Article from:

ARCH 2014.1 Proceedings

July 31-August 3, 2013

On improving pension product design

Agnieszka K. Konicz[1] and John M. Mulvey[2]

[1] Technical University of Denmark, DTU Management Engineering, Management Science
[2] Princeton University, Department of Operations Research and Financial Engineering, Bendheim Center for Finance

ARC, Philadelphia
August 2, 2013

DTU Management Engineering
Department of Management Engineering

On improving pension product design

- Focus on DC pension plans:
  - Quickly expanding,
  - Easier and cheaper to administer,
  - More transparent and flexible so they can capture individuals' needs.
- However,
  - If too much flexibility (e.g. U.S.), the participants do not know how to manage their saving and investment decisions.
  - If too little flexibility (e.g. Denmark), the product is generic and does not capture the individuals' needs.

On improving pension product design

- Asset allocation, payout profile and level of death benefit capture the individual's personal and economical characteristics:
  - current wealth, expected lifetime salary progression, mandatory and voluntary pension contributions, expected state retirement pension, risk preferences, choice of assets, health condition and bequest motive.

- Combine two optimization approaches:
  - Multistage stochastic programming (MSP)
  - Stochastic optimal control (dynamic programming, DP).

On improving pension product design

- Asset allocation, payout profile and level of death benefit capture the individual's personal and economical characteristics:
  - current wealth, expected lifetime salary progression, mandatory and voluntary pension contributions, expected state retirement pension, risk preferences, choice of assets, health condition and bequest motive.

- Combine two optimization approaches:
  - Multistage stochastic programming (MSP)
  - Stochastic optimal control (dynamic programming, DP).

Optimization approaches

stochastic optimal control (DP) - explicit solutions

✓ ideal framework - produce an optimal policy that is easy to understand and implement

✗ explicit solution may not exist

✗ difficult to solve when dealing with details

stochastic programming (MSP) - optimization software

✓ general purpose decision model with an objective function that can take a wide variety of forms

✓ can address realistic considerations, such as transaction costs

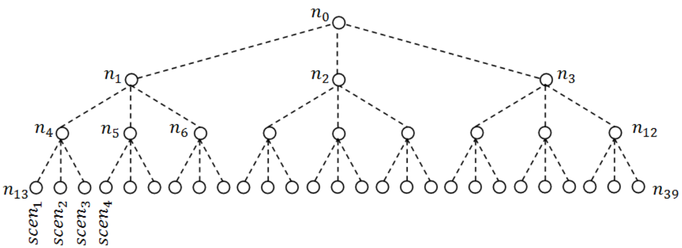
✓ can deal with details

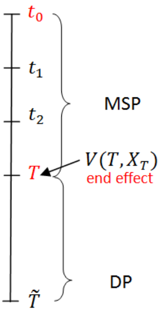
✗ difficult to understand the solution

✗ problem size grows quickly as a function of number of periods and scenarios

✗ challenge to select a representative set of scenarios for the model

Combined MSP and DP approach





n_0, x_0 = 550		Benefits	34.4
	Purchases	Sales	Allocation
Cash			
Bonds	300.6		0.58
Dom. Stocks	177.3		0.34
Int. Stocks	37.7		0.08

n_1		Benefits	31.6	
	Purchases	Sales	Allocation	Returns
Cash				0.030
Bonds		98.8	0.49	-0.039
Dom. Stocks	8.3		0.44	-0.093
Int. Stocks		4.4	0.07	-0.169

Objective

Maximize the expected utility of total retirement benefits and bequest given uncertain lifetime,

$$\max \sum_{s=\max(t_0,T_R)}^{T-1} \sum_{n\in\mathcal{N}_s} {}_s p_x u\left(s, B^{tot}_{s,n}\right)\cdot prob_n$$
$$+ \sum_{s=t_0}^{T-1} \sum_{n\in\mathcal{N}_s} {}_s p_x\; q_{x+s} K u\left(s, I^{tot}_{s,n}\right)\cdot prob_n$$
$$+ {}_T p_x \sum_{n\in\mathcal{N}_T} V\left(T, \sum_i X_{\vec{i},T,n}\right)\cdot prob_n$$

Parameters:

T_R	retirement time,
T	end of decision horizon and beginning of DP,
${}_t p_x$	probability of surviving to age $x+t$ given alive at age x,
q_x	mort. rate for an x-year old,
$prob_n$	probability of being in node n,
K	weight on bequest motive.

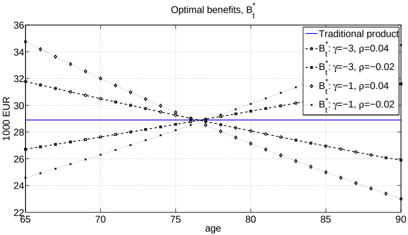
Variables:

$B^{tot}_{t,n}$	total benefits at time t, node n,
$I^{tot}_{t,n}$	bequest at time t, node n,
$X_{\vec{i},t,n}$	amount allocated to asset i, period t, node n.

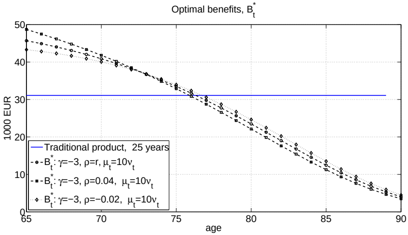
Richard, S. F. (1975),
Optimal consumption, portfolio and life insurance rules for an uncertain lived individual in a continuous time model.
Journal of Financial Economics, 2(2):187–203.

Conclusions I

- Equally fair payout profiles given CRRA utility:
  $u(t, B_t) = \frac{1}{\gamma} w_t^{1-\gamma} B_t^{\gamma}, \; w_t = e^{-1/(1-\gamma)\rho t}$



Sensitivity to risk aversion $1-\gamma$ and impatience (time preference) factor ρ.



Subjective mortality rate $\mu_t = 10\nu_t$: 30% chances to survive until age 75, <1% chance to survive until age 85.

$$\bar{a}^*_{y+t} = \int_t^{\widetilde{T}} e^{-\int_t^s \left(\bar{r}+\bar{\mu}_\tau\right) d\tau} ds, \qquad B^*_t = \frac{X_t}{\bar{a}^*_{y+t}},$$

$$\bar{r} = \frac{1}{1-\gamma}\rho - \frac{\gamma}{1-\gamma} r$$

$$\bar{\mu}_\tau = \frac{1}{1-\gamma} \underbrace{\mu_\tau}_{subj.} - \frac{\gamma}{1-\gamma} \underbrace{\nu_\tau}_{obj.}$$

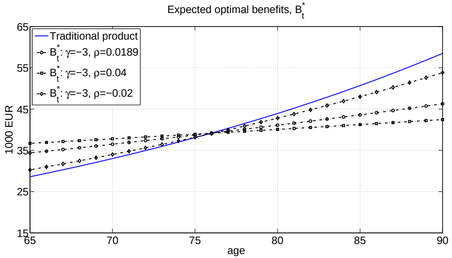
Savings upon retirement $X_{T_R} = 550,000$ EUR, $b^{state}_{T_R} = 0$, risk-free investment, no insurance.

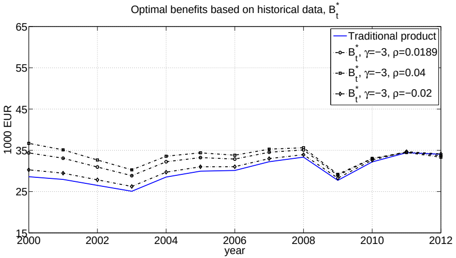
Conclusions II

- More aggressive investment strategy and higher benefits given state retirement pension $b^{state}_{T_R}$

	$b^{state}_{T_R} = 0$	$b^{state}_{T_R} = 5$					
Expected asset allocation\Age	65-90	65	70	75	80	85	90
Cash	20%	4%	5%	6%	7%	7%	7%
Bonds	44	53	52	52	51	51	51
Dom. Stocks	25	30	30	29	29	29	29
Int. Stocks	11	13	13	13	13	13	13

Expected benefits\Age	65	70	75	80	85	90
Benefits B^*_t, $b^{state}_{T_R} = 0$	32,7	34,8	36,9	39,1	41,5	44,1
Benefits B^*_t, $b^{state}_{T_R} = 5$	34,4	36,5	38,7	41,1	43,6	46,3



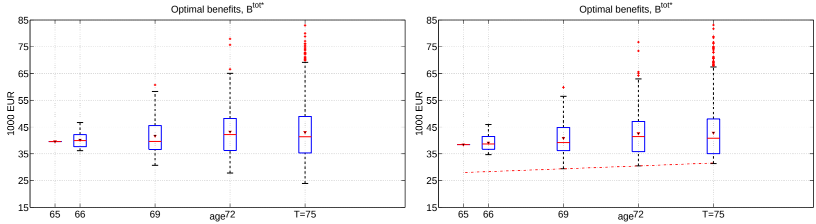


Left plot: expected optimal benefits. Right plot: optimal benefits based on historical returns: 3-m U.S. T-Bills, Barclays Agg. Bond, S&P500, MSCI EAFE.

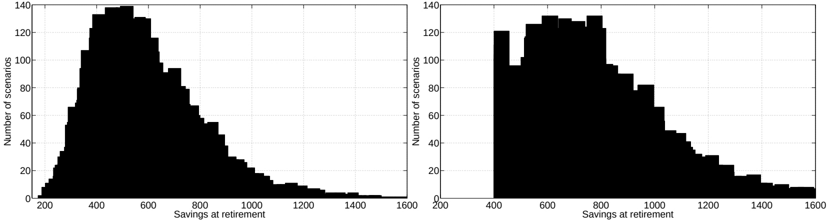
Conclusions III

- Possible to adjust the investment strategy such that $B_t^{tot*} \geq b_t^{min}$
- Possible to adjust the investment strategy such that $\sum_i X_{i,t,n}^{\rightarrow} \geq x_t^{min}$

(a) immediate annuity, $age_0 = 65$, $x_0 = 550$, $b_{T_R}^{state} = 5$



(b) deferred annuity, $age_0 = 45$, $x_0 = 130$, $l_0 = 50$, $p^{fixed} = 15\%$, $p^{vol} = 10\%$ (right plot only), $b_{T_R}^{state} = 5$, $ins_0^{fixed} = 150$

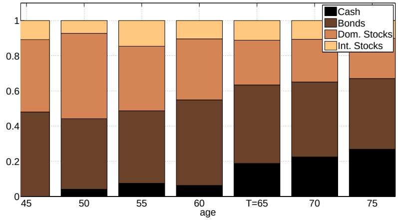


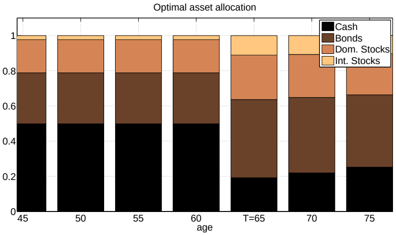
Conclusions IV

- Possible to include individual's preferences on portfolio composition,

$$X_{i,t,n} \geq d_i \sum_i X_{i,t,n}, \quad X_{i,t,n} \leq u_i \sum_i X_{i,t,n}$$

  e.g. $d_{bonds} = 50\%$ and $u_{bonds} = 70\%$.
- Though any additional constraints lead to a suboptimal solution ($\Longrightarrow$ lower of more volatile benefits).
- Optimal investment vs. optimal fixed-mix portfolio:





Deferred life annuity. 20% lower expected benefits given the same risk level.
Left: optimal investment, $E[B_t^{tot*}] = 46,200$ EUR. Right: fixed-mix portfolio, $E[B_t^{tot*}] = 37,700$ EUR.

Selected references

Høyland, K., Kaut, M., and Wallace, S. W. (2003).
A Heuristic for Moment-Matching Scenario Generation.
Computational Optimization and Applications, 24(2-3):169–185.

Kim, W. C., Mulvey, J. M., Simsek, K. D., and Kim, M. J. (2012).
Stochastic Programming. Applications in Finance, Energy, Planning and Logistics, chapter Papers in Finance: Longevity risk management for individual investors.
World Scientific Series in Finance: Volume 4.

Konicz, A. K., Pisinger, D., Rasmussen, K. M., and Steffensen, M. (2013).
A combined stochastic programming and optimal control approach to personal finance and pensions.
http://www.staff.dtu.dk/agko/Research/~/media/agko/konicz_combined.ashx.

Milevsky, M. A. and Huang, H. (2011).
Spending retirement on planet Vulcan: The impact of longevity risk aversion on optimal withdrawal rates.
Financial Analysts Journal, 67(2):45–58.

Mulvey, J. M., Simsek, K. D., Zhang, Z., Fabozzi, F. J., and Pauling, W. R. (2008).
Assisting defined-benefit pension plans.
Operations research, 56(5):1066–1078.

Richard, S. F. (1975).
Optimal consumption, portfolio and life insurance rules for an uncertain lived individual in a continuous time model.
Journal of Financial Economics, 2(2):187–203.

Appendix

Constraints I

Budget equation while the person is alive, $t \in \{t_0, \ldots, T-1\}$, $n \in \mathcal{N}_t$:

$$B_{t,n}\mathbf{1}_{\{t \geq T_R\}} + \nu_t I^{tot}_{t,n} + \sum_i X^{buy}_{i,t,n} = P^{tot}_{t,n}\mathbf{1}_{\{t<T_R\}} + \sum_i X^{sell}_{i,t,n} + \nu_t \sum_i X^{\rightarrow}_{i,t,n}$$

Value of the savings at the beginning of period t:

before rebalancing in asset i, $t \in \{t_0, \ldots, T\}$, $n \in \mathcal{N}_t$, $i \in \mathcal{A}$,

$$X^{\rightarrow}_{i,t,n} = x_{i,0}\mathbf{1}_{\{t=t_0\}} + (1 + r_{i,t,n})X_{i,t^-,n^-}\mathbf{1}_{\{t>t_0\}},$$

after rebalancing in asset i, $t \in \{t_0, \ldots, T-1\}$, $n \in \mathcal{N}_t$, $i \in \mathcal{A}$,

$$X_{i,t,n} = X^{\rightarrow}_{i,t,n} + X^{buy}_{i,t,n} - X^{sell}_{i,t,n},$$

Purchases and sales, $t \in \{t_0, \ldots, T-1\}$, $n \in \mathcal{N}_t$, $i \in \mathcal{A}$,

$$X^{buy}_{i,t,n} \geq 0,\ X^{sell}_{i,t,n} \geq 0.$$

Constraints II

Premiums, $t \in \{t_0, \ldots, T-1\},\ n \in \mathcal{N}_t$**,**

$$P^{tot}_{t,n} = P_{t,n} + p^{fixed}\, I_t,$$

$$P_{t,n} \leq p^{vol}\, I_t,$$

Benefits, $t \in \{t_0, \ldots, T-1\},\ n \in \mathcal{N}_t$**,**

$$B^{tot}_{t,n} = B_{t,n} + b^{state}_t,$$

$$B^{tot}_{t,n} \geq b^{min}_t,$$

Insurance, $t \in \{t_0, \ldots, T-1\},\ n \in \mathcal{N}_t$**,**

$$I^{tot}_{t,n} = I_{t,n} + ins^{fixed}_t,$$

$$I_{t,n} \geq ins^{min} \sum_i X^{\rightarrow}_{i,t,n},$$

Portfolio composition, $t \in \{t_0, \ldots, T-1\},\ n \in \mathcal{N}_t,\ i \in \mathcal{A}$**,**

$$X_{i,t,n} \leq u_i \sum_i X_{i,t,n}, \quad X_{i,t,n} \geq d_i \sum_i X_{i,t,n},$$

Minimum savings, $t \in \{t_1, \ldots, T\},\ n \in \mathcal{N}_t$**,**

$$\sum_i X^{\rightarrow}_{i,t,n} \geq x^{min}_t.$$

End effect

- DP - very specific and simplified model: power utility, risk-free asset, risky assets following GBM, Gompertz-Makeham mortality rate model, deterministic labor income and state retirement pension, no constraints on portfolio composition and no constraints on the size of savings or benefits.

Utility:

$$u(t, B_t) = \frac{1}{\gamma} w_t^{1-\gamma} B_t^{\gamma}, \qquad w_t = e^{-1/(1-\gamma)\rho t}$$

Optimal value function (**end effect**):

$$V(t, x) \quad = \quad \frac{1}{\gamma} f_t^{1-\gamma} \left(x + g_t\right)^{\gamma}$$

Optimal controls:

$$\text{benefits:} \qquad B_t^* = \frac{w_t}{f_t}(X_t + g_t) - b_t^{state}$$

$$\text{sum insured:} \qquad I_t^{tot*} = \left(K \frac{\mu_t}{\nu_t}\right)^{1/(1-\gamma)} \frac{w_t}{f_t}(X_t + g_t)$$

$$\text{proportion in risky assets:} \qquad \pi_t^* = \frac{\alpha - r}{\sigma^2(1-\gamma)} \frac{X_t + g_t}{X_t}$$

- g_t - present value of future cashflows (labor income, retirement state pension, insurance price)
- f_t - optimal life annuity