

**“Undergraduate Research in
Actuarial Science and Financial Mathematics
At the University of Illinois”**

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Abstract

A program of undergraduate research in actuarial science and financial mathematics has been implemented at the University of Illinois over the last two years. This program has included two National Science Foundation-sponsored Research Experiences for Undergraduates (in Summer 2007 and Summer 2009) on the topic “Stochastic Modeling in Actuarial Science and Financial Mathematics,” as well as a variety of academic-year research projects undertaken by current actuarial science majors. This paper summarizes our evolving experiences in providing research opportunities to undergraduates, and highlights some of the topics and issues arising from these research experiences.

Section 1: Introduction

Providing opportunities for undergraduate research has been a recent priority for many organizations – for example the National Science Foundation (NSF), and several universities, including the author’s home institution, the University of Illinois at Urbana-Champaign (UIUC). There are many reasons for the interest in initiating programs of undergraduate research. Students benefit from the intellectual stimulation and enhanced experiences provided by such opportunities, as well as the chance to augment the technical skills involved. It is also good for

both future theoreticians and practitioners to be exposed to potential extensions of current knowledge, and to the process of exploring and communicating advances in their fields.

With respect to actuarial science specifically, the actuarial profession will certainly benefit from pushing research activity and familiarity into the classroom; it is easier and more effective to embed an appreciation for research and exploration relatively early in an actuary's career. Furthermore, to the extent that many actuaries conclude their academic careers after achieving their bachelor's degrees, undergraduate research may be one of the few, and perhaps the last, opportunity for an actuary to be exposed to, and convinced of their opportunities to contribute to, research which will advance the profession.

This paper discusses issues involved with providing research opportunities for undergraduate students in actuarial science and financial mathematics. In particular, these issues are discussed in the context of recent undergraduate research initiatives in UIUC's actuarial science program. Section 2 describes the UIUC research opportunities. Section 3 discusses some of the issues and procedural considerations involved in offering research opportunities to undergraduate students. Section 4 concludes, and the Appendix provides a summary of a recent summer research offering at UIUC.

Section 2: Research at the University of Illinois at Urbana-Champaign

Several research initiatives directed toward undergraduate students in the actuarial science program at UIUC have recently been implemented.

Undergraduate research associates program

During the last couple of years, several UIUC undergraduate students in the actuarial science program have been engaged as research associates, each working on a specific topic for either one or two semesters. The standard approach has been for each student to work on the research project for about ten hours per week, with a regularly-scheduled weekly meeting with a professor / mentor / guide. Depending upon the particular topic, the deliverable product emerging from the project could be a written report/article, a computer program, a written literature review, etc. In some cases, call papers have been attempted for different units of the Casualty Actuarial Society and the Society of Actuaries.

Some of the research topics undertaken in this program have included the following:

- **Neuroeconomics / cognitive biases** – the potential impact of recent research concerning human cognitive and behavioral “quirks” on insurance and risk management decisions.

- **Power laws** – the mathematical and conceptual underpinnings of power laws, and understanding their apparent frequent applicability throughout economics, finance, and other fields.
- **Predator-prey models** – the potential for the use of such models to reflect competitive economic and financial dynamics.
- **Agent-based models** – the technical and conceptual foundations of ABM, and their potential applicability to actuarial science and risk management.
- **Fuzzy modeling of risk** – the mathematical and conceptual underpinnings of fuzzy random variables, and their representations of qualitative characteristics in enterprise risk management.
- **Housing wealth** – emerging financial securities and processes such as reverse mortgages and equity releases.
- **Public pensions** – the status and economics of public pension funds, and an exploration of the possibility of securitizing pension claims.

State Farm Research Center

Several years ago, the State Farm Insurance Companies created a corporate Research Center, and placed it in the Research Park of the University of Illinois, in Champaign, IL. Each academic semester, many UIUC actuarial science students are hired for part-time internships, and assigned to work in teams on significant corporate research projects.

In-class research

To the extent possible, opportunities for research and real-world-type problem-solving and analyses are provided to UIUC undergraduate students in their classes. Such opportunities can take the form of current event reports (which are often made available publicly, and linked to the author's web page), case studies, or group projects associated with a realistic actuarial problem.

Research Experience for Undergraduates

In addition to the above-mentioned opportunities, which are directed toward UIUC actuarial science students, the University of Illinois Department of Mathematics has been running two NSF-sponsored Research Experiences for Undergraduates (REUs) per summer for several years. These REUs are open to applications from qualified undergraduate students across the country. In 2007 and 2009, one of the REUs offered at UIUC, and directed by the author, was "Stochastic Processes in Actuarial Science and Financial Mathematics."

Per the general NSF program description literature:

“The Research Experiences for Undergraduates (REU) program supports active research participation by undergraduate students... in ongoing research programs or in research projects specifically designed for the REU program... and to help ensure that they receive the best education possible... Research experience is one of the most effective avenues for attracting talented undergraduates to, and retaining them in careers in, science and engineering, including careers in teaching and education research... projects are also encouraged, when appropriate, to involve students at earlier stages in their college experience.”

Each summer for the last several years, UIUC has held two 8-week REU programs, each intended for six participants. Over the years, the subject matter of the REUs has rotated and included the following:

- Evolutionary Game Theory
- Geometric Group Theory
- Number Theory
- Harmonic Analysis
- Visualization in Virtual Environments
- Stochastic Modeling in Actuarial Science and Financial Mathematics

Per the UIUC REU program literature:

“Our students join the working environments of faculty mentors, their postdocs, graduate students and collaborators. Students work on individual or small group projects in their mentor's current research area.”

A report on the 2007 Stochastic Processes REU was given at the August 2007 Actuarial Research Conference, and published in the accompanying edition of the *Actuarial Research Clearinghouse*. As documented in that summary report, the titles of the six projects from the 2007 REU were:

- “A Model for Increasing Black Life Expectancy by Reducing Infant Mortality, Homicide, and HIV”
- “A Markov Chain Approach to Modeling Cancer Risk”
- “A Regime-Switching Model for Foreign Exchange Rates”
- “On the Statistical Properties of Interest Rates and a Basic Regime-Switching Model”
- “Stochastic Modeling for Illinois Prepaid Tuition Contracts”
- “A Fractal-Geometric Approach to Risk Management and Mitigation”

A summary report of the 2009 Stochastic Processes REU is presented in the Appendix at the end of the present paper.

Section 3: Issues Concerning the Offering of Undergraduate Research Opportunities

In the process of implementing these research opportunities for undergraduate students, the author has encountered a number of considerations and issues, the responses to which can help to construct a framework for the research.

- (1) What is the overall *objective* associated with an undergraduate research project or program – with respect to both the student and the mentor / professor? Generally, the goal will be to provide an opportunity to the student, early in her/his career, to be exposed to the research process, and to experience its inherent value. Another possible function of undergraduate research could be to assist the faculty member with her/his research efforts. In either case, the underlying motivation for the research should be thoroughly identified and understood from the beginning.
- (2) What form of *deliverable* is expected of the student? Identifying the specific end-product and the delivery date (and, ideally, some intermediate deadlines on the way to the final product) at the beginning of the project will help the student to focus, and to gain a feel for the degree to which different activities can advance the project's cause. Possible deliverables could include a paper (possibly in response to a call for papers, which can involve subject matter that might be within the realm of undergraduate abilities), a literature review, a presentation, data analysis, or some combination of these or other items.
- (3) What is the specific *nature* of the research? In particular, is the research an isolated project, or part of a longer, continuing or emerging effort? The answer will likely impact how the student is brought up to speed and introduced to the material.
- (4) How is the research *topic* determined? Assignment of a topic to the student by the mentor has several potential advantages: greater interest and involvement by the mentor; a topic that is appropriate within the context of the broader research field, and the student's abilities; quicker involvement by the student in the meat of the project. Probably most undergraduate research project topics are determined in this way. On the other hand, there can be some benefits to the student's selecting a research topic: if the professor has several students and projects to mentor, it can be difficult to identify multiple and simultaneous topics; the project may better sustain the student's interest; the process of discovering and identifying a research topic is a significant

and integral part of the general research process, and exposure to this can be very valuable for the student.

- (5) What **resources** should be assigned to each topic? In particular, should a student work independently, or as part of a team of two or more students dedicated to a project? Both types of experiences can be valuable – and of course potential relative advantages and disadvantages apply to each.
- (6) What should be the **timing** of the project? Some projects can be successfully completed during a single semester (or even a partial semester); others may require more than one semester. For multi-term projects, whether the original student is continuing on the project, or another student takes over at an intermediate stage, the transition from one term to the next requires attention, and can potentially be the source of extreme inefficiencies. Another issue regarding timing could be whether the project is better undertaken during an academic year, or over a break such as the summer.
- (7) What form and frequency of **guidance** should the mentor employ? Different students may react more or less effectively to different approaches – step-by-step guidance with the project effectively broken up into reasonably-sized bites, or specifying the general topic or goal, and letting the student find creative ways to proceed.
- (8) Should **payment** be made for a student’s research activities? Often, the choice is between payment (creating an analogy to a graduate research assistant), and academic credit – a university may well be prohibit a student from receiving both.

In addition, and often related to one or more of the above items, the author has noted several issues associated with the mentoring process:

- Because faculty tend to be more involved with graduate students regarding research, it is important to recognize and remember that undergraduates can differ significantly from graduate students with respect to intellectual **maturity** level.
- A big issue, as alluded to under “guidance” above, involves how hard to “**push**” a student, and what form that pushing / encouragement / direction should take.
- As a mentor trying to engage and teach the student as much as possible, it is important to discuss with the student the broader **perspective and context** of the research – how the research fits into risk management and the actuarial profession as a whole. The broader economic / financial context may even be of greater long-term interest and benefit to the student than the analytical / mathematical / statistical aspects of the research.

Section 4: Conclusions

In the author's experience, the best undergraduate students in actuarial science are extremely capable individuals, for whom a productive research project is a completely reasonable goal. An appropriate interdisciplinary research experience can be a tremendous capstone opportunity for an undergraduate. The mentor can aid the process in many ways, including by encouraging the student to identify the research topic very early in the project (or become familiar with the topic as quickly as possible, if it has been assigned by the mentor), and to promulgate frequent interim deadlines throughout the project.

Appendix: 2009 NSF-Sponsored Research Experience for Undergraduates – Summary Report

From June 15 through August 7, 2009, the University of Illinois at Urbana-Champaign (UIUC) Department of Mathematics hosted an NSF-sponsored Research Experience for Undergraduates on the topic "Stochastic Modeling in Actuarial Science and Financial Mathematics." This Experience, provided to 6 undergraduate mathematics students from universities across the United States, was directed by Professor Rick Gorvett, the Director of the UIUC Actuarial Science Program. This report summarizes the program, identifies the participants, and describes the research projects undertaken.

Overview of Program

Actuarial science and financial mathematics have evolved significantly over the last couple of decades. The level of sophistication of their mathematical and financial content, along with their potential range of applications, have increased enormously. Recent economic and financial events and uncertainties reflect the fact that we live in a stochastic and ever-riskier world – with risks both natural and man-made – and that mathematical, financial, and analytical skills are critical for identifying, quantifying, understanding, and managing the impact of those risks.

This summer research experience provided a select group of undergraduate students with an opportunity to explore and research the mathematical modeling of stochastic processes in an actuarial and/or financial context. The program began with the participants obtaining a theoretical background in several key areas: stochastic processes (including Brownian motion, stochastic differential equations, and Poisson jump processes); risk theory (beginning with the

classic Cramér-Lundberg model); and Monte Carlo simulation. Opportunities to explore both mathematical theory and hands-on use of simulation models and techniques were provided.

Then, building upon their exposure to foundational concepts, participants explored and pursued several possible research directions. Participants, depending upon their specific interests, undertook both mathematical and data-analytical types of research projects. The program culminated with each participant making progress in writing a research paper, and making a presentation to a general admission audience at UIUC.

Program Participants

Six students, from six U.S. colleges and universities, were chosen from an extremely competitive applicant pool (which numbered 60). Participants were from the following universities:

- North Carolina State U.
- Otterbein College
- University of Illinois at Urbana-Champaign
- University of Oregon
- University of South Carolina
- Whittier College

Subject Matter and Materials Introduced

The first two or three weeks of the REU involved an intensive introduction to relevant subject matter, and the suggestion and exploration of possible research projects. Specific reading materials, data sources, and software tools that were introduced to all participants during the REU are documented in a later section of this report.

Participants were provided “lecture notes” and other materials as subjects were covered. Topical highlights from this front part of the REU included:

- Review of statistics and probability theory
- Markov chains and Markov processes
- Introduction to actuarial science and finance
- Poisson processes
- Brownian motion
- Risk and ruin theory
- Monte Carlo simulation

- Interest rate modeling
- Economic scenario models

Participant Research Projects

After the introductory materials described above, the participants each identified a research topic (with the help of the program director) and read the recent relevant research literature, refining and exploring their topics. While joint-participant projects were allowed, in the end each student chose to work on their own individual project. The program director generally met individually at least once per week with each student. Group gatherings also generally occurred each week; these meetings provided opportunities for the participants to share their research experiences, bounce ideas off of each other, and to communicate items of general interest.

With the exception of two projects with a similar theme (but with different specific foci), the projects undertaken in this REU were quite varied. Briefly:

- One project modeled the reliability of components of complex systems, and quantified the potential impact of different levels of use and reliability of those components. This project has potential and interesting applications for the actuarial pricing and simulation of warranty coverages, for example on automobiles.
- One project, and the potential applications of the latter to situations in actuarial science. This could be an important area of future actuarial modeling.
- Two projects considered the dynamics and stochastic modeling of employment and unemployment. This modeling could be useful in the broader context of generating possible future economic and financial scenarios.
- One project considered the information provided by different types of risk measures with respect to simulated stock portfolios. This work has potential implications for a better understanding of how best to measure investment risk.
- One project involved a creative exploration of the stochastic dynamics of foreign exchange rate movements before, during, and after events of global significance. In particular, the project involved an event study, around Summer and Winter Olympic Games, of FX rate changes in host versus other countries.

Collectively, these REU participants instigated research and performed analyses that have potential to impact and advance at least the areas of actuarial science, dynamic financial analysis (modeling an insurance organization as being impacted by both stochastic underwriting and investment processes), and enterprise risk management (holistic modeling of risk and risk interactions across an enterprise).

Written Materials, Lectures, etc.

Book Read by Participants

The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward, 2004, by Mandelbrot and Hudson

Lecture Topics (Lecture Notes Provided)

- Review of statistics and probability theory
- Markov chains and Markov processes
- Introduction to actuarial science and finance
- Poisson processes
- Brownian motion
- Risk and ruin theory
- Monte Carlo simulation
- Interest rate modeling
- Economic scenario models

Papers

Ahlgrim, D'Arcy, and Gorvett, 2004, Report of "Modeling of Economic Series Coordinated with Interest Rate Scenarios," on the Casualty Actuarial Society website at <http://www.casact.org/research/econ/> (includes links to the various sections of the report, presentations and papers presented in association with the research, and an Excel-based scenario generator model)

Boyle, 2006, "The Magnificent Seven" (of a financial engineer), *North American Actuarial Journal* (April), pp. iii-v.

Jones, 1997, "Stochastic Models for Continuing Care Retirement Communities," *North American Actuarial Journal* (January)

Stix, 2009, "The Science of Economic Bubbles and Busts," *Scientific American* (July)

Watts, 2009, "Too Complex to Exist," *Boston Globe* (June 14)

Software and Computing

- (1) TeX and/or LaTeX – mathematical word processing
- (2) Mathematica – demonstrated to the REU participants during a visit to Wolfram Research
- (3) CUBE virtually reality environment – demonstrated in a visit to the Illinois Simulator Laboratory, University of Illinois at Urbana-Champaign

Financial Data Source Introduced

FRED – Federal Reserve Economic Data –the research database on the St. Louis Fed website, at <http://research.stlouisfed.org/fred2/> .