Answers to exercises

for the report

Complexity science – an introduction (and invitation) for actuaries

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Introduction

This document provides answers to the exercises in the report titled *Complexity Science – an introduction (and invitation) for actuaries*. It is arranged in chapter order. If you come up with better answers, please let me know.

Chapter one: Complexity science

Exercise 1

Because the identification of system types is somewhat subjective, your answers may differ from mine:

Item	Set	System	Dynamic system	Simple system	Random system	Complex system	Complex adaptive system
a ball, a blanket, and a bird	X						
a traffic light	X	X	X	X			
air molecules in a room	X	X	X		X		
a cloud	X	X	X			X	
the weather	X	X	X			X	
the solar system	X	X	X	X			
a soccer team	X	X	X			X	X
a termite colony	X	X	X			X	X
the U.S. healthcare system	X	X	X			X	X

Item	Objects	Relationships	Behavior rules	Environment
a ball, a blanket, and a bird	3 objects mentioned	none	none	none
a traffic light	parts of the light	interlocking parts of a machine	on-off rules for the lights	none
air molecules in a room	air molecules	proximity	laws of physics	the room
a cloud	water droplets	proximity	laws of physics	the sky
the weather	air molecules and water droplets	proximity	laws of physics	the sky
the solar system	planets and the sun	proximity	laws of physics	space
a soccer team	soccer players	team relationships	rules of game, individual movement rules	the soccer field and the ball
a termite colony	termites	working and social relationships	termite behavior rules	physical location of the colony
the U.S. healthcare system	consumers, physicians, hospitals, insurance companies, etc.	family, doctor-patient, patient-hospital, consumer-insurance carrier, physician-insurance carrier, etc.	human and institutional behavior rules	economic evironment

Exercise 2

a) Simple system: a score with one note for all players, played continuously.
   Random system: a score with random notes generated individually for each player.
   Complex system: Beethoven's sixth symphony.

b) A score that provides a theme, and then directs the players to riff on the theme, like a jazz group.

Chapter one: Complexity science continued

Exercise 3
The orchestra performs a computation by each musician playing the notes of his or her music (the information) and adding the player's own musical talent and sensibility. The program is the way each musician plays the music. The computation result is the resulting performance, which is an emergent property of the orchestra.

If the orchestra is large enough and the score long enough, it could be a universal computer.

Exercise 4
The traditional actuary might analyze historical data regarding aggregate changes in physician prescribing behavior when the company changed its test reimbursement policies. Using the results of this analysis, together with judgment regarding the current policy change, the actuary would calculate the potential impact on profit and present the results in a technical report.

By contrast, the complex system actuary would first identify the relevant complex system related to the problem, together with its agents, their relationships and behavior rules, and the environment. For example, the relevant agents might be physicians, their patients, health insurance consumers in general, and even insurance brokers. The reaction of all these agents could affect the company's profit. The complex system actuary would then determine the agent behavior rules related to reimbursement policy change. To do this, the actuary would search the scientific behavioral science literature, interview representative agents, and perhaps even conduct simple controlled experiments, all to understand the agents' individual behavior rules. Then the actuary would build an agent-based model incorporating these elements, and might even build the model as a serious game (for more about this approach, see Chapters seven and eight). From the bottom-up simulation results, the actuary would better understand the potential impact on company profit and would communicate conclusions to company executives, using the model to help them understand. The actuary would participate in implementing the policy and monitoring its impact.

Chapter two: Agent-based models

Exercise 1
Please let me know if you find a counter-example.

Exercise 2
Any time you are trying to solve a problem associated with a complex system, agent-based modeling may prove helpful.

Exercise 3
Think of the complex systems related to your work. Do you understand how they work, from the bottom up? Are there any problems with these systems that, traditionally, actuaries have not addressed?

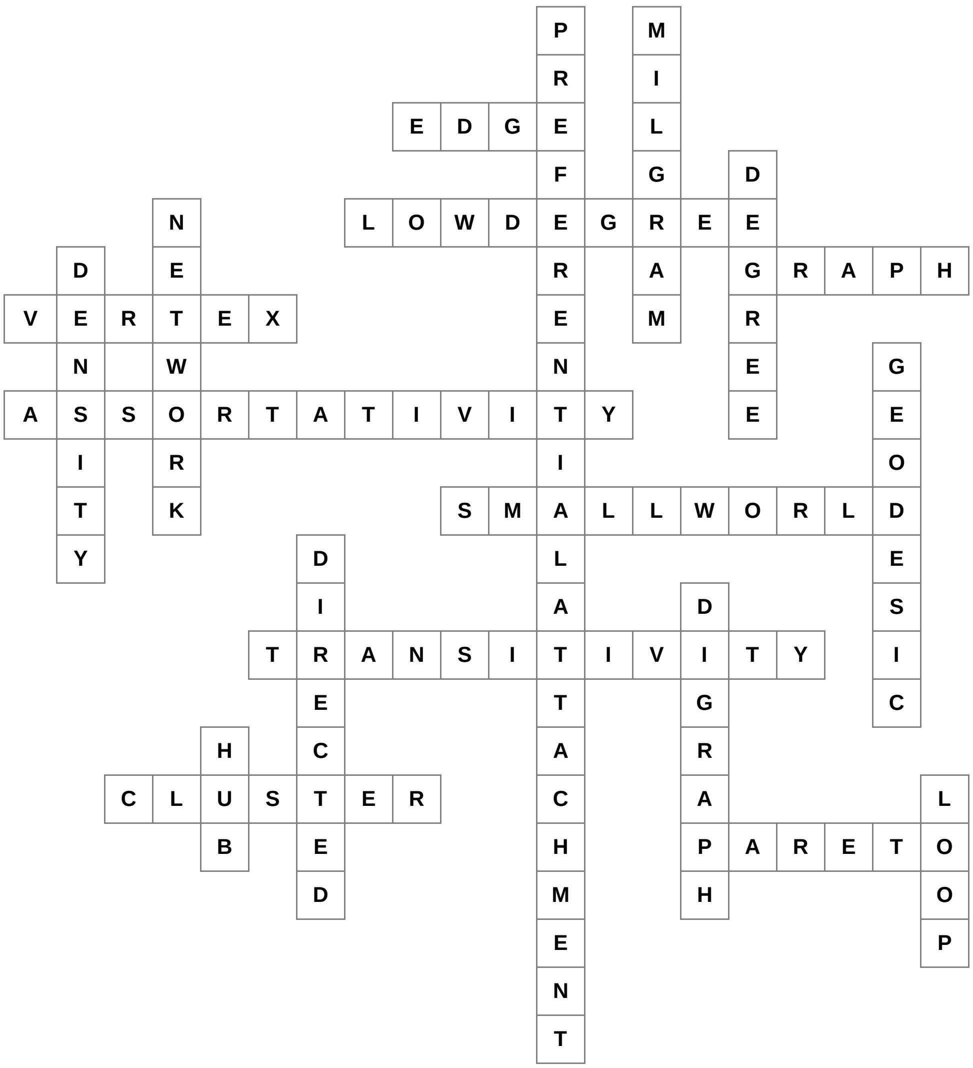
Exercise 4
Iterate the process for determining the three weights. See if the weights converge to 0.3.

Chapter four: Networks

Exercise 1

Use the footnotes to understand the igraph functions needed to produce the figures, then run the functions in R.

Exercise 2



Note: ‘assortative mixing’ is the tendency of vertices to connect to vertices with similar properties.

Exercise 3

$\binom{n}{2}$ where n is the number of vertices.

Exercise 4

To generate the graph: gGrgGame = grg.game(1000, .0575, FALSE, FALSE)

To plot the graphs in interactive mode: PlotGraph(gGrgGame, “”, TRUE, 7, “Console”, “”, 4, 4)
It may take several minutes to plot the graph.

Chapter four: Networks continued

Exercise 4 continued

The gGrgGame graph in the 'FR' layout from the interactive mode:

Following are the results from AnalyzeGraph(gGrgGame)

	Geometric random graph	Random graph
Directed	FALSE	FALSE
Vertices	1000	1000
Edges	4887	4963
Diameter	31	5
Mean degree	9.8	9.9
Maximum degree	21	21
Minimum degree	1	2
Mean geodesic	12.6	3.3
Density	0.01	0.01
Transitivity	0.60	0.01
Correlation	0.533	-0.009
Clusters	1	1

As the table shows, the geometric random graph's diameter, mean geodesic, transitivity, and correlation are much larger than the random graph. The primary reason for these differences is that the relationship between two vertices in the geometric random graph is based on proximity, whereas in the random graph the relationship is random. It is therefore more likely that a few vertices in a local neighborhood of the random graph will have a few connections that reach far beyond the local neighborhood, and reduce the mean geodesic and the diameter. The geometric random graph is highly correlated and has high transitivity again because the relationship rule favors vertices that are close to one another.

Exercise 5

The following ratio is one such measure: transitivity/mean geodesic. For a small world, the ratio is much larger than for a random graph.

Exercise 6

Please let me know the results of your project.

Chapter five: Cellular automata

Exercise 1
There are $2^8 = 256$ such behavior rules, but only $256/4 = 64$ unique rules.

Exercise 2
If you find a method to make these predictions, please let me know.

Exercise 3
A model with periodic boundaries produces results that are similar to results with the model with cliff boundaries. Per Bak probably chose cliff boundaries to make the model more like a real pile of sand on a table.

Exercise 4
The results are largely the same as for the classic model.

Exercise 5
Use the two-dimensional model, and set the number of rows to 1.

Exercise 6
The results would not be materially different.

Chapter six: Artificial societies

Exercise 1
Changing these parameters does not affect the model's results.

Exercise 2
The results are not materially different.

Exercise 3
The results are not materially different.

Exercise 4
The results are not materially different.

Exercise 5
The results are not materially different.

Exercise 6
Your model should reproduce the waves of migration shown in chapter two of Epstein and Axtell's book.

Chapter seven: Serious games

Exercise 1

1. Create a large (perhaps 10 x 10) grid like large checkerboard.
2. Divide a group of 60-80 people in half; one half is assigned the color blue, and one half is red. To do this, give each person a large card with his or her assigned color.
3. Assign each person a number from 1 to the number of people in the group.
4. Ask each person to determine his or her preference percentage, and keep it for the duration of the game.
5. Distribute the people randomly on the grid.
6. At regular time intervals, ask each person to assess his or her happiness according to their preference percentages, and move to a cell that increases their happiness. Do this in order of each person's assigned number. Once a person moves, the person cannot move again until the next time interval.
7. After about ten rounds, assess the degree of segregation.
8. See if anyone is surprised, and discuss the results.

Exercise 2

To implement and monitor a complex adaptive system in Second Life, you might set up an insurance company to insure Second Life residents against loss of their real estate (a problem that appears to be relatively common). The company, its members, and its eventual competitors may become a complex adaptive system.

Exercise 3

Please let me know what you design.

Chapter eight: Complex system actuaries

Exercise 1

For this exercise, it is useful to think beyond traditional actuarial problems. For instance, how would you model a hospital's risks related to a pandemic.

Exercise 2

Please let me know what you propose.