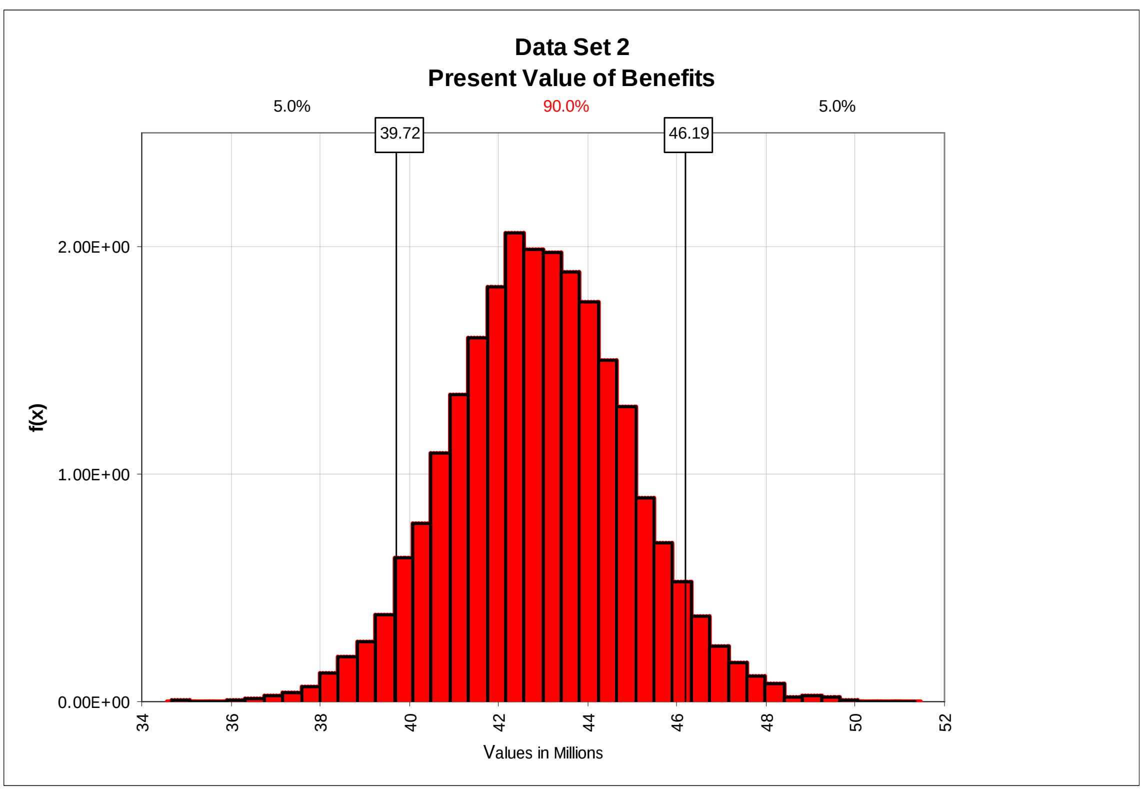
In an attempt to separate the impact of the shorter durations from the smaller sample size, the original data set was modified.

In the following example, dates of disability have been modified so that all claims are within 24 months of disability. This scenario shows a much higher variance and much longer tails than the original (mature) data set. Results are shown in Table 6 and Figure 7. The 80th percentile is 3.8% higher than the mean and the 95th percentile is 7.6% higher.

Table 6

Modified Data Set 2 (<2 years Duration) Summary Statistics				
Statistics		**Percentile**		**Ratio to Mean**
Minimum	$ 34,667,098	**50%**	$ 42,892,794	99.94%
Maximum	$ 51,313,034	**75%**	$ 44,228,787	103.05%
Mean	$ 42,918,130	**80%**	$ 44,555,856	103.82%
Std Dev	$ 1,973,855	**85%**	$ 44,923,530	104.67%
Median	$ 42,892,794	**90%**	$ 45,417,999	105.82%
Mode	$ 42,527,560	**95%**	$ 46,188,021	107.62%

Figure 7



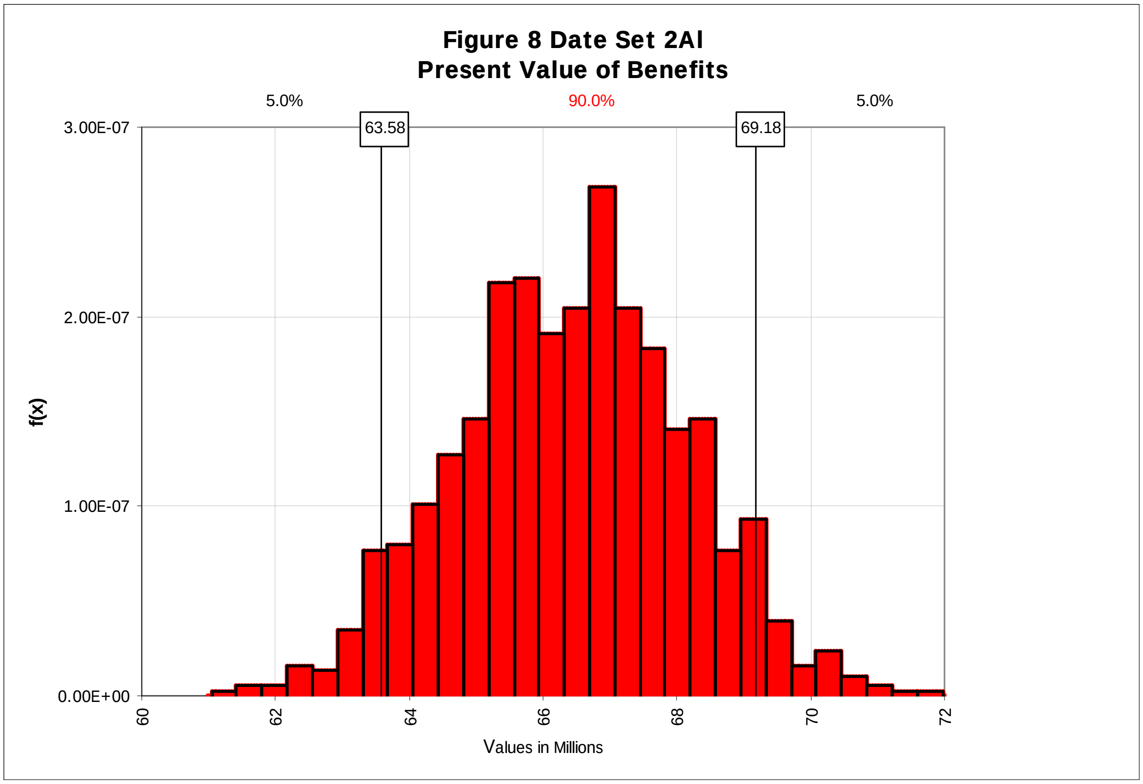
Comparing results based on the same sample size but different risk characteristics, the results based on the modified data set 2 (in Table 6 and Figures 7) show much longer tails than the results based on the unmodified data (in Table 2 and Figure 3). In particular, although the mean for the shorter duration claims is approximately 30% smaller than the mean of the mature block, the standard deviation has increased by approximately 10%. These results show the impact of the proportion of shorter duration claims on the volatility of aggregate results.

Comparing results based on the differing sample sizes but similar risk characteristics, the results based on larger modified sample (in Table 7 and Figure 8) show smaller tails than the results based on the smaller unmodified data set (in Tables 4 and Figure 5). As to be expected, increasing the sample size decreases the variance.

In the next example, dates of disability have been modified so that all claims have been disabled for 2 years or more. Results are shown in Table 7 and Figure 8. These results are similar those shown Table 5 and Figure 6 for a smaller data set with similar risk characteristics. However the inherent volatility of the LTD benefit can be seen. Even with the mature sample and larger sample size, the spread of results is significant.

Table 7 Modified Data Set 2A (>2 years Duration)
Summary Statistics

Statistics		Percentile		Ratio to Mean
Minimum	$ 61,042,114	**50%**	$ 66,491,305	100.08%
Maximum	$ 71,969,138	**75%**	$ 67,610,565	101.77%
Mean	$ 66,437,606	**80%**	$ 67,864,206	102.15%
Std Dev	$ 1,706,313	**85%**	$ 68,266,021	102.75%
Median	$ 66,491,305	**90%**	$ 68,590,214	103.24%
Mode	$ 65,828,626	**95%**	$ 69,178,049	104.12%



Modified Models to Simulate Dependency of Risks

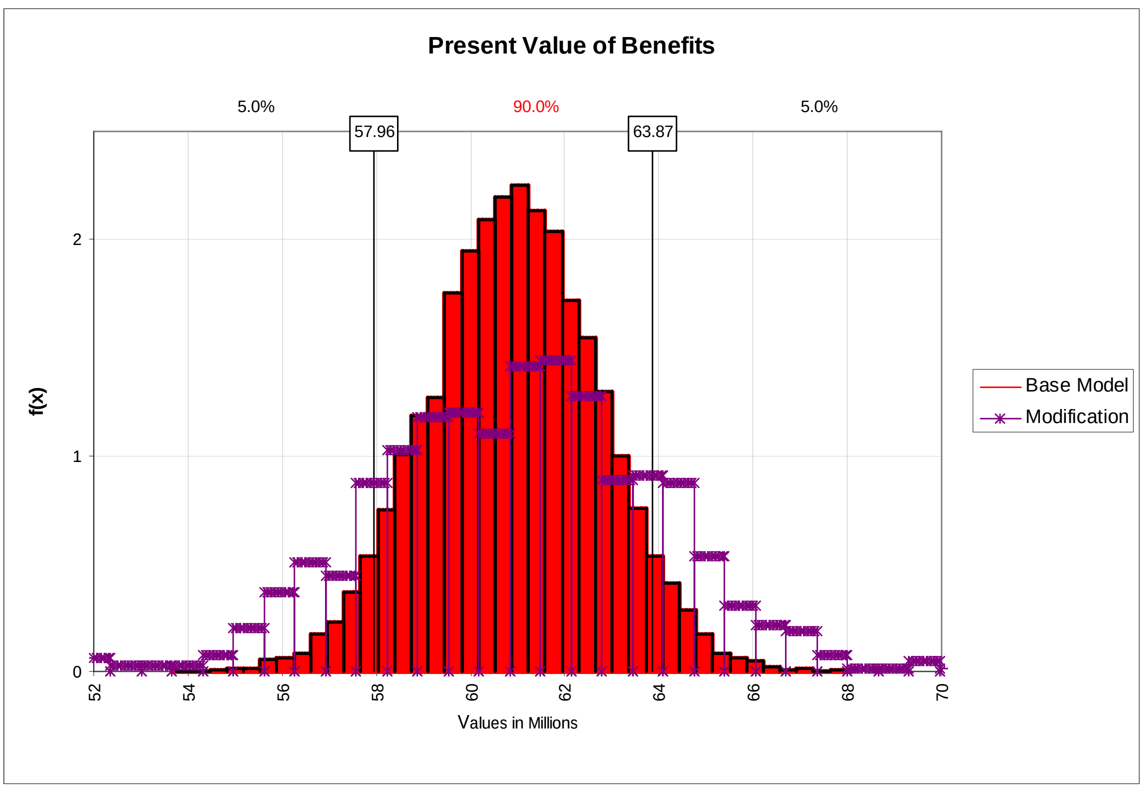
When the modifications for inter-dependencies of risk of added to the model, the additional variation in results is marked. Since the simulation becomes much more complex and running times extend considerably, only 1000 trials were used for illustration. The full unmodified data set was used for these simulations.

In the first modification the termination rates are modified as follows $q'_{[x]+d} = q_{[x]+d} \cdot m_j$ where m_j is a normal random variable with mean 0 and standard deviation 0.1. Statistical Results are shown in Table 8and the histogram in Figure 9. The equivalent output for the base model is shown for comparison. In Figure 9 the approximately normal shape remains but the variance and increased dramatically and thus the tails are much fatter.

Table 8

Summary Statistics Using Full Data Set (First Modified Model)					
Statistic	**Base Model**	**Modified Model**	**Percentile**	**Base Model**	**Modified Model**
Minimum	$54,180,455	$51,697,289	**50%**	$60,918,227	$61,011,923
Maximum	$67,423,964	$70,614,394	**75%**	$62,151,413	$62,870,914
Mean	$60,922,683	$60,989,952	**80%**	$62,443,907	$63,558,263
Std Dev	$1,805,768	$2,902,786	**85%**	$62,809,934	$64,085,395
Median	$60,918,227	$61,011,923	**90%**	$63,236,550	$64,646,125
Mode	$61,256,273	$60,992,841	**95%**	$63,886,737	$65,538,258

Figure 9

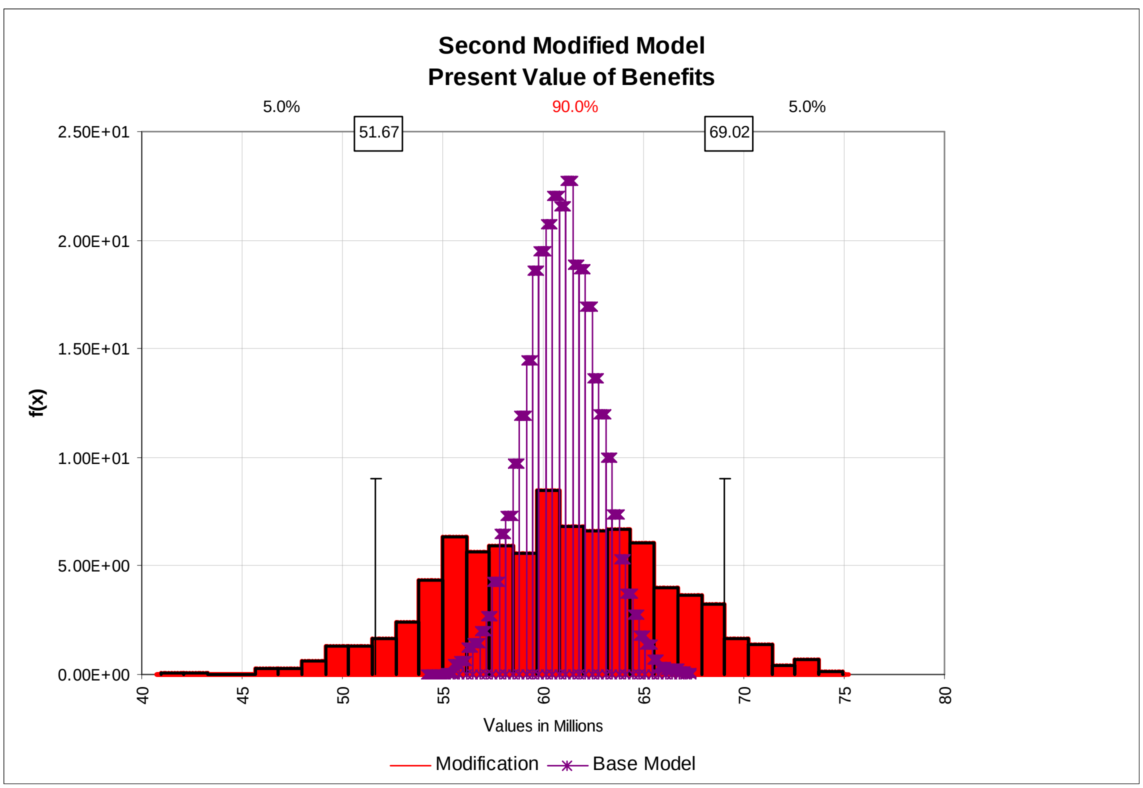


In the second modification, the survival function is modified as follows $S'(t) = S(t)^{m_j}$ where m_j is a lognormal random variable with mean 1 and standard deviation 0.1. Results are shown in Table 9 and Figure 10 (again the output for the base model is shown for comparison). With the modification, the changes in the distribution are quite dramatic. Figure 10 shows that distribution has been flattened considerably, has lost its symmetry and the tails are considerably fatter.

Table 9

Summary Statistics Using Full Data Set (Second Modified Model)					
Statistic	**Modified Model**	**Base Model**	**Percentile**	**Modified Model**	**Base Model**
Minimum	$ 40,956,179	$ 54,180,455	**50%**	$ 60,707,663	$ 60,918,227
Maximum	$ 74,897,099	$ 67,423,964	**75%**	$ 64,325,593	$ 62,151,413
Mean	$ 60,608,893	$ 60,922,683	**80%**	$ 65,152,956	$ 62,443,907
Std Dev	$ 5,294,871	$ 1,805,768	**85%**	$ 66,083,213	$ 62,809,934
Median	$ 60,707,663	$ 60,918,227	**90%**	$ 67,421,475	$ 63,236,550
Mode	$ 60,676,478	$ 61,256,273	**95%**	$ 69,018,936	$ 63,886,737

Figure 10

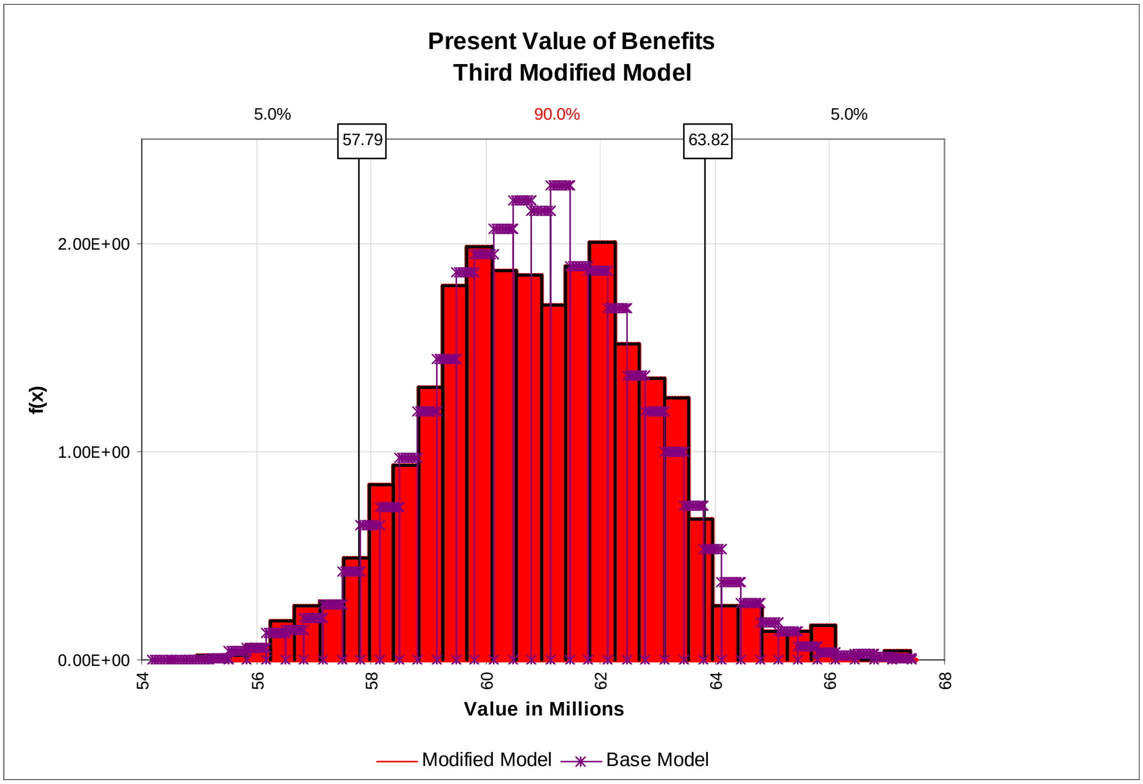


Since the results of the second modification are very dramatic, this model was rerun with a less severe modification. In this case the random variable m_j is a log-normal random variable with mean 1 and standard deviation 0.01. Results are shown in Table 10 and Figure 11 (again the output for the base model is shown for comparison). With the modification, the changes in the distribution are less dramatic. Table 10 shows that most of the statistics for the modified model are very close to the equivalent statistics for the base model. However, Figure 11 shows that distribution has been flattened considerably and the tails are fatter.

Table 10

Summary Statistics Using Full Data Set (Third Modified Model)					
Statistic	**Modified Model**	**Base Model**	**Percentile**	**Modified Model**	**Base Model**
Minimum	$ 54,961,827	$ 54,180,455	**50%**	$ 60,931,877	$ 60,918,227
Maximum	$ 67,390,671	$ 67,423,964	**75%**	$ 62,239,808	$ 62,151,413
Mean	$ 60,922,273	$ 60,922,683	**80%**	$ 62,527,758	$ 62,443,907
Std Dev	$ 1,903,582	$ 1,805,768	**85%**	$ 62,889,700	$ 62,809,934
Median	$ 60,931,877	$ 60,918,227	**90%**	$ 63,293,344	$ 63,236,550
Mode	$ 61,489,856	$ 61,256,273	**95%**	$ 63,824,888	$ 63,886,737

Figure 11



Conclusions and Recommendations

The following conclusions and recommendations follow from the analysis described above.

With respect to modeling LTD liabilities, the following results can be seen

.

1) The underlying distribution for the cost of LTD benefits for an individual claimant has a long right tail and a trigger point after 2 years of benefits as shown in Figures 1 and 2. Therefore an actuary can gain insights into LTD risks by using a stochastic treatment to supplement existing deterministic models.

2) A range of external and internal forces can significantly impact termination experience and therefore the cost of an LTD plan. The inter-dependencies between individual LTD risks are best modeled using stochastic techniques.

3) A stochastic model for LTD experience can be developed using common Windows-based applications. The model used for the illustrations contained in this paper was created in MS Excel using the @Risk add-on.

4) The additional variance and risks arising from the external forces can be added to a basic stochastic model but at a cost of complexity and computing time. Two such methods are presented as options to achieve this goal. While the choice of methodology and nature of the random variables used for this simulation are highly subjective, these techniques do provide valuable insight into the potential increase in risk arising from external forces.

5) The large sample sizes mitigate the extreme cliffs and long tails seen in the single life loss survival models. However, the resulting aggregate model retains a high variance. While the simulated aggregate distribution function retains a bell shape, the bell is quite flattened and extreme results are possible.

This paper was intended to illustrate the potential use of stochastic models for LTD valuation. Therefore, detailed recommendations with respect to management of these risks are beyond its scope. However, the following generalizations can be made.

1) As expected, a larger sample size can considerably dampen the potential for upside risk. This can be seen from comparing Data Sets 1A and 2, and to a lesser extent by comparing Data Sets IC and 2A. In both cases, we are comparing samples of differing sizes but similar risk characteristics. The difference is most apparent between the two groups within the Own Occ period both due to the differences in samples and the additional variation for these claims.

Data Set	**Description**	**Number of Lives**	**Mean**	**Ratio 90^{Th} Percentile to Mean**
IA	Durations <2 years	97	$11.0 m	114.1%
2	Durations <2 years	488	$42.9 m	105.8%
IC	Durations >2 year	391	$50.4 m	103.4%
2A	Durations >2 year	488	$66.4 m	103.2%

2) A new group of LTD claims has significantly higher risks than a mature block of LTD claims obtain a larger block of claims. This can be seen from

comparing the results for Data Sets 1, 2 and 2A, which are based on the samples of equivalent size but differing characteristics.

Data Set	Description	Number of Lives	Mean	Ratio 90^{Th} Percentile to Mean
I	All Durations	488	$60.9 m	103.8%
2	Durations <2 years	488	$42.9 m	105.8%
2A	Durations >2 year	488	$66.4 m	103.2%

3) Unfortunately, while the insurer's ability to pool LTD claims blocks will be severely limited in many cases by the financial underwriting arrangements in place. Even if the insurer's total LTD block may be both large and mature, this may have to be evaluated as a series of separate blocks of varying size and maturity. Therefore the impact of a few new LTD blocks on the total bottom line is magnified.

Appendix A Summary of Data Used

Table A1 Base Data Set by Age at Disability and Gender

Age at Disability		Male	Female	Total
20	25	0	3	3
25	30	2	13	15
30	35	9	34	43
35	40	25	56	81
40	45	20	78	98
45	50	30	71	101
50	55	30	73	103
55	60	6	33	39
60	65	3	2	5
Total		125	363	488

Table A2 Base Data Set by Attained Age and Gender

Attained Age		Male	Female	Total
20	25	0	0	0
25	30	0	1	1
30	35	0	13	13
35	40	6	22	28
40	45	13	43	56
45	50	18	78	96
50	55	45	70	115
55	60	28	86	114
60	65	15	50	65
Total		125	363	488

Table A3 Base Data Set by Duration of Disability and Gender

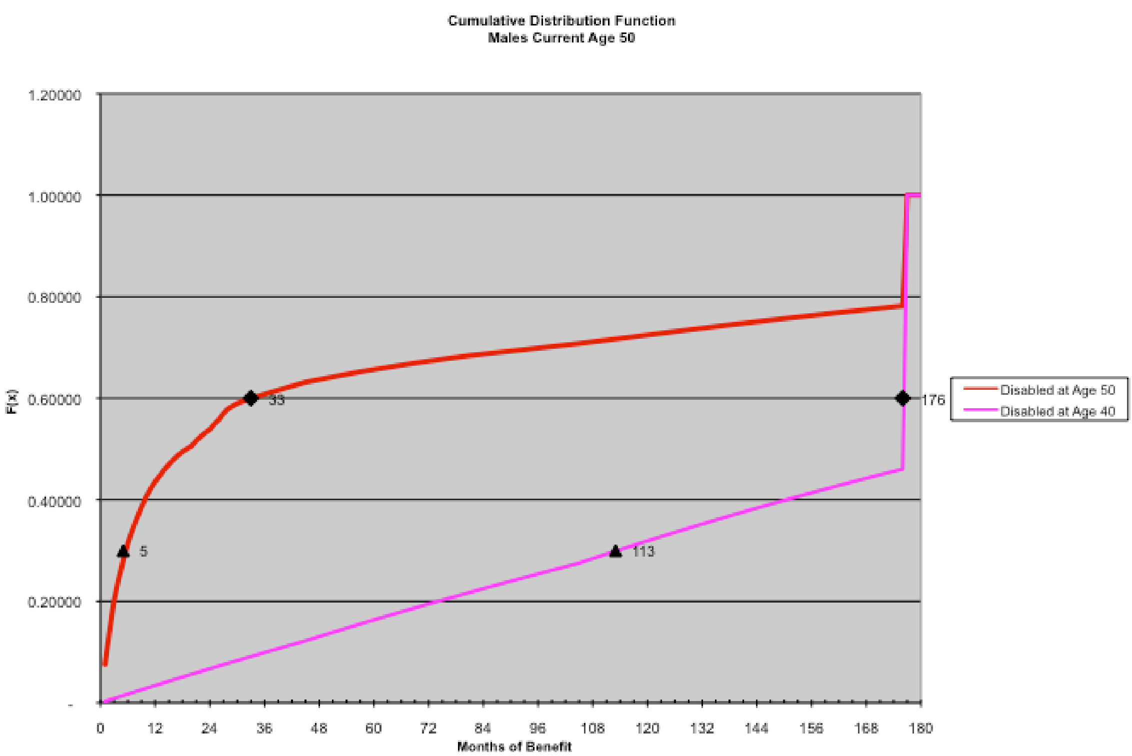
Duration in Months		Male	Female	Total
3	12	6	28	34
12	24	17	46	63
24	36	17	44	61
36	48	14	28	42
48	60	9	25	34
60	72	12	32	44
72	84	6	21	27
84	96	6	27	33
96	or More	38	112	150
Total		125	363	488

Table A4 Base Data Set by Benefit Amount and Gender

Benefit Amount		Male	Female	Total
$0	$100	2	0	2
$100	$500	1	6	7
$500	$1,000	2	13	15
$1,000	$1,500	49	156	205
$1,500	$2,000	33	91	124
$2,000	$3,000	32	92	124
$3,000	$4,000	4	4	8
$4,000	$5,000	2	0	2
$5,000	Or More	0	1	1
Total		125	363	488

Appendix B Simulation Examples

The following chart shows the cumulative distribution function for two male individuals currently aged 50 years and 3 months. One individual (A) was disabled at exact age 40 (i.e. duration of claim 10.25 years) and the other (B) was disabled at exact age 50 (duration 0.25 years) and has just satisfied a 3 month elimination period. The horizontal axis illustrates the number of months that the individual remain disabled up to a maximum of 177 (age 65 in each case). The vertical axis shows the cumulative distribution function F(x).



For example consider the case where the simulated uniform random variables were 0.3 for claimant A was 0.6 for claimant B. The inversion technique shows that the for A, F(5)=0.2802<0.3< F(6) =0.3190. Therefore, for this trial A is assumed to remain disabled for 6 months. Similarly for B, F(176)=0.4604<0.3<F(177)=1 so B is assumed to remain disabled for 177 months (age 65). If A receives a benefit of $500 and B a benefit of $600 per month, the simulated present value of benefits at i=5% is

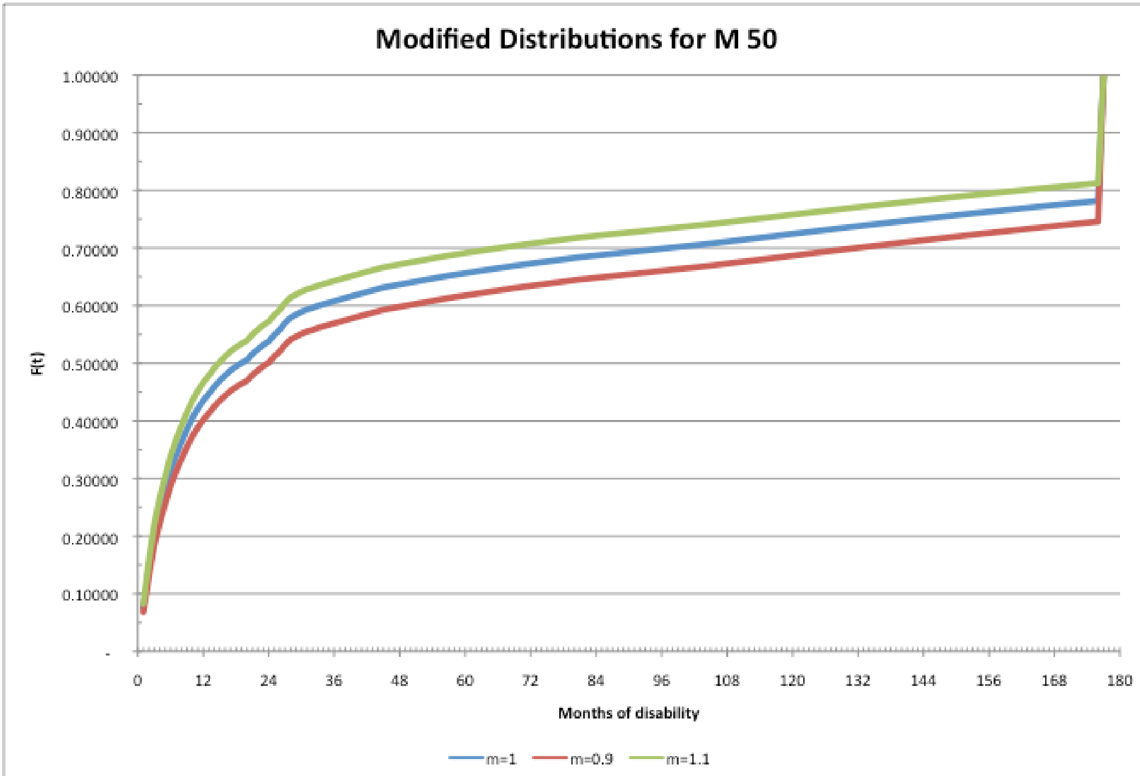
$$PV = 500 a_{\overline{6|}0.41\%} + 600 a_{\overline{177|}0.41\%} = \$78,519.$$

Now consider the case where the simulated uniform random variables were 0.6 for claimant A was 0.3 for claimant B.. For A, F(33)=0.5996<0.6< F(34)=0.6024 so for this trial A is assumed to remain disabled for 34 months. For B, F(113)=0.2988<0.3<F(114)=0.3017 so B is assumed to remain disabled for 114 months. Now, the simulated present value of benefits at i=5% is

$$PV = 500 a_{\overline{33|}0.41\%} + 600 a_{\overline{113|}0.41\%} = \$69,659.$$

This process is repeated for the entire block of claims (up to 500 individuals of varying ages and durations of disability) for each of 10,000 trials.

The modifications for risk dependences are shown below, using the second modification (the modification takes the form $S'(t) = (S(t))^m$). This shows the cumulative distribution function for claimant A (a male disabled at 50 and at current age 50 years and 3 months), with the random variable ***m*** =0.9, 1.0 and 1.1. The 30th percentile of the modified distribution is 5 months when ***m*** =1.1 and 7 months when



m =0.9, compared to 6 months when ***m***=1.0 (i.e. the unmodified distribution). Similarly, the 60th percentile of the modified distribution is 27 months when ***m*** =1.1 and 50 months when ***m*** =0.9, compared to 34 months when ***m***=1.0 (i.e. the unmodified distribution).

Appendix C Sources

Actuarial Models

General information can be found in the following standard actuarial texts.

- *Actuarial Mathematics* (Second Edition), by Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A. and Nesbitt, C.J.
- *Loss Models: From Data to Decisions,* (Third Edition), by Klugman, S.A., Panjer, H.H. and Willmot, G.E.
- *Simulation* (Fourth Edition) by Ross, Sheldon M.

Long Term Disability

Background on Long Term Disability Insurance and historical experience can be found in the following published morbidity studies.

- *Group Long-Term Disability (GLTD) Valuation Tables* Originally published in *Transaction of the Society of Actuaries, 1987 Volume 39* and available online at http://www.soa.org/library/research/transactions-of-society-of-actuaries/1987/january/tsa87v3913.pdf. Although based on US data that is over 20 years old, the 1987 GLTD remains the base valuation table used by most Canadian insurance companies.

- *Canadian Group Long-Term Disability termination Experience 1988-1994* (1998) available online to members of the Canadian Institute of Actuaries. The 1998 CIA table represents the most recent publically available study of Canadian LTD experience, although a study based on 1988-1997 data is expected to be released in 2009.

- *Group Life Waiver Study Based On 1988-1994 Canadian Group Long-Term Disability termination Experience* (2001) available online to members of the Canadian Institute of Actuaries. This study used the same data as the previous LTD study, but treated mortality and recovery as separate contingencies (the prior study grouped these together as terminations).