

Post-Retirement Financial Strategies from the Perspective of an Individual Who Is Approaching Retirement Age

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“Executing these proposals efficiently is no small task. That said, I see this issue as a tough engineering problem, not one of new science.” -- Robert C. Merton

Abstract

This research provides a literature review on the topic of post-retirement financial strategies (PRFS) from the perspective of an individual who is approaching retirement age. The body of knowledge surveyed is the PRFS literature that is related to the risks of longevity, inflation, and investment. The literature sources are both academic and practitioner-focused. The end product is a documentation of and commentary on the state-of-the-art in the PRFS area. The intended audience is retirement professionals, and others, who need to be well informed about the current state of post-retirement financial strategies, as reported in the literature.

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1 Introduction

What post-retirement financial strategy should be suggested for an individual who is approaching retirement age? The answer to this question must accommodate a number of risks, the most obvious of which are related to longevity, inflation, and investment. The longevity risk is that assets will not last as long as the retiree; the inflation risk is that the purchasing power of retirement assets will be eroded; and the investment risk takes the form of market risk for the assets invested in equities and interest rate risk for the assets invested in fixed income products. What makes this question especially interesting is that it is relevant to many pension plan participants, its solution must be molded to the wealth, features and agenda of a particular individual and/or family, and standard market and demographic metrics may have limited relevance.

Bodie et al. (2006) reformulate the foregoing question into four basic questions individuals are confronted with:

- How much of their income should they save for the future?
- What retirement risks should they insure against?
- How should they invest what they save?
- When should they plan to retire?

The purpose of this study is to document the extent to which the literature provides answers to these questions.¹

¹ While not addressed in this study, another critical consideration when it comes to the feasibility of the various post-retirement financial strategies, is the profile of the individual or family unit involved. Two recent studies that speak to this issue are Abkemeier and Hamann (2009) and Bryck and Rappaport (2009). The former focuses on the middle income group, segregated by wealth and income, age groupings, and household type, while the latter was a survey that explored how retirees make decisions about investing their assets and purchasing financial products.

2 Longevity Risk

“[T]here is a 50% chance of radical life extension therapies arriving in 25-30 years. If it does, lifespans could theoretically be extended almost indefinitely, and it is feasible that the first person to live for a thousand years might actually be alive today”

-- Aubrey De Grey, Chairman & Founder of the Methuselah Foundation

Longevity refers to the lifetime of an individual. It obviously is important in the context of PRFS because the longer the lifetime of an individual the greater the retirement fund needed to support that individual. In this section, literature associated with longevity risk is reviewed. We begin with an introduction to longevity, which is followed by the subsections: estimates of the upper limits to human longevity, measuring shortfall risk in retirement, and optimal investment strategy to minimize probability of lifetime ruin.

2.1 An introduction to longevity

This subsection provides an introduction to longevity. The topics discussed are longevity and post-retirement lifetime, the longevity risk problem, and longevity risk management.

2.1.1 Longevity and post-retirement lifetime

Much of humankind has begun to reap what MacGregor (2003) describes as “one of the most valued harvests of its scientific and technological pursuits: a significant increase in human longevity.” Due mainly to advances in medicine and health care, we live longer than ever before and there is a relatively high probability that the last survivor of a retired couple will live to age 90 or more.

We see evidence of the consequences of this phenomenon for post-retirement lifetime. AAA (2006), for example, noted that Americans are entering the workforce later and retiring earlier than was previously the case. This, combined with an increased life expectancy, has resulted in retirees spending an average of 20 years in retirement today, compared to 13 years in 1940. Moreover, between 1940 and 2006, the ratio of retirement years to years in the workforce has increased by 79 percent.

Similar statistics hold abroad. Kapur and Orszag (2002) reported that the expected future lifetime of a 60 year old female in the UK increased from 18 years in the early 1950s to 22 years by the early 1990s. This increase in longevity, which is paralleled across other OECD² countries, coupled with the trend towards more early retirement in these countries, means that these workers will be spending a substantial amount of time in retirement.

² Organization for Economic Co-operation and Development, www.oecd.org

2.1.2 The longevity risk problem

The longevity risk, in the context of PRFS, is that a retiree will outlive his or her retirement funds. Hazards, such as early retirement or a family history of longevity, aggravate this risk.

Obviously, as individuals live longer in retirement, how they invest during their retirement becomes more pertinent [Kapur and Orszag (2002)].

Retirees face two major problems related to sufficient income in their later retirement years. First, all demographic subgroups and income levels are expected to live longer as mortality rates continue to decline. Second, retirees generally need more income in the second half of their retirement to meet nondiscretionary needs due to price inflation and the increased utilization of medical and long-term care at older ages [Orth (2000)].

Based on the National Retirement Risk Index³ of the Center for Retirement Research at Boston College, if long-term care expenses are included in the calculation, more than 60 percent of working-age households are at risk of being unable to maintain their standard of living in retirement [Munnell et al. (2006), Tomlinson (2008)].

For many workers, the impetus for the problem is the profound shift in the way that employment-based retirement benefits are delivered to workers. Defined benefit (DB) plan pensions have been largely replaced by lump-sum distributions from defined contribution (DC) plans, mainly 401(k) plans.⁴ Not only has this shifted the investment risk to the employee, but, since longevity risk is inherent in all retirement systems, it too has been shifted to the employee [Orth (2004)].

The harsh reality is that longer lifetimes increase the likelihood that retirement assets will not last long enough [Orth (2004)].

2.1.3 Longevity risk management

Retirees must make optimal strategic decisions about the management of their retirement funds to cope with the three major post-retirement risks: longevity, investment returns, and inflation [WISER (2003)].

Retirement income management involves choosing the combination of annuity, withdrawal, and investment strategies that minimizes the probability of running out of money during the retiree's lifetime, while still realizing his or her financial agenda. The retiree must manage both market and longevity risk during this decision-making [Kaplan (2005: 1)].

³ The National Retirement Risk Index, developed by the Center for Retirement Research at Boston College, is a percentage measurement of how many working-age American households are at risk of being unable to maintain their standard of living in retirement. It is updated semiannually.

⁴ Workers have embraced the flexibility offered by 401(k) plans. Employers are attracted to them as well, because they are less costly and easier to administer than DB plans [Orth (2004)].

Section V of Crawford et al. (2008: 20-23) presents a short literature review from the perspective of the individual exposed to longevity risk and mentions ways in which individuals can manage their own risks.⁵

Clients need to be aware of their longevity risk and they need to have a longevity-risk management strategy [Milevsky (2007e)].

2.1.4 Commentary

An unresolved question is whether the lure of 401(k) plans will diminish as baby boomers retire and are confronted with the prospects of managing their retirement assets over an unknown, but extended period of time [Orth (2004)].

2.2 Estimates of the upper limits to human longevity

This subsection addresses three considerations when it comes to estimating the upper limits to human longevity: causes of longevity, the debate regarding future trends of mortality, and techniques for modeling mortality.

2.2.1 Causes of longevity

In the past, longevity has been attributed to reduced infant mortality. Sinha (1986), for example, noted that “[i]n most countries, a rise in life expectancy has been caused by the lowering of [the] mortality rate of babies and children ... [while] ... for higher age groups, this increase has been extremely small.” Thus, in his analysis, he modeled uncertain lifetimes using survival probabilities that were increasing for the lower age group and a probability of survival that remained unchanged for the oldest age group.

In recent years, much of the interest in mortality models can be traced to the recognition that actuaries in the past have tended to pay insufficient attention to aggregate mortality risk. The Equitable Life fiasco⁶ is a stark reminder of this [Dowd et al. (2006)].

Stallard (2006) reviewed the literature on the sources and characteristics of historical changes in longevity and current deterministic and stochastic forecasting based on these data. He found that plausible alternative sets of forecasting assumptions have been derived from the same sets of historical data, and he concluded that additional methodological development is needed to integrate the various assumptions into a single coherent forecasting model. Illustrations based on existing forecasts suggest that the survival functions of older cohorts will have manageable ranges of uncertainty. However, the uncertainty ranges for younger cohorts will be larger and Stallard speculated that the desire for greater precision likely will motivate further model development.

⁵ Crawford et al. (2008) present a literature review on the topic of longevity risk that is intended to be a resource for actuaries and other professionals. However, its primary focus is from the perspective of insurers, not individuals.

⁶ In this case, the insurer offered guaranteed annuity options based on 1950s mortality tables, and its failure to take proper account of mortality risk was a key factor contributing to its being forced to close to new business in 2000.

2.2.2 Future trends of mortality (the debate)

There is a debate about whether the recent improvement of human longevity will continue into the future. The optimists include demographers, such as Vaupel et al. (1998), Tuljapurkar et al. (2000), Oeppen and Vaupel (2002), and Tuljapurkar (2005), who, based on an extrapolative forecasting approach, see no natural upper limit for human lifetime. The pessimists include demographers, such as Olshansky et al. (1990, 2001, 2005), and Loladze (2002), who anticipate that future life expectancy might plateau or even decline due to lifestyle influence, such as obesity, and environmental factors. Other demographers, like de Grey (2006), reject the extrapolative forecasting of the optimists, but feel that scientific advances and the sociopolitical responses to them might significantly extend life expectancies over the next century [Blake et al. (2008)].

Turner (2006), in his study of longevity risk issues, concluded that there is considerable uncertainty about future projections of average life expectancy and these expectancies explode the further the planning horizon. Moreover, there are inherent uncertainties, not precisely modelable risks, so that the associated confidence intervals are judgments, not mathematical facts.

2.2.3 Techniques for modeling mortality

Cairns et al. (2006) investigate the evolution of the post-age-60 mortality curve in the United Kingdom and its impact on the pricing of longevity risk. They introduce a two-factor stochastic model for the temporal development of this curve. The mortality-rate dynamics resulting from the first factor affect all ages in the same way, while that resulting from the second factor is weighted more heavily at the higher ages.

Guo and Wang (2007) discuss issues and techniques with respect to an advanced age mortality study using hybrid data mining techniques, summarize the preliminary results of their study, and illustrate the information discovery process for the old age mortality model.

Lin and Liu (2007) model human mortality using a finite-state Markov process with one absorbing state, and use the model to formulate a health index called physiological age. Their model results in closed-form expressions for many quantities of interest including conditional survival probabilities and actuarial present values. Their model also is able to explain some stylized facts of observed mortality data.

Blake et al. (2008) used survivor fan charts to illustrate the uncertainty associated with future male survival probabilities. These are charts of prospective probability densities over each year in a specified forecast period and show the likely confidence interval to which a dynamic quantity may belong in a particular future year. Their study indicated that survivorship uncertainty is greatest for males aged a little over 90 and that taking account of uncertainty in the parameters of the underlying mortality model leads to major increases in estimates of the widths of the fan charts.

2.2.4 Commentary

Hári et al. (2008) differentiate between two categories of mortality risk, micro-longevity risk and macro-longevity risk. Micro-longevity risk, which quantifies the non-systematic deviations from an individual's expected remaining lifetime, is related to uncertainty in the time of death if survival probabilities are known with certainty. In contrast, macro-longevity risk, which results from survival probabilities changing over time, is due to uncertain future survival probabilities. It is the macro-longevity risk that is so problematic.

Forman and Chen (2008) discuss the implication of defining old age as years-from-birth and of failing to index early and normal retirement ages for longevity increases. They also discuss alternative approaches for relating retirement age to future lifetime.

MacGregor (2003) sought to identify how people can find meaning and fulfillment throughout their whole lifespan, and to distinguish trends in psychology that can be emphasized and strengthened to improve theoretical frameworks that cast longevity in a positive light. To this end, he reviewed current developments in research on cognition, emotion and aging, as a means to investigate the optimistic perspective of the psychological implications of increased lifespans.

2.3 Measuring shortfall risk in retirement

This subsection provides a brief overview of the literature on issues related to measuring shortfall risk in retirement.

2.3.1 Introduction

With the rapid growth of DC plans, additional education and research is needed on the disposition of retirement assets, because more and more retirees are electing to self-manage longevity risk [Rubin (2000)].

One of the major conclusions of Murphy (2000) is that these individuals risk outliving their retirement funds, either because they live too long or poor investment performance.

Milevsky and Abaimova (2005a), in their discussion of financial risk management during retirement, make the point that “longevity risk, which is a financial risk, should be hedged and insured like all other risks faced during the human lifecycle.”

2.3.2 Analysis

Young (2004) exemplifies the type of analysis that has been done with respect to measuring shortfall risk in retirement. In her study, she determined the optimal investment strategy for a retiree seeking to minimize the probability of lifetime ruin, subject to a specific level of consumption. Dually, she determined the maximum consumption rate to target a given probability of lifetime ruin. She also showed that if the consumption of the retiree was based on targeting a

given probability of ruin during the expected lifetime, the consumption will be too high, and, conversely, if the possibility of death prior to the expected lifetime is ignored, too little will be consumed.

Other articles that have focused attention on longevity risk and the probability of outliving wealth include Ameriks, Veres and Warshawsky (2001), Duff (2001), Bengen (2001) and Goodman (2002).

2.3.3 Consumer Software

Sondergeld et al. (2003) reported on a study (jointly sponsored by LIMRA and SOA) of software used to model retirement. He noted that nearly all of the 19 programs reviewed use a deterministic approach to the treatment of longevity risk.

Young (2004) demonstrated the fallacy of the deterministic approach.

Turner and Witte (2009), a follow-up study of retirement planning software, built on Sondergeld et al. (2003). They examine twelve nonrandomly selected retirement planning software programs. Five of the programs were available for free over the internet, one program was available to consumers at a fee, and six programs were designed for use by financial planners for their clients.

Their analysis indicated that while programs offer planning tools to consumers that they would not otherwise have, they do not adequately help users in answering critical questions they are facing. Moreover, better treatment is needed for key program inputs, such as longevity assumptions, rates of return, Social Security benefits, housing, and target consumption for survivors.

2.4 Optimal investment strategy to minimize the probability of lifetime ruin

This subsection discusses literature related to investment strategies that minimizes the probability of lifetime ruin. The topics addressed include risk factors, the Markowitz single-period paradigm, and longevity models.

2.4.1 Risk factors

Chen and Milevsky (2003) found that the factors that influence the optimal allocations across assets include age, risk aversion, subjective probability of survival, utility of bequest, and the expected risk and return tradeoffs of different investments. They also found that the global allocation between risky and risk-free assets is influenced only by the investor's risk tolerance, and is not significantly affected by the subjective probability of survival or the utility of consumption vis-à-vis bequest.

2.4.2 Markowitz single-period paradigm

Researchers, such as Bodie (2002) and Chen and Milevsky (2004), question the effectiveness of the Markowitz single-period paradigm of efficient diversification when dealing with asset allocations for retirees, since longevity risk is not considered.

2.4.3 Longevity models

Chen and Milevsky (2003) used a one-period model to investigate the theory and practice of constructing an optimal asset allocation during retirement, taking into consideration both financial market risk and longevity risk. They concluded that there is a two stage process to a well-balanced retirement plan. In the first stage a suitable global mix of risky and risk-free assets is developed based strictly on the investor's risk tolerance. Then, given the existing asset mix, the annuitization decision is made, based on bequest motives, subjective health assessments, and so on.

Menoncin (2008) investigated the optimal amount of longevity bond for a retiree who wants to maximize the expected utility of his or her intertemporal consumption, given a perfectly competitive, arbitrage free and frictionless financial market. He started with a financial market made up of a riskless asset, a stock, a bond as a derivative on the stochastic interest rate, and a longevity bond whose coupons were proportional to the population survival rate. Then, assuming a force of mortality instantaneously uncorrelated with the interest rate, he demonstrated that the wealth invested in the longevity bond must be taken from the ordinary bond and the riskless asset proportionally to the duration of the two bonds.

2.4.4 Longevity risk

The Second International Longevity Risk and Capital Market Solutions conference had as its focus the extent of the aggregate longevity risk problem and the possible capital market solutions. Papers from that conference were subsequently published in Volume 73 Issue 4 of the *Journal of Risk and Insurance*. MacMinn et al. (2006) provides a synopsis of those papers as well as an overview of the longevity problem.

2.5 Comments

Orth (2000) identifies a number of legislative changes, educational efforts, and changes in annuity products and asset allocation software that could assist workers and retirees to better prepare for and respond to the challenges of longevity and price changes.

AAA (2006) noted that the changing demographics mean that people may not have adequate resources for retirement. Options it suggested to deal with this situation included: modifying the IRS regulations to allow employees to work at a reduced schedule while receiving partial retirement benefits, a practice commonly referred to as phased retirement; utilizing more flexible hybr-

id pension plan designs; increasing retirement ages for both private pension and Social Security; and curbing the use of early retirement subsidies.

3 Inflation Risk

This section reviews the literature on inflation risk, as it related to post-retirement financial strategies. After an introduction, the topics include the consumer price index, annuities, and advanced-life delayed annuities.

3.1 Introduction

According to WISER (2003), few retirees realize how expensive it is to maintain their standard of living for 20, or more, years after they retire, especially when confronted with inflation.

Most of the old private DB plans did not have inflation protection.

An early actuarial paper dealing with inflation was Richmond and Rosen (1982). It discussed the intricacies of measuring inflation and indicated the wide range of options available to lessen the impact of inflation on pensioners. While the focus was on corporate plan sponsors, the paper is a good source of insights into consequences of inflation and the cost impact of various approaches to inflation.

3.2 The Consumer Price Index

Retirees face a unique type of inflation risk (CPI-Elderly index) and there is considerable probability that (nominal) income will likely be worth much less than it was at the time of retirement [Milevsky and Abaimova (2005a)].

Milevsky (2007d) commented that we all have slightly different and personal inflation rates based on our desired basket of expenditures, and anonymous macroeconomic projections between 2 percent and 4 percent do not capture the personal nature of inflation for retirees. To emphasize this, he coins the acronym CPI-ME to denote the “unique risk-label for a very personal adversary that must be constantly battled.”

Siegel (1995) documents the 200 year history of North American financial markets, and concludes that equities consistently outperform inflation and fixed income securities over the long run.

The interaction between equity markets, interest rates and inflation in the short run is still debated in the academic literature [Milevsky (1998)].

3.3 Annuities

Kaplan (2005) found that the role annuities play in an optimized strategy depends largely on how inflation was modeled.

Babbel and Merrill (2007) list erosion of purchasing power due to inflation as one of the deterrents to purchasing an annuity.

An annuity with direct inflation protection or increasing future payments would be considered as more attractive to those with longer life expectancies [Balls (2004)].

Mitchell (2001) stressed the role of inflation-indexed annuities and related products for protecting real consumption streams until very old ages.

3.4 Advanced-life Delayed Annuity

The advanced-life delayed annuity (ALDA) is a deferred life annuity, purchased at retirement, or earlier, that is deferred to the advanced age of 80, 85 or even 90, and is linked to, and adjusted for, consumer price inflation [Milevsky (2004a)].

ALDAs can be traced to Stephenson (1978). A critic of the annuity products at the time, he argued in favor of designs that contain high ratios of “protection to investment.” Using his concept of index of protection, he demonstrated that properly designed deferred annuities could provide greater inflation-protected value to annuitants.

3.5 Comments

Greenwald et al. (2006) found important misconceptions concerning longevity and the long-term impact of inflation among focus group⁷ participants.

Ways of coping with inflation have been addressed by Campbell and Viceira (2001) and Brennan and Xia (2002) [Horneff et al. (2008b)].

See inflation linked products.

⁷ The focus groups had considerable retirement income management responsibilities, in the sense that they had at least \$100,000 of investable assets and depended on that money to maintain their lifestyles because their income from Social Security and defined benefit plans was insufficient to meet all of their expenses.

4 Portfolio Risk

“I have all my portfolio in stocks. In fact I've borrowed to invest more in stocks, so I'm actually 150% in equities ... the last few months have not been pretty” [Moshe A. Milevsky, in an interview with Schurenberg (2008)].

This section presents an overview of the portfolio risk literature as it pertains to post-retirement financial strategies. The topics include individual versus institutional investing, life-cycle models, conserving portfolios during retirement, investing retirement wealth, and optimal investment strategy.

4.1 Introduction

Prudential (2007) notes that the 5-year period before and the 5-year period following retirement, which they call the “Retirement Red Zone,” may represent the most critical time of a retiree’s investing life. Poor market performance during this period can have serious effects on a retiree’s portfolio, with the result that retirement lasts longer than their assets.

One solution to this problem, suggested by Prudential, is an add-on to a variable annuity that, for an added fee of .60% assessed daily against the variable account value, guarantees a 5% annual stream of income from the retiree’s Protected Withdrawal Value⁸ for life [Prudential (2007)].

Milevsky and Salisbury (2006) also mention a “retirement risk zone”.

4.2 Individual versus institutional investing

Markowitz (1991) discussed how a game-of-life simulation⁹ is a parallel approach for modeling individual financial management. He believed that simulation, as opposed to analytic methods, has more chance of producing credible decision rules for practice. He noted that the development of a realistic simulator requires identifying the goals that are essential to the family planning process, formulating optimizable sub-problems, using technology to interpret and record decisions, and developing robust decision rules that can be implemented in practice.

4.3 Life-cycle models

Life-cycle models are models that involve multi-period hedging. This subsection discusses key aspects of these models, including their origin and characteristics, human capital, life-cycle consumption, life-cycle portfolio, and their relationship to annuities.

⁸ Prior to the first 10 years from the benefit election date or until the first withdrawal, the protected withdrawal value is the larger of the accumulated fund value at 5 percent per annum and the highest anniversary value. After withdrawals have begun, a ratchet provision is available to lock in investment gains.

⁹ “game” in the sense of von Neumann and Morgenstern (1944).

4.3.1 Introduction

The key attribute of the life-cycle approach is that it is based on a “new Mertonian paradigm that takes account of multi-period hedging, labor supply flexibility, and habit formation,” rather than the “old Markowitz single-period paradigm of efficient diversification.” [Bodie (2002)]

Samuelson (2006) presents some observations on personal life-cycle finance.

4.3.1.1 Modigliani

Bodie et al. (2006) survey the conclusions of economic theorists, which are based on their models of rational optimizing behavior and competitive market equilibrium. They believe that this theory offers important insights and guidelines to the financial service firms that produce life-cycle financial products and to the advisors who suggest which of these products their clients should buy.

The basis for Modigliani Life cycle Hypothesis of Saving (LCH) were the articles Modigliani and Brumberg (1954 and 1979), which he and a student of his wrote [Modigliani (1986)].

“Lifecycle finance à la Franco Modigliani” [Samuelson (2006)].

4.3.1.2 Characteristics of life-cycle model

Bodie et al. (2006) highlight six concepts from the life-cycle model that are directly germane to the practice of life-cycle planning: (i) the notion of a lifetime budget constraint, (ii) the relevance of contingent claims in life-cycle planning, (iii) the trade-off imposed by varying costs of consumption over one’s lifetime, (iv) asset allocation over the lifecycle, (v) the role of portfolio constraints, and (vi) the phases of the life cycle.

4.3.1.3 Comment

The two competing views with respect to optimization goals for retirees are captured in the following statements.

“The goal of these retirees should be to maximize income with an acceptable probability of ruin” [Rubin (2000)].

“I go on to write articles ... to rebut the many pure mathematicians who believe that all of us should seek only to maximize our portfolio's longtime growth rates” [Samuelson (2006)].

4.3.2 Human capital

4.3.2.1 Introduction

Campbell et al. (1999) explain the relevance of human capital as follows:

The theoretical literature on [the effect of labor income] can be loosely summarized as follows. A household with labor income has an implicit holding of a nontradable asset, human capital, that represents a claim to the stream of future labor income. This nontradable asset can “crowd out” explicit asset holdings. If labor income is literally riskless, then riskless asset holdings are strongly crowded out and the household will tilt its portfolio towards risky assets (Bodie, Merton, and Samuelson 1992). If the household is constrained from borrowing to finance risky investments, the solution may be a corner at which the portfolio is 100% risky assets. If labor income is risky but uncorrelated with risky financial assets, then riskless asset holdings are still crowded out but less strongly; the portfolio tilt towards risky assets is reduced (Viceira 1997). If labor income is positively correlated with risky financial assets, then risky assets can actually be crowded out, tilting the portfolio towards safe financial assets.

Bodie et al. (1992) examined the effect of the labor-leisure choice on portfolio and consumption decisions over an individual's life cycle. Their main goal was to investigate how the presence of labor flexibility affects consumption, saving, and portfolio investment decisions over the life cycle. Their analysis showed that labor and investment choices are intimately related and led to the proposition that “Under 'normal' circumstances, an individual will tend to exhibit more conservative investment behavior as he nears retirement.”

Gomes et al. (2008) investigated optimal consumption, asset accumulation and portfolio decisions in a realistically calibrated life-cycle model with flexible labor supply. Their analysis reinforced prior findings that equities are the preferred asset for young households, with the optimal share of equities generally declining prior to retirement except that variable labor materially alters pre-retirement portfolio choice by significantly raising optimal equity holdings.

Benzoni et al. (2007) studied portfolio choice when labor income and dividends are cointegrated (have long-run dependence). They concluded that cointegration causes the young worker's human capital to effectively become “stock-like,” which suggests that young investors should take substantial short positions in the stock market. Conversely, for older workers with shorter times to retirement, cointegration does not have sufficient time to act, and their human capital becomes more “bond-like.”

Milevsky (2009) provides a novel, yet insightful, account of the role of human capital in personal financial planning.

4.3.3 Life-cycle consumption

Bodie et al. (2006) stress the central role of consumption in life-cycle planning and showcase the use of financial assets as a vehicle for transferring consumption from points in the individual's life-cycle at which consumption is relatively less valuable to points at which consumption is relatively more valuable.

4.3.4 Life-cycle portfolio

Horneff et al. (2008b) derived the optimal consumption and portfolio choice pattern over the life-cycle for households facing uninsurable labor income risk, ruin risk, stochastic capital markets, and uncertain lifetime. The investor underlying their model was assumed to be a dynamic utility maximizer with constant relative risk aversion (CRRA) and Epstein/Zin preferences, who has access to liquid stocks, bonds, and illiquid life annuities.

Dominitz and Hung (2006) simulate the welfare implications of alternative retirement plan investment options given that households invest according to simple decision rules like lifestyle, lifecycle, and the 1/n rule. They conclude that while it does tend to be conservative, life-cycle investing may induce some investors to take on more risk than they would otherwise and to invest more efficiently than when left to their naive strategies.

A complicating factor is that many households, particularly younger and poorer ones, do not appear to hold equities. This is contrary to simple frictionless models of optimal portfolio choice, but may be due to a fixed cost for participating in equity markets. Such a fixed cost would deter households from buying equities until their wealth levels are high enough to justify paying the cost [Campbell et al. (1999)].

4.3.5 Humped-shaped

4.3.5.1 Humped-shaped equity position

While evidence on household portfolio allocation is fragmentary, a few empirical papers found that household portfolios over the life cycle have hump-shaped equity positions and U-shaped positions in safe assets, as anticipated by the theoretical literature [Campbell et al. (1999)].

Bodie and Treussard (2007) concluded that in order to reflect gradual changes in human capital risk over the life cycle from predominantly stock-like to mostly bond-like, the proportion invested in equity should be hump-shaped rather than a linear function of age.

When labor income and dividends are cointegrated, the effects associated with younger and older individuals create hump-shaped life-cycle portfolio holdings, consistent with empirical observation. These results also hold when asset return predictability is accounted for [Benzoni et al. (2007)].

4.3.5.2 Humped-shaped consumption function

Hansen and İmrohoroğlu (2008) used a calibrated general equilibrium model with social security to assess the extent to which a lack of annuity markets, in and of itself, can account for the hump shape that consumption exhibits over the life cycle in empirical studies. They found that consumption over the life cycle in their calibrated model displays a hump shape, but the timing of the hump is later than what has been estimated from US data.

4.3.6 Annuities

Yaari (1965) concluded that a life-cycle consumer with no bequest motive will always choose to fully annuitize, provided that the market for annuities is actuarially fair. His conclusion was based on the observation that the actuarial bonds pay the market interest rate plus a mortality credit during the lifetime of the annuitant.¹⁰

Brown (2001) uses a life-cycle model of consumption and dynamic programming to examine household annuitization decisions. While some of the variation in the expected annuity decision is explained by the life-cycle model, he found that health status and an individual's time horizon for financial decision making are significant determinants. There was no evidence that bequest motives are a material factor when making marginal annuitization decisions.

Hansen and Imrohoroglu (2008) explore the quantitative implications of uncertainty about the length of life and a lack of annuity markets for life cycle consumption in a general equilibrium overlapping generations model in which markets are otherwise complete.

4.3.7 Comment

Siegel (2007), and its roundtable transcripts¹¹, and Bodie et al. (2007) provide a good review of the state-of-the-art and prognosis for the life-cycle concept.

4.4 Conserving portfolios during retirement

Two important risk factors retirees must consider when making [investment] decisions are financial market risk and longevity risk. Financial market risk is the volatility in the capital markets which induces portfolio values to fluctuate up and down. If the value of the securities in the portfolio of a retiree drops in the early years of retirement, the portfolio may not be able to cushion the added stress of systematic withdrawals. This may deplete the portfolio to such an extent that it is unable to provide the necessary income for the desired lifestyle or it may simply run out of money too soon [Chen and Milevsky (2004)].

4.5 Investing retirement wealth

It often is suggested that the portfolio of retirees should include equities. Murphy (2000), for example, presented a model that projects withdrawals and investment returns for a hypothetical retirement account. Essentially, his model uses a Monte Carlo simulation process to compute the probability that the retirement fund will be exhausted under different withdrawal and asset mix scenarios. Over a long time horizon, his analysis indicated that the difference in downside risk of having the funds exhausted early because of poor returns is minimal between asset classes, while

¹⁰ Yaari's model did not involve a risky asset.

¹¹ www.soa.org/research/pension/research-post-retirement-needs-and-risks.aspx

there is more upside potential for significant gains with the more risky portfolios. He concluded that future retirees should consider investing a portion of their funds in the stock market.

This view is not unanimous. Bodie (2006), for example, notes that a retiree can “have a long time horizon and be very averse to risk. If so, [the retiree] will prefer to invest in safe assets that have a long maturity, such as Treasury Inflation-Protected Securities (TIPS) and Inflation-adjusted Savings Bonds (I-Bonds).”

4.6 Optimal investment strategy

4.6.1 Lifetime ruin

Young (2004) investigated how retirees should participate in a risky financial market to minimize the probability that they outlive their wealth, also known as the probability of lifetime ruin.

4.6.2 Suitable and optimal

Bolster and Warrick (2000) describes a technique for asset allocations that are both suitable and optimal. They begin with a model of suitability that uses the Analytic Hierarchy Process (AHP) to create unique asset allocations for individual investors based on their personal attributes. They then compare the mean-variance performance of these suitable portfolios with independent portfolios generated using traditional mean-variance optimization (MVO) methodology. Their main goal is to reconcile suitable and optimum portfolios by adjusting AHP and mean variance parameters and they argue that sequential application of the two approaches provides superior results when compared to those generated by AHP alone.

5 Investment Withdrawals During Retirement

One of the most important challenges that must be faced at retirement is determining how to convert retirement funds into a sustainable stream of retirement income, given the uncertainty associated with future lifetime and future expenditures [Brown (2004)]. In this section, we review the literature pertaining to five aspects of this issue: withdrawal goals, sustainable investment withdrawals, optimum withdrawals, social security, the role of home equity, and adjusting withdrawal rates for taxes and expenses.

5.1 Introduction

Pre-retirement financial strategies focus on issues related to accumulating wealth, such as how much to save, asset allocation, the advantages of tax-deferred accounts, and so on. An understanding of such topics is important because wealth accumulation is imperative if pre-retirement standards of living are to be sustained after retirement. However, a downside of this emphasis on wealth accumulation is that many retirees find it difficult to transition from an accumulation mindset to a decumulation mindset [Brown (2008)].

5.2 Withdrawal goals

Levenson and Traub (2007: slide 6) define the retirement number as the amount of money needed at retirement to fund post-retirement life. Noting that everyone's number is different, they give a simplified representation of the number as the amount planned to be spent each year in retirement, reduced by income from Social Security and/or other pensions. Current investment balances, adjusted for taxes upon withdrawal, are sufficient if they exceed that amount.

Penner (2008) uses data from the 2004 Survey of Consumer Finances to explore how well baby boomers are prepared for retirement based on estimates of the ratio of post- to pre-retirement consumption. He shows that some of the poorest households are best prepared because they can maintain consumption by relying almost solely on Social Security while many of the most affluent households are poorly prepared because they will experience a decline in consumption upon retiring. He also questioned the appropriateness of the equalization of pre- and post-retirement consumption as a yardstick at all income levels.

Vernon (2005: 131, 316-7) also questions the appropriateness of the equalization of pre- and post-retirement consumption as a yardstick. He advocates, instead, focusing on what he terms the $I > E$ methodology, where I is annual income and E is annual living expenses.

5.3 Sustainable investment withdrawals

In keeping with the concern for safe withdrawal rates, a primary goal of a PRFS is to provide a sustainable stream of retirement income for a retiree. Following an introduction, this subsection

focuses on five aspects of sustainable investment withdrawals: fixed withdrawals, annuity-equivalent withdrawals, considerations, phased withdrawal and analysis.

5.3.1 Introduction

There are two basic ways to convert retirement savings into retirement income [WISER (2003)]:

- Make systematic withdrawals at a rate sustainable for the retiree's lifetime, keeping the remaining funds fully invested; or
- Buy a life annuity with some of the funds, in order to provide a stream of guaranteed income payable for the retiree's lifetime or for the joint lifetime of the retiree and his or her spouse.

Brown (2004) comments that for many retirees financial security in retirement will prove to be an elusive goal. Even those who have accumulated sizeable retirement funds may find that retirement brings a host of new financial challenges. Perhaps the most important of those challenges is determining how to convert retirement funds into a sustainable stream of retirement income, given the uncertainty about future lifetime and future expenditure.

Financial commentators, such as Arnott (2004), have emphasized the need for more research on sustainable spending rates from diversified portfolios, a topic that is of relevance to individual retirees.

5.3.2 Fixed withdrawals

Huang et al. (2004) used numerical partial differential equations (PDE) to compute the probability of lifetime ruin, as measured by the probability that a fixed retirement consumption strategy will lead to financial insolvency under stochastic investment returns and lifetime distribution. Their paper then goes on to compare the PDE-based values with moment matching and comonotonic-based¹² approximations that have been proposed in the literature.

Gerrard et al. (2004) investigated the income drawdown option and optimal investment strategies to be adopted after retirement, when allowing for periodic fixed withdrawals from the fund. The indices they analyze include the risk of outliving the assets before annuitization occurs (risk of ruin), the average time of ruin, the probability of reaching a certain pension target (that is greater than or equal to the pension that the member could buy immediately on retirement), the final outcome that can be reached (distribution of annuity that can be bought at limit age), and how the risk attitude of the member affects the key performance measures mentioned above. They find that there is a natural target level of the fund, interpretable as a safety level, which can never be exceeded when optimal control is used.

¹² A set of n -dimensional vectors is said to be comonotonic if for any two of its vectors, one is greater than or equal to the other, componentwise.

5.3.3 Considerations

Milevsky and Abaimova (2005a) conclude that the probability of ruin for retirees is substantial if more than 5% of initial capital, adjusted for inflation, is spent during retirement; the first seven to ten years of retirement is the most critical investment period, because the combination of withdrawals and a bear market during that time may so devastate the retirement fund that drastically reduced payments, or even ruin, will result. There are substantial benefits to annuitization and this should be encouraged as a risk management strategy.

5.3.4 Phased withdrawal

Dus et al. (2004) evaluated several alternative designs for phased withdrawal strategies, allowing for endogenous asset allocation patterns and worker flexibility with respect to both when to retire and when to switch to an annuity. They extended the prior literature by considering not only the probability of a consumption shortfall, but also the size of the shortfall when it occurs. They concluded that delayed annuitization is appealing since it offers relatively low expected shortfall risk, good expected payouts during the retiree lifetime, and some bequest potential for the heirs.

5.3.5 Analysis

Milevsky and Robinson (2000) used option pricing to compute the probability of ruin for a retiree who wishes to consume a fixed periodic amount from an initial endowment invested in a portfolio earning a stochastic rate of return. This probability of ruin corresponds to the probability that a suitably parameterized Asian call option¹³ will expire in-the-money. Using realistic market values for equity and fixed-income investments, they demonstrate that the conditional [on death] probability of ruin is minimized when there is a relatively high allocation to equity until quite late in life.

Milevsky (2004b), as an alternative to resorting to opaque and often irreproducible Monte Carlo simulations, uses a simple analytic formula for the probability that a portfolio earning a lognormal investment return subject to a constant (after inflation) withdrawal rate will be sustainable over a random lifetime horizon. The formula merges asset allocation parameters, mortality estimates and spending rates. He confirms that a 65-year old retiree with a stochastic lifetime horizon faces a 10% chance of ruin if he/she consumes more than \$4-per-\$100 principal of an equity-based portfolio with an expected real return of 7% and volatility of 20%.

5.3.6 The 4% rule

In his seminal article on safe withdrawal rates, Bengen (1994) wrote:

¹³ The payoff of an Asian option is determined by the average underlying price over some predetermined averaging period [Hull (2003: 443)].

For a client [beginning retirement at] age 60-65, [the “safe” withdrawal rate] ... will usually be about 4 percent ... adjusted up or down for inflation every succeeding year ... [and] as long as the client's goals remain the same, there is no need to change the initial asset allocation. ... You should do all you can to dissuade the client from being too “frisky” with spending early in retirement. An initial five-percent withdrawal rate is risky; six percent or more is “gambling.”

In contrast, Guyton (2004) using decision rules that govern portfolio management, sources of annual income withdrawals, impact of years with investment losses and withdrawal increases to offset ongoing inflation, produces a maximum “safe” initial withdrawal rate as high as 5.8 percent to 6.2 percent depending on the percentage of the portfolio that is allocated to equities.

Guyton and Klinger (2006) used stochastic (Monte Carlo) analysis to validate the decision rules established by Guyton (2004), which lead to higher initial withdrawal rates than reported in previous research. Then, using two new decision rules, the capital preservation rule and the prosperity rule, which act as “financial guardrails” when market conditions cause the initial withdrawal rate to rise or fall significantly, they conclude that initial withdrawal rates of 5.2–5.6 percent are sustainable at the 99 percent confidence standard for portfolios containing at least 65 percent equities. They write that “[c]onsistently applying the two new decision rules effectively eliminates the risk of exhausting retirement assets.”

Milevsky (2007g) points to the futility of using this type of betting strategies to improve the retiree’s solvency position. He admonishes retirees that adopt such strategies, by observing that “... you have neither reduced nor mitigated financial risk but simply taken a bet on economic scenarios you believe will not happen. Safety is just a mirage.”

Two recent studies have revisited the 4% rule. In the first, Scott et al. (2009), Sharpe’s study,¹⁴ concluded that “Despite its ubiquity, it is time to replace the 4% rule with approaches better grounded in fundamental economic analysis.” Their concerns with respect to the rule were that retirees accumulate unspent surpluses, they overpay for the distributions they receive, and many retirees may actually prefer an alternative, cheaper plan.

In the second study, Milevsky and Huang (2010), concurring with Scott et al. (2009), assert that the “start by spending x%” strategy has absolutely no basis in economic theory. Then, using what they characterize as a rational utility-maximizing model of consumer choice, a Gompertz mortality curve, and simplifying assumptions, they conclude that risk aversion, a component missing in the 4% rule, is a key consideration when developing optimal portfolio withdrawal rates.

These studies notwithstanding, the state-of-the-art is encapsulated in the statement of Scott et al (2009) that “Many practical issues remain to be addressed before advisors can hope to create in-

¹⁴ This study often is referred to as “Sharpe’s study” because all 3 authors are employed at William F. Sharpe’s Financial Engines, and he is a coauthor.

dividualized retirement financial plans that maximize expected utility for investors with diverse circumstances, other sources of income, and preferences.”¹⁵

5.4 Optimum withdrawals

This subsection discusses the literature as it pertains to six considerations with respect to optimal withdrawals: exaggerated need, the Merton model, investment-consumption constraints, continuous time models, pooled annuity funds, and lockboxes.

5.4.1 Introduction

Palmer (2000) cautions that the amount needed for retirement should not be exaggerated, either by inappropriate assumptions or overlooking income sources, because it might frustrate the individual and cause him/her to behave sub-optimally. WISER (2003) also discusses this issue.

Kitces (2008) echoes this concern with his study of how stock market valuation affects withdrawal strategies and his query as to whether the safe withdrawal rate of Bengen (1994) sometimes is too safe. An interesting feature of his analysis is his use of the inverse relationship between price-to-earnings ratios and projected returns for anticipating the level of sustainable withdrawal rates.

5.4.2 Merton

Merton (1969) derives the optimality equations for a multi-asset problem when the rate of returns are generated by a Brownian-motion process. The two-asset model with constant relative risk-aversion was examined in detail and an explicit solution was found for the case of constant absolute risk-aversion. His general technique has since been employed to examine a wide class of intertemporal economic problems under uncertainty.

In contrast, advisors try to incorporate intertemporal considerations into their analysis by rebalancing [Merton (1969)].

There were a number of other noteworthy articles on the same general topic during that period of time, including Samuelson (1969), who developed a similar model in discrete-time for more general probability distributions; Tobin (1965), who did a multi-period analysis; Phelps (1962), whose model could be used to determine the optimal consumption rule for a multi-period situation where income is partly generated by an asset with an uncertain return; and Mirrlees (1963), who developed a continuous-time optimal consumption model of the neoclassical type with technical progress a random variable.

¹⁵ These and other practical concerns were reported by Powell (2010), who documented a rebuttal to Scott et al. (2009) that occurred in the financial-adviser community.

5.4.3 Investment-consumption

Li et al. (2007) investigates three investment-consumption problems for a risk averse investor: the first involves utility from only terminal wealth; the second involves utility from only consumption; and the third involves utility from both consumption and terminal wealth. To this end, he uses a multiperiod analysis that incorporates the uncertainties due to the economic environment, the asset return, and mortality risk. Dynamic programming is used to obtain analytical expressions of the optimal investment consumption strategies.

5.4.4 Continuous time model

Merton (1969) examines the combined problem of optimal portfolio selection and consumption rules for an individual in a continuous-time model whose income is generated by returns on assets and these returns, or instantaneous “growth rates,” are stochastic.

Babbal and Merrill (2007) focus on rational decumulation decisions for a newly retired individual who receives a lump-sum settlement from a retirement savings program, and has accumulated private savings and Social Security credits. In the spirit of Merton (1969, 1971) and Richard (1975), they develop a continuous-time model to study the asset allocation choices, where irrevocable life annuities are included along with fixed income and equity as the asset classes. The inflation-protected life annuity is the riskless asset in an intertemporal context with an uncertain lifetime.

Milevsky (2009: chapter 10) elaborates on this idea with his assertion that “product allocation is the new asset allocation.”

5.4.5 Pooled annuity funds

Stamos (2007) investigates the optimal continuous time dynamic consumption and portfolio choice for pooled annuity funds, where the population of the pool is modeled using a Poisson process with time-dependent hazard-rate, and the solution of the optimization problem is reduced to the numerical solution of a set of ordinary differential equations (ODEs). He found that even pools no bigger than a family can self-insure against mortality effectively and reach more than 50% of the utility gains of a perfect pool, which may help to explain the low demand for private life annuities.

5.4.6 Lockbox

The lockbox strategy divides the initial retirement wealth among separate accounts, one account for each future year of spending. The assets in each account are dynamically managed according to the account's exogenous investment rule. When an account reaches its target year, the retiree cashes out its investments, closes the account, and uses the proceeds for that year's consumption. The accounts can be real or virtual, and are collectively called lockboxes, in recognition of the

retiree's implicit obligation to yearly spend all the assets from the target account and to never commingle or spend the assets of any of the remaining accounts [Sharpe et al. (2007: 12)].

5.4.7 Comments

Babbel and Merrill (2007) find that high levels of annuitization are rational under a wide range of risk aversion levels, even when stock market returns and annuity price loadings are assumed to be much greater than is generally the case.

The first study to model individual behavior under the possibility of default by the insurer issuing annuities, they find that even a little default risk can have a very large impact on annuity purchase decisions, and that state insolvency guaranty programs can have a big impact upon the levels of rational life annuity purchases, particularly large annuities [Babbel and Merrill (2007)].

Post et al. (2008) use micro level data from the United States and Germany to compare the optimal expected lifetime utility computed using a realistically calibrated model with the actual utility as reflected in empirical asset allocation choices. Their results show large welfare losses as a consequence of suboptimal investment behavior for many combinations of individual characteristics. They conclude that there is considerable room for improvement and their model identifies population subgroups that would benefit most from a better asset allocation.

5.5 Social Security

Claiming Social Security benefits and retiring are separate decisions.

MetLife (2007) discussed the optimal time to begin Social Security benefits. They noted that Social Security benefits are a major component of retirement income for most Americans. They cautioned, however, that while benefits can begin at age 62, deferring receipt until age 67 or even age 70, results in considerably higher payments. Moreover, with respect to a retiree and a non-working spouse, during the lifetime of the retiree, the spousal benefit is 50% of the smaller of the retiree's benefit or the full retirement amount. However, subsequent to the death of the retiree, the spouse receives 100% of the retiree's benefit.

Sass et al. (2007) investigated why most married men claim Social Security benefits at age 62 or 63, well short of both Social Security's Full Retirement Age and the age that maximizes the household's expected present value of benefits (EPVB). Using data of married men who retired prior to becoming eligible for Social Security at age 62, they found that the average age such men maximize household EPVB is 66, but the great majority claim at 62, which results in a survivor benefit that on average is nearly 20 percent less than its value at the maximizing age. Using regression analysis, they found no association between early claiming and caddishness, the ability of husbands to make claiming decisions independently, and financial-numerical literacy, but there was a statistically significant association between college education and later claiming.

5.6 The role of home equity (reverse mortgages)

“... my 1958 point that, money aside, people's homes are an ideal contrivance for converting working age savings into retirement day dissaving” [Samuelson (2006)].

Van Harlow (2007) notes that reverse mortgages offer the advantage of regular cash flows that are not taxable income. However most retirees find the high initial costs of reverse mortgages daunting and the market needs to mature. She warns against regarding home equity as a significant retirement funding source because recurring cyclical downturns in real estate can severely restrict a retirement income that relies too heavily on home equity. Moreover, investment returns on residential real estate have been lower historically than on stocks and bonds. She notes also that emotional aspects of homeownership can also present significant barriers to the conversion of home equity into income for retirees.

Van Harlow (2007: 16-17) mentions that another way to capitalize on home ownership is to downsize the home. In this case, the net difference in house values is the source of additional income in retirement. Once again, cyclical downturns can have a significant effect on the amount of income generated.

SOA (2009), a recent Society of Actuaries' monograph on “Housing in Retirement,” provides perspectives on many housing and retirement issues. The topics addressed include retirement implications of housing wealth and spending, how housing wealth affects retirement planning, reverse mortgages, and home equity as a long-term care funding source. Rappaport and Siegel (2009) provide an overview of the monograph, and summarize its key findings and observations about aspects of housing in retirement, and options for using housing value to help support retirement.

5.7 Adjusting withdrawal rates for taxes and expenses¹⁶

Grant et al. (1991) provide a dated, but informative, discussion of the taxation of pension income.

McGill et al. (2010: Chapter 17) present an overview of the current taxation situation for retirement benefits.

Milevsky (2007h) compares what he labels static (old) thinking, based on current tax law, to dynamic (new) thinking, based on possible tax law change. He makes the point that taxpayers should contemplate managing their future tax risk, not just their current taxes. Not unlike the conventional rules of investment diversification, the diversification of tax risk includes adopting strategies that will benefit the investor under alternative states of nature.

¹⁶ One interesting commentary, noted by Brown (2004), is that “while public policy has provided tax incentives for households to increase retirement savings, virtually no tax or other policy incentives exist to encourage households to convert their nest eggs into streams of lifetime retirement income.”

6 Retirement Annuities and Annuitization

This section presents an overview of the literature related to retirement annuities and annuitization. The subtopics addressed include: the life annuity, demand for annuities, annuitization decisions, timing of annuity purchases, systematic withdrawals and ruin in retirement, the insurer's perspective, and mandatory annuitization.

6.1 The life annuity

6.1.1 Introduction

Future retirees will need access to an alternate, reliable source of guaranteed retirement income for life. One financial product that meets this need is the life annuity, since it provides a guaranteed source of monthly income that cannot be outlived. Because they provide insurance against a drop in the retiree's standard of living at older ages, life annuities ought to play a central role in the portfolios of retirees [Brown (2004)].

Brown (2008) demonstrates that a life annuity is the most cost effective way of providing any given level of guaranteed income.

Life annuities are well designed to fill in the gap that retirees usually find between the amount of income they receive from guaranteed sources such as Social Security and the expenditures they consider essential for maintaining their standard of living [Brown (2008)].

WISER (2001) provides information and guidelines for people on how to make their savings last. It helps explain the important role that immediate annuities can play by stretching out retirement money to make sure it will last for the retiree's lifetime.

6.1.2 Bond-based investment perspective

The constant payout life-annuity is a bond-based investment with longevity insurance protecting the retiree from outliving his resources [Mitchell et al. (1999)].

6.1.3 Mortality credits

Milevsky (2004a) sees the acquisition of mortality credits as the *raison d'être* of life-contingent annuities. He notes that during the early years of retirement the magnitude of these mortality credits is quite small compared to the cost of survivor benefits, insurance fees and antiselection charges, but that, at advanced ages, they are substantial and unavailable from any competing asset class.

As a benchmark, we can compare the level annuity yields (payment as a fraction of initial premium) for 65-year-old males against the returns on long-term, high grade, fixed income securities, as a proxy for the underlying investment vehicles for insurance companies issuing level annuities. There clearly is a significant annuity premium as long as only income in retirement is involved. This premium is not surprising as it arises from the capital sacrifice [Kapur and Orszag (2002)].

6.2 Demand for annuities

“... it is a well known fact that annuity contracts, other than in the form of group insurance through pension systems, are extremely rare. Why this should be so is a subject of considerable current interest and debate Undoubtedly, “adverse selection”, causing an unfavorable payout, and the fact that some utility may be derived from bequests ... are an important part of the answer.” Franco Modigliani, December 9, 1985 Nobel Prize acceptance speech in Stockholm, Sweden.¹⁷

The demand for annuities is discussed in this subsection. Topics covered include reasons for the low levels of annuitization, the bequest motive, annuity equivalent wealth, and the consumption perspective.

6.2.1 Reasons for low levels of annuitization

Observed levels of annuitization are generally far below those considered optimal by economists. Possible justifications for this behavior, both supposed and real, that have appeared in the literature include: bequest motive, health shocks, self-insurance through family or other networks, social security crowding out private annuitization, premiums above actuarially fair prices,¹⁸ adverse selection, regulatory barriers for retirement plan sponsors, high-profile failures of insurance companies, irrevocability (illiquidity), imperfect information, and erosion of purchasing power due to inflation [Babbel and Merrill (2007: 3)].

Tomlinson (2008) adds complexity of the sale and clients' aversion to facing reality. Insofar as the former, he stresses the need for comprehensive recommendations with respect to age to annuitize, features, amount, asset allocation for remainder, wealth, survival outlook, and bequest goals.

Brown and Scahill (2007) reported that, at the time, in the United States and Canada, persons who applied for a life annuity were generally assumed to be in extremely good health.

¹⁷ Reprinted in the *American Economic Review*. Modigliani (1986).

¹⁸ While product delivery systems are beyond the scope of this report, it is worth noting that to the extent such systems improve competition and provide greater transparency of fees in annuity products, the price of annuities may become less of a deterrent in the future. One recent example of this is Income Solutions' platform for the purchase of institutionally priced immediate annuities by individual consumers [McGuire (2008)].

6.2.2 Bequest motive

Friedman and Warshawsky (1990) and Bernheim (1991) found that the bequest motive, coupled with the perception of actuarially unfair premiums, was a significant deterrent to annuitization.

Purcal and Piggott (2008) explored the puzzle of thin voluntary life annuity markets. To this end, they used an optimizing financial planning model in the tradition of Merton's (1969, 1971) optimal lifetime asset allocation model and Richard's (1975) theoretical extension to longevity insurance markets to investigate how individuals should determine their life insurance and annuity choices. The model explicitly recognized the role of consumption and bequests in an individual's preference function, as well as social security benefits and premium loading. Calibrated to the case of Japan, they found that the bequest motive was the most important deterrent to purchasing annuities, followed by the social security provision. Loadings were found to be relatively unimportant.

Other researchers disagree, contending that the bequest motive had only marginal impact on annuity purchases. These include Hurd (1989) and Brown and Poterba (2000). These last studies compared married and unmarried subjects.

In countries that have mandatory annuitization, such as the UK,¹⁹ the bequest motive does not inhibit purchases with respect to the assets that are required to be annuitized. It does result in a social conflict, however, since the state is the contingent beneficiary, rather than the heirs [Orszag (2000)].

6.2.3 Annuity equivalent wealth

Bütler and Teppa (2007) used micro data from 10 Swiss employer-based pension plans to study individual decisions to annuitize or to cash out pension wealth. The essential feature of the Swiss case is that the occupational pensions are mandatory. They found, consistent with other empirical studies, that the annuity equivalent wealth is the most important determinant of the annuitization decision. They also found that individuals often choose the standard option offered by the company or seem to follow their peers, and that small stocks of old age capital are much more likely to be withdrawn as a lump sum.

6.2.4 The consumption perspective

Brown et al. (2008b) found that re-framing the annuitization decision in consumption terms rather than investment terms significantly increases the relative attractiveness of life annuities.

Brown et al. (2008a) tested the relative effectiveness of these two framing contexts when different reference points are introduced. They found the previous result to be robust even when the

¹⁹ In the UK about 85%, or so, of individuals purchase a simple level annuity when they retire with a personal pension [Kapur and Orszag (2002)].

reference points include such things as annuity purchase price, level of principal protection, or the level of habitual spending. They concluded that there was little evidence that reference points are an important determinant of annuity demand.

6.3 Annuitization decisions

This subsection discusses annuitization decisions. The topics addressed include Yaari's full annuitization hypothesis and annuity payout strategies.

6.3.1 Introduction

At, or about, the age of retirement, DC plan participants must decide what fraction of their retirement funds, if any, should be annuitized. This decision is complicated, since it is irreversible and can prove costly in hindsight [Milevsky (2001)].

Participants in 401(k), 403(b), and IRA plans, and variable annuity contracts, have considerable discretion as to the annuitization of their retirement funds [Milevsky (2001)].

Due to adverse selection, acquiring a life annuity is an irreversible²⁰ transaction. Consequently, there is an incentive to delay annuitization [Milevsky and Young (2007a)].

A retiree's consumption strategy leads to lifetime ruin if it results in zero wealth²¹ while the retiree is still alive. Milevsky et al. (2004) derived the optimal investment and annuitization strategy for a retiree whose objective is to minimize the probability of lifetime ruin.

Mitchell (2001) reviews the usefulness of life annuities in providing protection against longevity risk.

6.3.2 Yaari's full annuitization hypothesis

As mentioned previously, Yaari (1965) concluded that a life-cycle consumer with no bequest motive will always choose to fully annuitize, provided that the market for annuities is actuarially fair.

Davidoff, Brown and Diamond (2005) proved that full annuitization is optimal in a complete market. In so doing, they proved that the original results of Yaari (1965) hold true without assuming exponential discounting, or relying on the expected utility axioms and intertemporal separability. Moreover, their result does not require actuarially fair prices, but only that there is no bequest motive and that the rate of return for the annuities is greater than that of conventional

²⁰ This is an overstatement, of course, since the SEC has allowed annuitants to sell their annuities since 1988, and firms such as Woodbridge Investments are in the business of buying annuities.

²¹ Although the "zero wealth" criterion often is used in analysis, a more appropriate criterion is the subsistence level.

assets of matching financial risk. In incomplete markets²², even if there is a strong bequest motive, they show that it may still be optimal to annuitize a substantial amount of the retiree's wealth. Even with optimal consumption trajectories very different from available annuity income streams, they still found a preference for considerable annuitization.

6.3.3 Annuity payout strategies

This subsection presents three annuitization strategies that have been discussed in the literature: all-or-nothing, gradual annuitization, and a combination of the two.

6.3.3.1 All-or-nothing amortization

Kaplan (2005) used Monte Carlo simulation to investigate the trade-offs between income level, portfolio volatility, success probability, and estate size, given a set of asset mixes and single premium immediate annuities. His analysis focused on strategies in which the retiree makes a one-time annuitization decision and selects a single asset mix for non-annuitized assets. He found that the role annuities play in an optimized strategy depends largely on how inflation was modeled.

Gupta and Li (2007) develop a retirement planning model that merges annuity purchase decisions with consumption–investment selections in order to maximize lifetime utility of consumption and wealth. Using a multi-period wealth evolution model, optimal decisions are determined as a trade off between consumption and investment among an annuity, a risky and a risk-free asset. They found that if the investment return from the annuity was not competitive, because of the premium loading, then there would be a single annuity premium payment at the optimal age of annuitization. The choice of this optimal age is influenced by the retiree's wealth, income, and remaining lifetime.

6.3.3.2 Gradual annuitization

The analysis of Horneff et al. (2008b) lead to the finding that a considerable fraction of wealth is gradually annuitized to skim the mortality credit, with the remaining liquid wealth invested mainly in stocks to offset the riskless investment in life annuities. Their model allows them to assess the economic importance of common explanations for the empirically low annuity demand by analyzing participation rates, annuitization fractions, and welfare effects.

6.3.3.3 Combination

Milevsky and Young (2007a) examined the optimal annuitization, investment and consumption strategies of a utility-maximizing retiree facing a stochastic time of death under a variety of institutional restrictions. Their focus is the impact of aging on the optimal purchase of life annuities

²² While the financial market model might be complete, in the sense that any contingent claim is replicable by a trading strategy, the product space (of the two individual spaces, financial and actuarial) will be incomplete unless the actuarial risk (mortality risk) is hedgeable in the stock market [Argesanu (2004)].

and, to this end, they differentiate between the institutional all-or-nothing arrangement, whereby annuitization must take place at one distinct point in time, and an open market structure, where individuals can annuitize any fraction of their wealth at anytime.

Under the all-or-nothing arrangement, they derive the optimal age at which to annuitize and develop a metric to capture the loss from annuitizing prematurely. Under the openmarket structure, they locate a general optimal annuity purchasing policy, and find that an individual will initially annuitize above a minimum level, and then buy annuities to keep wealth to one side of a separating ray (barrier) in wealth-annuity space [Milevsky and Young (2007a)].

Milevsky and Young (2007a) claim that their article was the first in the portfolio choice literature to use both life annuity products and realistic institutional restrictions that are unique to the market for mortality-contingent claims.

Kapur and Orszag (2002) investigated the optimal portfolio strategies for retirees who can invest in assets of different risk classes and levels of capital sacrifice on the death of the annuitants. One key result was that when suitable equity-linked annuities are not available, retirees wait to annuitize in order to capture the higher returns from equities. However, because the annuity premium increases with age and the equity premium does not, retirees focused on income typically will shift their portfolio towards fixed-income annuities as they age.

6.3.3.4 *Overlapping generations model*

Strawczynski (1999) remarked that two well-known results pertaining to the demand for annuities in an overlapping generations model are: (i) egoistic agents should annuitize all their wealth; and (ii) altruistic agents should segment their savings between riskless bonds, for their bequests, and annuities, for their own consumption.

6.3.3.5 *Comment*

Milevsky (2007a) notes that the guaranteed minimum income benefits (GmIBs) and guaranteed minimum withdrawal benefits for life (GmwBfL), which are attached to variable annuity (VA) policies, present an alternate solution to the annuitization dilemma. In this instance, these riders provide protection against the underlying investments being devastated.

Given that there should be some level of lifetime income that cannot be outlived, he notes that the issues to be resolved include the overall portfolio's optimal allocation to this product class, the proportion of retirement income that should emanate from SPIAs, compared to that from a systematic withdrawal plan (SwiP) and/or a VA with a guaranteed living benefit (GLB), and the age when these types of longevity insurance should be acquired [Milevsky (2007)].

6.4 Timing of annuity purchases

The high price sensitivity of the life annuity makes the implicit option to time the purchase valuable [Milevsky and Young (2002), Frehen et al. (2008)]. After an introduction to the topic, four

related issues are discussed: specific age purchases, gradual purchases, interest rate risk, and the lookback option.

6.4.1 Introduction

WISER (2003) points out that retirees need to steer a safe path between the two pitfalls of running out of money during their lifetime or, perhaps a lesser evil, living below the standard they could afford. A way out of this dilemma is to manage their own retirement funds until an optimal time to convert some assets to an annuity, and then tailoring the amount and type of annuity to their own situation.

Milevsky (2001) focuses on the question of when and if to annuitize. His main conclusions are: annuitization of assets should be actively encouraged at higher ages; standard microeconomic utility-based arguments suggest that retirees would be willing to purchase a life annuity in spite of a substantial premium loading; large adverse selection costs, which range from 10% to 20%, might be a strong deterrent to full annuitization; the retiree with a (strong) bequest motive, should self-annuitize²³ during the early stages of retirement (until age 75 to 80), and then annuitize; and variable immediate annuities (VIAs) are currently underutilized (and not available in certain jurisdictions) and can only grow in popularity

6.4.2 Specific age purchases

Milevsky and Young (2002) develop a normative model of when, and if, an immediate annuity should be purchased, based on the qualitative argument that there is a real option embedded in the decision to annuitize. Using standard continuous-time technology to solve the optimal asset allocation and annuitization timing problem, they define the value of the real option to defer annuitization as the compensating utility loss from being unable to behave optimally. Then, based on reasonable capital market and actuarial parameters, they estimate that this real option is quite valuable until the mid-70s or mid-80s. However, they also show that low-cost variable immediate annuities greatly reduce the option value to wait as well as create substantial welfare gains.

Milevsky and Young (2007b) use mean-variance dominance arguments (which do not involve utility-based models of consumption and bequest motives) to develop a framework for locating the optimal age (time) at which to purchase an irreversible life annuity. They show that annuitization prior to age 65-70 is dominated by self-annuitization even in the absence of any bequest motives, and that for retirees who are willing to accept some risk in exchange for liquidity and potential bequests, the optimal age can be even higher.

²³ As indicated throughout the text, although some current authors have used the term self-annuitization in situations involving a single retiree, others have used terms such as systematic withdrawal and phased withdrawal, which seem more consistent with conventional terminology. Just as with self-insurance, self-annuitization involves the spreading of risk among many units of exposure, and not merely the retention of risk [Huebner et al (1996: 452-453, 561-563)].

6.4.3 Gradual purchases

Horneff et al. (2007) compute the optimal dynamic asset allocation policy for a retiree who can choose between consumption and investment in stocks, bonds, and annuities. Three cases are considered: complete switching, partial switching, and gradual annuitization. Pricing the annuities to account for asymmetric mortality beliefs and administration expenses, they show that it is optimal for the retiree to use gradual annuitization. Insofar as the case in which the retiree is restricted to a one-time purchase of annuities, accompanied by a (complete or partial) switching strategy, they find that this restriction reduces both the utility and the demand for annuities.

Milevsky and Salisbury (2006a) refer to this notion of combining annuities and wealth management into an overall package as “product allocation.”

Pang and Warshawsky (2009) bring a consultant’s view to product allocation. Specifically, they compare several wealth management strategies for retirees: systematic withdrawals from mutual funds, one-time complete or partial conversion to fixed or variable payout annuities, year-long phased conversion to fixed life annuities, and variable annuities with a guaranteed minimum withdrawal benefit.

Prominent among the standardized retirement payout strategies of financial advisors are phased withdrawal plans offered by mutual funds including the self-annuitization or default rules encouraged under US tax law, and fixed payout annuities offered by insurers. Horneff et al. (2008a) use a utility-based framework, stochastic capital markets and uncertain lifetimes to evaluate these payout strategies on both a stand-alone, and integrated basis for consumers with different degrees of risk aversion. They report that the integrated strategies can enhance retirees’ well-being by 25-50 percent for low and moderate levels of risk aversion. They also examine welfare changes associated with switching to a fixed annuity sometime after retirement, which affords retirees the chance to initially benefit from the equity premium, and then to exploit the mortality credit later in life. For moderately risk-averse retirees, they find that the optimal switching age lies between 80 and 85.

Milevsky (2007a), discussing what the insurance industry does to help promote annuitization, feels that annuitization should be a gradual default strategy, where a portion of the pension wealth is annuitized at age 65, age 70, etc.

6.4.4 Interest rate risk

One form of annuity risk is the risk that at the time of annuitization the prevailing interest rates are low. This risk can be reduced using the “income drawdown option” whereby the retiree is allowed to choose when to annuitize within a certain period of time after retirement. Some countries impose a compulsory annuitization age. In the UK, as mentioned previously, this age is 75, and, prior to this age, the retiree can withdraw periodic amounts of money to provide for daily living, subject to the limits imposed by the plan’s rules (or by law).

6.4.5 Lookback option

Frehen et al. (2008) develop and price a lookback option on a life annuity contract which would allow retirees to look back at retirement and buy a life annuity for the minimum forward price in the lookback period. The option would be purchased in the last years before retirement. They find that the price of a lookback option, with a maturity of three years, amounts to 8%–9% of the retirement fund at the option issuance date. They report that the option price is highly sensitive to the exercise price of the option, the time to maturity, and interest rate volatility, but that asset allocation decisions and initial interest rates hardly affect the option price.

6.5 Systematic withdrawals and ruin in retirement

The terms systematic withdrawal, phased withdrawal, and self-annuitization are used interchangeably in the literature.

Rubin (2000), motivated by concerns over the longevity risk, stressed the advantages of annuitization over systematic withdrawals. Applying Social Security mortality improvement factors to the Annuity 2000 table, he found that for a couple both age 65 the probability that one will be alive past age 94 is over 50%. He noted that TIAA-CREF seeks to make annuitization the decumulation method of choice of a participant, rather than the exception.

The “do-it-yourself” strategy of investing retirement wealth and withdrawing the exact same consumption stream as an annuity would have provided, risks under-funding retirement in the event of below average investment returns with above average longevity [Milevsky (1998)].

Goodman and Heller (2006) examine the financial efficacy of receiving income through a life annuity, in contrast to systematic withdrawals from a retiree's account. For comparison purposes, the payments from a life annuity are contrasted with withdrawals of the same amount made from an investment account, assuming that both the life annuity and the investment account reflect the same net investment return. Their analysis shows that for both a fixed and variable life annuity, a retiree who uses systematic withdrawals is projected to run out of funds some time before reaching his or her life expectancy. That is, there is more than a 50% chance that the retiree would run out of money

For both fixed and variable annuities, they show that the cost to a 65 year old of delaying annuitization for five years, during which time there are the foregoing systematic withdrawals, is about a 5% reduction in future income, while a delay of ten years might result in a 15% reduction in lifetime income. In contrast, however, they note that if interest rates rise by 0.25% each year, a five-year delay in annuitization results in a 7% gain in income.

6.6 Insurer's perspective

Turner (2006) concluded that there is no inherent lack of capital to support the growing demand for longevity coverage in, say, the last twenty years before an expected death, and the price for

coverage will still look attractive to many annuity purchasers. However, when contemplating longevity risk for extended periods, for instance, forty or fifty years before the expected age of death, the uncertainties explode, and the price that risk capital will require for absorbing that risk is unattractively high, and would not be paid by any significant number of individuals making their own rational decisions.

6.7 Mandatory annuitization

Mandatory annuitization requires that some portion of retirement assets be annuitized on or before some stipulated age(s). The goal of mandatory annuities is to insure individuals against longevity risk by providing an income guarantee until the end of life [Orth (2004: 7)]. They have the added benefit that they tend to reduce the adverse selection difficulties that motivated annuity mandation in the first place [Doyle and Piggott (2000)]. This subsection discusses literature that deals with the design of mandatory annuitization systems and their distributional implications.

6.7.1 Design

Orth (2004) suggests a possible framework for the mandatory annuitization of U.S. retirement savings, considers the experience of other countries, and analyzes the advantages and disadvantages of mandatory annuitization. She concludes that if properly structured, the benefits of an annuity mandate could outweigh the drawbacks.

Doyle and Piggott (2000) explore the implications of various mandatory annuity designs in a privatized Australian Social Security system for broad contours of retirement provision policy and several different annuity products. They find that while a “standard” actuarially fair life annuity is likely to be attractive from both an individual and social perspective, products that offer only partial insurance against the major retirement risks may dominate, and they conclude that it would be advantageous to allow some flexibility in mandatory annuity design.

“In most cases, income adequacy could be achieved by annuitizing less than all retirement assets. In particular, because Social Security benefits are paid as an annuity, retirees with sufficient retirement plan accumulations would not need to annuitize all of their other retirement assets” [Orth (2004: 7)].

6.7.2 Distributional Implications

Brown (2003) examines the distributional implications of mandatory longevity insurance. To this end, his model is based on a population with mortality heterogeneity and a life-cycle framework that allows distributional effects to be examined on a utility-adjusted basis. He finds that a mandatory annuity program results in substantially lower redistribution effects on a utility adjusted basis than on a purely financial one. He concludes that in a simple life-cycle model with no bequests, complete annuitization is welfare enhancing even for those with higher-than-average expected mortality rates, so long as administrative costs are sufficiently low.

Fehr and Habermann (2007) investigated the intergenerational welfare effects of life annuities. They noted that while young retirees experience significant welfare gains from annuitization, future generations are hurt by lower bequest. Moreover, for interest rates that are sufficiently higher than the population growth rate, long-run welfare will decrease although aggregate efficiency rises.

Without mandatory annuitization, bequests are higher for future generations. Conversely, with mandatory annuitization, the surviving annuitants receive the mortality credits, not future generations.

7 PRFS Involving Products

Longer lifespans generally are seen as a positive outcome of economic growth, but they also mean that more retirees face the risk of outliving their assets and means of support. A range of financial products exists currently or can be envisioned for the future that would help to protect retirees against having to dramatically curtail consumption in old age [Mitchell (2001)]. In this section, the literature that deals with a number of these products include: longevity insurance, tiered annuities, variable annuities, equity-indexed annuities, market-value adjusted annuities, guaranteed minimum benefits, enhanced annuities, life-cycle (target-date) funds, inflation linked products, long-term care insurance, and life-care annuities.²⁴

7.1 Longevity insurance

This subsection discusses longevity insurance and a related product, the advanced-life delayed annuity.

7.1.1 Introduction

“Longevity insurance is a form of annuity with guaranteed payments and a long deferral period.” Tomlinson (2007) provides a user-friendly introduction to longevity insurance, covering topics such as its basic characteristic, how it compares with alternate strategies, and the impact of withdrawal rates.

Chen and Milevsky (2004) review the need for longevity insurance and explore an asset allocation decision in retirement that involves both conventional asset classes and immediate payout annuity products.

Cairns et al. (2006) studied longevity bond pricing with different terms to maturity and different cohorts. They found that longevity risk over relatively short time horizons is very low, but at horizons in excess of ten years it begins to increase very rapidly.

To the extent that there is inflation protection, it doesn't start until the payouts start; inflation until then is not covered.

7.1.2 Advanced-Life Delayed Annuity

As mentioned previously, one conceptual solution to the longevity problem is the advanced-life delayed annuity (ALDA). Essentially, the ALDA entitles the holder to insurance against the risk of outliving assets, but only when the assets actually run the risk of being depleted, which is later

²⁴ The focus of this section is on insurance products, as opposed to structured capital market products. Readers interested in an overview of the latter may find the Oesterreichische Nationalbank handbook [Thonabauer,G.(Ed.) (2004)] on the topic helpful.

in life. In its purest form, it would have no cash value, no survival or estate benefits, and be linked to inflation.²⁵

Gong and Webb (2007) use numerical optimization techniques to show that the ALDA would provide a substantial portion of the longevity insurance provided by an immediate annuity, at a small fraction of the cost. They find that at plausible levels of actuarial unfairness, households should prefer the ALDA to both immediate and postponed annuitization, and an optimal decumulation of unannuitized assets. They conclude that few households would suffer significant losses if it was a 401(k) plan default.

7.2 Tiered annuities

The two-tiered annuity, which is based on two interest rates, rewards deferred-annuity policyholders for taking their accumulation as retirement income and penalizes them if they withdraw their money in a lump sum. This is accomplished by crediting a relatively high interest rate (above that paid by a single-tier policy) to their account in the case of the former, and a relatively lower interest rate (below that paid by single-tier contracts) in the case of the latter. Moreover, the interest credited under the lower tier of rates is recalculated from the contract's inception [Richter (1990), Pfeifer (2004)].

In the past, there have been concerns over disclosure and the differential between the two values. Of course, this is not a problem after annuitization if it is a fixed annuity, since it will not be cashed in.

7.3 Variable annuities

A variable annuity (VA) is an annuity contract whose cash values and benefit payments vary directly with the experience of assets designated to back the contracts. Assets backing VA policies are maintained in a separate account of the insurer, and the VA values directly reflect the account investment experience [Black and Skipper (2000: 174)].

This subsection provides literature citations for an introduction to variable annuities. The equity premium, the notions of the VA as a put option, and taxation of VAs also are discussed.

²⁵ Milevsky (2004a) reported that pure ALDAs were unavailable. However, an ALDA-type product was offered by IDS Life of Minneapolis in the 1980s [Pechter (2006)], and they are currently offered by insurers such as Metlife and The Hartford [Tomlinson (2007)]. While no CPI-linked ALDAs exist as mentioned in the Milevsky paper, some have step-up provisions, where the payments can increase 1%-5% per year [Correspondence from Kristian Baney, PIMCO].

7.3.1 Introduction

Variable annuities were first offered in 1952 by the College Retirement Equities Fund (CREF), a subsidiary of the Teachers Insurance and Annuity Association (TIAA) [Black and Skipper (2000: 174)].

A good, albeit dated, introduction to variable annuities is given by Campbell (1969), in which the general concept of the variable annuity is introduced through a discussion of retirement security and inflation.

Milevsky (2002) noted that although he still believed that a broadly diversified portfolio of equities is a prudent asset allocation even during retirement, variable payout annuities (VPAs) are a more efficient method to avoid outliving retirement funds for those who are solely concerned with maintaining a standard of living during retirement. He went on to explain why he considered these instruments so important and timely for a properly funded retirement.

There is a tendency to value a variable payout equity-linked annuity more highly than a real annuity because the additional real returns associated with common stocks more than compensate for the volatility of prospective payouts [Brown et al. (1999)].

7.3.2 The equity premium

The equity premium is the rate by which risky stocks are expected to outperform safe fixed-income investments, such as bonds or bills. Welch (2000), who characterized it as “the single most important number in financial economics,” concluded, on the basis of a survey of the views of financial economists on the equity premium, that the “average arithmetic 30-year equity premium consensus forecast hovers around 7%.”

Fama and French (2002) studied the risk premium on the S&P 500 from 1872-2000 using fundamental data. They found that the ex ante risk premia is between 2.55% and 4.32% for 1951-2000 period.

Kapur and Orszag (2002) concluded that the equity premium of equity over bonds, which has been estimated at roughly 6% for the UK and the U.S., suggests a greater role for equity-linked retirement income solutions.

Arnott and Bernstein (2002) wrote in the same year that “the long-term forward-looking risk premium is nowhere near the level of the past; today, it may well be near zero, perhaps even negative.”

In rebuttal to such notions, Ibbotson and Chen (2001) wrote that “[c]ontrary to several recent studies that declare the forward looking equity risk premium to be close to zero or negative, we find the long-term supply of equity risk premium is only slightly lower than the pure historical return

estimate. The long-term equity risk premium is estimated to be about 6% arithmetically, and 4% geometrically.”

Two subsequent publications involving Ibbotson provided further insight into the nature of the equity premium. The first, Goetzmann and Ibbotson (2005), dealt with the history of the equity premium; and the second, Goetzmann and Ibbotson (2006), was a collection of their papers on the topic.

Recently, however, Welch (2009), in an update of his 2000 survey, found that the 2008 decline in the stock market has made economists mildly more bullish about future stock market rates of return. Typical expected equity premia were between 5% and 6% per year.

A multi-year study of the equity risk premium also was done by Graham and Harvey (2009), who analyzed its history using surveys of U.S. Chief Financial Officers conducted every quarter from June 2000 to March 2009. In their case, the risk premium was the expected 10-year S&P 500 return relative to a 10-year U.S. Treasury bond yield. They noted that there appeared to be relatively little time-variation in the risk premium, and that it appeared to be higher during recessions. They reported that the then current risk premium of 4.74% was the highest in the history of their survey, and that the overall average ten-year risk premium return was 3.46%, with a standard deviation is 2.67%.

Thus, while the historical equity premium may have been around 6 percent, it may be somewhat less now, which could have significant implications insofar as the forecasting of PRFS.

7.3.3 Put option

Milevsky and Promislow (2001) note that an insurance company that issues a VA has essentially granted the policyholder put options on the future interest rates and future mortality rates. Heuristically, they treat the underlying life annuity as a defaultable coupon-bearing bond, where the default occurs at the exogenous time of death. Their focus is the insurance company perspective, and in that context, they observe that both the mortality and interest rate risk of a VA can be hedged, and the option to annuitize can be priced by locating a replicating portfolio involving insurance, annuities, and default-free bonds. However, this view also has implications for the modeling of the life annuity from the annuitant’s perspective.

U.S.-based insurance companies generally offer variable annuity (VA) policyholders the long-term option to annuitize their policy at a pre-determined rate over a pre-specified period of time. This is in addition to any guaranteed minimum death benefit. At the turn of the century, there was approximately one trillion dollars invested in such policies [Milevsky and Promislow (2001)].

7.3.4 Taxation of VAs

Gains in a VA grow tax-deferred, but they are taxed as income when the money is withdrawn. That contrasts with other investments such as stocks and mutual funds, which generally qualify

for lower capital gains treatments [Weston (2008)]. The difference can be substantial, since capital gains tax rates range from 5% to 15%, while income tax rates are currently 10% to 35% [Weston (2008)]. Critics of VAs argue that this tax disadvantage coupled with the extra fees and expenses generally make it suboptimal relative to mutual funds. Similarly, since IRAs already receive favorable income-tax treatment, investing IRA contributions in a VA results in duplication of fees and expenses [Rejda (2005: 445)].

7.3.5 Commentary

Condron (2008) discusses the pros and cons of variable annuities.

Weston (2008) argues that VAs are the worst retirement investment an individual can make. He argues that high fees, low flexibility, and 'horrendous' tax treatment make variable annuities less attractive than ever, except to the people who sell them.

“Nobody who's in the fee-only (planning) business is going to recommend [variable annuities]” [Armstrong (2004)].

7.4 Equity-indexed (EI) annuity

Equity-indexed annuities offer both a guaranteed minimum interest rate and a guarantee against loss of principal if held to term. Interest credited in excess of the minimum guaranteed rate is linked to the upward movement of a designated equity index. If the index moves upward, the interest rate includes some percentage of the increase; if the index moves downward, the guaranteed minimum rate is credited.

They are generally considered to be fixed annuities. As with other fixed annuities, surrender charges may reduce the principal amount if the equity-indexed annuity is surrendered prematurely.

7.5 Market-value adjusted (MVA) annuity

The market-value adjusted (MVA) annuity is a fixed annuity product with a fixed interest rate and a market-driven feature that only applies if the contract is surrendered before the contract period expires.

If an MVA annuity is surrendered early, a surrender charge and a market value adjustment will apply. If interest rates decreased (increased) during the contract period, the market value adjustment will be positive (negative), which would act to decrease (increase) the contract's surrender charge.

For example, suppose the owner of a 10-year MVA annuity contract decides to cash it in after the sixth year. Cashing it in early automatically subjects the owner to a surrender charge. Moreover, if interest rates rose since the time the contract was purchased, the market-value adjustment would be negative, resulting in an even higher surrender charge.

7.6 Guaranteed minimum benefits

This subsection discusses literature related to guaranteed minimum benefits. The topics addressed include: type of VA guarantees, enhanced earnings benefits, guaranteed minimum death benefits, guaranteed minimum income benefits, guaranteed minimum withdrawal benefits, ruin-contingent life annuity, and VA fees, expenses, and commissions.

7.6.1 Introduction

Features in the newer variable annuity product designs encourage retirees to remain invested in equities at older ages by cushioning them from some of the consequences of market volatility. As a result, retirees are more apt to maintain a higher equity-weighted investment portfolio [Condrón (2008)].

Condrón (2008) reported that “more than \$1.35 trillion [was then] invested in variable annuities in the U.S.”

Milevsky (2007c) suggests that clients must protect against the risk that they get the seven year famine before the feast. In his opinion, if there is not an adequate amount of guaranteed pension income for life, the simplest and easiest way to protect against this risk is to spend a fraction of the retirement funds on a variable annuity that contains some sort of guaranteed living benefit. In that way the retiree can hedge both longevity risk and sequence-of-returns risk.

7.6.2 Types of VA guarantees

Once in retirement, the dominant causes of failure are longevity risk, inflation and a bear market early in retirement. Milevsky counsels that no one kind of investment protects against all three of these contingencies, but one strategy is to diversify not only with respect to the market portfolio, but with respect to insurance products as well. His suggestion with respect to the latter is to purchase variable annuities with living benefits [Milevsky, in an interview with Schurenberg (2008)].

Stone (2003) provides a short overview of many of the VA guarantees.

Variable annuity financial guarantees embrace the guaranteed minimum death benefit and guaranteed minimum living benefits. The latter includes the guaranteed minimum accumulation benefit, the guaranteed minimum income benefit, and/or the guaranteed minimum surrender benefit [Gaillardetz, (2008)].

Chapter 12 of Hardy (2003) contains a comprehensive discussion on these guarantees.

7.6.3 Guaranteed minimum death benefits (GMDBs)

This subsection discusses literature related to guaranteed minimum death benefits (GMDBs). The topics discussed include: the ratchet feature, option to transfer funds, and the cost of the guarantee.

7.6.3.1 Introduction

Upon the death of the annuitant, the contingent beneficiary receives either the account balance, or the value of the initial investment, whichever is greater [Wang (2008)].

GMDBs received considerable attention in 2002, especially with the well-publicized reserve increase announced by CIGNA due to the runoff of existing GMDB liabilities [Stone (2003: 3)].

7.6.3.2 GMDB with the ratchet feature

While VAs with GMDBs generally are very popular in the U.S., the version that is particularly attractive is the one with the ratchet feature, since it periodically will be adjusted to reflect growth in the account. Under this provision, the death benefit will be the larger of the amount invested or the ratcheted account value, minus any withdrawals.²⁶ From a contingent claim perspective, this embedded option corresponds to a look-back basket put with a floating strike²⁷ [Guermazi and Abid (2009)].

Milevsky and Posner (2001) priced the GMDB with a ratchet feature in a contingent claim framework. In their model, as is frequently found in the literature, the ratchet clause is modeled as a continuous feature. That is, the maximum of the account value is assumed to be continuously updated, as opposed to being updated, as in real-life contracts, at discrete contract anniversary intervals. This assumption, while convenient since it allows the use of analytical formulas, overvalues the guarantee and results in an upper bound for the value of the GMDB.

Guermazi and Abid (2009) model this ratchet feature using discrete updates of the maximums. As a part of their model, they include the basket feature of the underlying account assuming that the initial premium is invested in a basket of correlated assets. Using finite differences methods and real mortality data, they show that the expected present value rises with the frequency of measurements of the maximum. They conclude that including discrete updates of the maximum and the basket feature of the underlying account, the ratchet clause is less valuable than when using the continuous model.

²⁶ http://www.variableannuityfacts.org/fees/types_fees.jsp

²⁷ Guermazi and Abid (2009) actually used the term “discretely measured floating strike” with their discrete model.

7.6.3.3 Option to transfer funds

Ulm (2006) considers the effect of the real option to transfer funds between fixed and variable accounts of a GMDB. If the GMDB rider is considered in isolation, it is sometimes in the policyholder's interest to transfer to the fixed fund if the fixed fund earns less than the variable fund on a risk-neutral basis, in order to "lock in" an in-the-money GMDB position. On the other hand, the option to transfer will not be used if the entire annuity and rider are considered together. In this case, the value of the fund and GMDB in total has been shown to be highest when the GMDB is ignored entirely and the policyholder invests in the funds that have the largest risk-neutral return.

7.6.3.4 Cost of guarantee

Milevsky and Posner (2001) investigating the GMDB, compared theoretical estimates against a cross-section of insurance risk charges, as reported by Morningstar, Inc. Their main conclusion was that a simple return-of-premium death benefit is worth between one and ten basis points, depending on gender, purchase age, and asset volatility, while the median mortality and expense risk charge for return-of-premium VAs is 115 basis points.

Wang (2008) provides a quantitative, albeit academic, analysis of hedging models for guaranteed minimum death benefits within variable annuities.

7.6.4 Guaranteed minimum income benefits (GMIBs)

Bauer et al. (2007) investigated VAs using a model that permits a consistent and extensive analysis of all types of current guarantees. They found that some guarantees, such as the guaranteed annuities within the GMIB, are offered significantly below their risk-neutral value.

7.6.5 Guaranteed minimum withdrawal benefits (GMWBs)

This subsection discusses literature related to guaranteed minimum withdrawal benefits. The topics discussed include: guaranteed lifetime withdrawal benefit, the value of the GMWB, and the adequacy of the fees.

7.6.5.1 Guaranteed lifetime withdrawal benefit (GLWB)

The guaranteed lifetime withdrawal benefit (GLWB) provides that the annuitant can withdraw up to a set percentage of the original annuity deposit each year for life. Percentages range from 4% to 7%, depending on the purchaser's age at the first withdrawal, with higher percentages at older ages. The two main advantages of the GLWB are that it provides a lifetime income and it preserves the purchaser's control over assets. A disadvantage is that it does not benefit from mortality pooling effects, so the guaranteed income amounts will be less than amounts available through annuitization [Tomlinson (2008)].

Regardless of the market performance of the underlying asset portfolio, the GLWB provides for the return of the entire initial investment, with withdrawals, at the contractual withdrawal rate, spread over the term of the contract. No penalty is imposed when the policyholder chooses to withdraw at or below the contractual rate. There is a penalty, however, if the policyholder withdraws at a rate higher than the contractual rate or surrenders the policy prematurely [Dai et al. (2008)].

7.6.5.2 *The value of the GMWB*

Milevsky and Abaimova (2005b) quantify the value of having access to variable payout annuities with downside protection inside an individually-controlled DC plan, such as a 401(k) plan, and conclude that the value of an implicit annuitization put option (APO) on such a portfolio can be substantial.

Holz et al. (2007) analyze guaranteed minimum withdrawal benefits for life. Within the general framework presented in Bauer et al. (2007), they price the embedded guarantee for different product designs and parameters under deterministic and optimal client behavior. They found that such products are risky for the insurer. While they are much less sensitive to client behavior than other guarantees typically embedded in variable annuities, as discussed in Bauer et al. (2007), their sensitivity to changes in interest rates, fund volatilities, and mortality rates is significant.

7.6.5.3 *The adequacy of the fees*

Milevsky and Salisbury (2006b) discuss a variety of methods for assessing the cost and value of a GMWB. Their main conclusion is that the no arbitrage hedging cost of a GMWB ranges from 73 to 160 basis points of assets, in contrast to a charge of 30–45 basis points for most products on the market. The options exchange, which is the only other place an investor can buy similar protection, often charges five to 10 times that amount during periods of market stress [Milevsky (2007b)].

Chen et al. (2008) study the no-arbitrage fee for GMWB VA riders. They find that several unrealistic modeling assumptions are required to duplicate GMWB fees in the normal range of charges, and conclude that typical fees are not enough to cover the cost of hedging these guarantees.

Dai et al. (2008) use a singular stochastic control model to price the GMWB. Using a finite difference algorithm and the penalty approximation approach, they explore the optimal withdrawal strategy for a rational policyholder who maximizes the expected discounted value of the VA cash flows. As one would expect, they find that the insurance fee increases with increasing equity volatility and the contractual withdrawal rate, but decreases with a higher penalty charge. In this regard, if the policyholder has the flexibility of *dynamic* withdrawals, a substantially higher insurance fee should be charged. Finally, as a validation of their discrete approach, they note that their solution values from the discrete model converge to those of the continuous model.

According to a Lehman Brothers (2005) market survey, often only delta risk²⁸ is hedged [Holz et al. (2007)].

7.6.6 Ruin-contingent life annuity (RCLA)

This subsection discusses literature related to ruin-contingent life annuities (RCLAs). The topics addressed include the definition of an RCLA and valuing the RCLA.

7.6.6.1 Introduction

Huang et al. (2007) proposed a ruin-contingent life annuity (RCLA), under which two contingencies must both occur before the annuitant gets paid. First, the individual (or their spouse) must be alive. Second, there must have been a bear market (weak investment performance) during the early critical years of their retirement. The payments are adjusted for inflation.

Phoenix companies and Lockwood Capital Management have come out with a similar product, but it is not available as a standalone and it does not offer inflation protection [Tomlinson (2008)].

Huang et al. (2007) observe that an RCLA is embedded within each and every guaranteed living benefit. A GMWB, for example, can be visualized as a combination of a systematic withdrawal plan together with an RCLA.

In a subsequent article, Huang et al. (2009), reported that RCLA were embedded in approximately \$800 billion worth of U.S. VA policies.

7.6.6.2 Valuing the RCLA

Huang et al. (2009) derive the PDE and relevant boundary conditions satisfied by the RCLA value assuming no arbitrage. They then apply numerical techniques to provide estimates of a typical RCLA under a variety of realistic parameters. However, since they use a simple valuation framework, their analysis only provides a rough indication of what these ruin-contingent life annuities might actually cost.

7.6.7 VA fees, expenses, and commissions

This subsection presents an overview of VA fees, expenses, and commissions.

²⁸ Delta indicates how much the theoretical price of an instrument or portfolio changes when the price of the underlying asset, currency, or commodity changes by a small amount. See Hull (2003), Chapter 14, for a discussion of risk measures associated with derivatives.

7.6.7.1 Introduction

The source of much of the following overview of VA fees, expenses, and commissions is Variableannuityfacts.org²⁹

7.6.7.2 VA fees and expenses

The VA fees and expenses include: a surrender charge of 10-15% of annual withdrawals [SEC (2007:4)], which may be assessed if contributions are withdrawn from a VA before a specified period has elapsed; a mortality and expense risk charge, which compensates the insurance company for providing guaranteed annuity purchase rates, for assuming related insurance risks, and for expenses associated with the contract; an account maintenance fee, to help pay the administration expenses associated with maintaining the account and reporting the information to the policyholder; and underlying fund fees, which are the expenses and management fees associated with the mutual funds that are the underlying investment options within the variable account component of the VA.

Based on its averages for mortality and expense risk charge, administrative fees, annual records maintenance fees, and total fund expense averages, Morningstar calculated the total weighted average expenses for the first quarter of 2006 as 2.335% of the variable account value.

As far as optional features, according to Morningstar, the fees charged in the second quarter of 2005 by a number of different insurance companies were as follows:

- Guaranteed Minimum Income Benefit had an average fee of 0.48%.
- Guaranteed Minimum Accumulation Benefit had an average fee of 0.43%.
- Guaranteed Minimum Withdrawal Benefit had an average fee of 0.47%, unless it includes a lifetime guarantee, in which case, the average would likely be more.
- The “ratchet” death benefit had an average cost of 0.58% of the account value.

7.6.7.3 Commissions

In addition to the VA fees and expenses, commissions are charged for each contribution to the VA, which generally are in the range of 4% to 7% of the contribution. These commissions are not a direct charge to the client, but are covered by the VA's fees and charges. There also may be an ongoing annual fee, called a trail commission, which usually is in the range of .25% of the account value.

Instead of a product-based agent or broker, a fee-based financial planner may be used. In this case, a flat fee will be charged, based on the amount of assets contributed to the plan.

²⁹ See http://www.variableannuityfacts.org/fees/types_fees.jsp

7.6.8 Comments

Milevsky (2007f) found that in policies purchased without living benefits, the amount of equity exposure tended to be much lower compared to the policies in which such riders were elected. He also found, consistent with the life-cycle theory of investing, older investors tended to allocate smaller amounts to risky sub-accounts within their variable annuities.

Milevsky and Salisbury (2006b, p 36, footnote 5) discuss how Coventry First (CF) arbitrages insurance companies that sell VAs with guaranteed benefits by buying the contracts from owners that want to lapse or surrender their policies, and later cash them in at maturity. The policyholder's enticement is that CF pays them a fraction of the embedded option value in addition to the market value.

7.7 Enhanced annuities

Enhanced annuities provide higher annuity payments for impaired lives. In the context of enhanced annuities, a life in poor health represents a “preferred risk” [Hamdan and Rinke (1998)]. They were developed by the insurance industry because numerous people reaching retirement age suffer from some form of illness or disease [Weinert (2006)]. In this subsection, we review literature pertaining to three topics related to enhanced annuities: enhanced annuities in the U.S. and Canada, enhanced annuities in the UK, and demographic risk.

7.7.1 Enhanced annuities in the U.S. and Canada

Brown and Scahill (2007) looked at the then present annuity marketplace in the United States and Canada. They also reviewed the underwriting and marketing of life annuities in the United Kingdom where annuitization has been mandatory and enhanced life annuities are available for a broader cross section of the marketplace. They conclude that the U.S. and Canadian annuity marketplace could be doing more to provide “fair value” annuities to substandard risks.

7.7.2 Enhanced annuities in the UK

Hamdan and Rinke (1998) present the different Enhanced Annuity product variants found in the United Kingdom as of mid 1997.

The UK pension environment is especially favorable for the marketing of Enhanced Annuities because, by law, a portion of the retirement fund must be invested in a life annuity [Hamdan and Rinke (1998)].

Weinert (2006) describes the development of enhanced annuities in the UK. Starting with their evolution, he reviews the different concepts and product advances. This is followed by a description of the current market environment and an explanation of underwriting processes and reinsurance options. Finally, possible upcoming changes are discussed.

7.7.3 Demographic risk

Levantesi and Menziatti (2007) investigate an enhanced life annuity product where the benefit increases once the insured becomes long-term care (LTC) disabled. Their focus is on the analysis of mortality and disability risk in enhanced life annuities using Italian national data on health and mortality. They show how the impact of demographic risk on risk-based capital requirements could be reduced through safety loading or capital allocation strategies.

Their analysis concentrates on the demographic risk, to the exclusion of the financial risk.

7.8 Target-date (life-cycle) funds

This subsection discusses literature related to target-date funds (TDFs), which also are known as life-cycle funds. The topics addressed include an introduction to target-date funds, welfare cost, humped-shaped proportion invested in equity, studies, and limitation of target-date funds.

7.8.1 Introduction

Standard investment advice is for individuals to reduce their allocation to risky assets as they age, or, more precisely, as the investment horizon shortens. Canner et al. (1997), for example, cite a rule of thumb that advises a stock allocation percentage of 100 minus the investor's age. More generally, life-cycle funds decrease the share in equities as the investor ages.

TDFs provide a simple solution to the investment task confronting participants in self-directed retirement plans. Essentially, a TDF is a "fund of funds" diversified across stocks, bonds, and cash with the feature that the proportion invested in stocks is automatically reduced over time. Based on empirical evidence, Bodie and Treussard (2007) conclude that a simple TDF strategy would be an improvement over the choices currently made by many uninformed plan participants.

These funds have become increasingly popular investment vehicles in the United States. In fact, President George W. Bush's proposal to reform Social Security called for allocating the entire investment portfolio at age 47 to a life-cycle fund, subject to an opt-out provision [Shiller (2005)].

7.8.2 Welfare cost

Gomes et al. (2008) investigated the welfare costs of constraining portfolio allocations over the life cycle to mimic popular default investment choices in defined-contribution pension plans. They found that target date (life-cycle) funds designed to match the risk tolerance and investment horizon of investors have small welfare costs. However, other choices, such as stable value funds, balanced funds, and life-cycle funds which do not match investors' risk tolerance, can have substantial welfare costs.

7.8.3 High risk aversion situations

Bodie and Treussard (2007) explored ways to improve TDFs for people who are very risk averse and have high exposure to market risk through their labor.

They found that people with relatively high risk aversion and a high exposure to market risk through their human capital would experience a substantial gain in welfare from being offered a safe TDF instead of a risky one.

7.8.4 Other Studies

Vigna and Haberman (2001) use dynamic programming techniques to analyze the financial risk in a defined contribution plan and to find an optimal investment strategy, given a final target linked to the net replacement ratio and a set of interim targets. Investment and annuitization risk faced by the individual were considered, as were lifestyle investment strategies. Their results seem to suggest the appropriateness of the lifestyle strategy to reduce the investment risk.

Bodie (2002) addresses important practical issues in personal finance of how much to save for retirement and how to invest those savings. He suggests ways that advances in finance theory combined with innovations in financial contracting technology might be used to design and produce a new generation of user-friendly life-cycle products for consumers.

Kintzel (2007) explores some of the theoretical and empirical foundations for life-cycle funds as discussed in the finance literature on optimal portfolio theory. He also discusses actual life-cycle plans and the advantages and disadvantages of these types of funds.

7.8.5 Limitation of target-date funds

Tomlinson (2008) discusses current and developing financial products individuals can purchase to reduce the risk of outliving their assets. He expresses concern that popular products, like target-date mutual funds and deferred fixed and variable annuities, purchased during the accumulation stage do not get annuitized during the decumulation stage, and, hence, do not help reduce the risk of outliving assets. He sees GLWB, the latest transformation, as a step in the right direction, but feels that there are existing and potential products capable of doing a better job of meeting retirement needs. Such products include income annuities, longevity insurance, life-care annuities, which is an income annuity with a pop-up long-term care benefit, and ruin-contingent life annuities.

7.9 Inflation linked products

In the past, as mentioned previously, most private defined benefit plans did not have inflation protection, and even public plans, which often had a cost of living adjustment (COLA), generally were capped at a relatively low rate, such as 2 percent. Nonetheless, it is now common for pension professionals to view inflation protection as an important component of PRFS. This

subsection discusses two areas of the literature related to inflation linked products: the contingent claims approach and the role of real annuities and indexed bonds.³⁰

7.9.1 Contingent claims approach

Bodie (1990) noted that a contract to insure a future payment against inflation is equivalent to a European call option on the consumer price index. He went on to use option theory to explore annuities that offer partial protection against inflation at a cost acceptable to retirees. Applying contingent claims analysis to the production and pricing of inflation-protected annuities that have deductibles and caps, the method synthesizes a CPI call option using a dynamic trading strategy of borrowing and investing in CPI-linked bonds. The net price of the insurance is the cost of implementing the strategy.

7.9.2 The role of real annuities and indexed bonds

Brown et al. (1999) explore four issues related to annuitization options: the operation of both real and nominal individual annuity markets in the UK; the then-current structure of two inflation-linked insurance products available in the U. S.; the potential of assets such as stocks, bonds, and Treasury bills, to protect retirees from inflation; and a simulation model to assess potential retiree willingness to purchase real, nominal, and variable payout equity-linked annuities.

They noted that the widespread availability of real annuities in the UK dispels the notion that private insurance markets could not, or would not, provide real annuities to retirees. They argue that the “inflation insurance” aspect of equities is mainly due to the high average return of stocks, and not because stock returns move in tandem with inflation. Finally, they conclude that, for plausible degrees of risk aversion, inflation protection appears to have only modest value.

7.10 Long-term care insurance

Long-term care (LTC) includes a wide range of services for people who need assistance on a regular basis because of chronic illness or physical or mental disabilities. In this subsection, we provide an overview of the literature as it pertains to the role of LTC in a post-retirement financial strategy. The topics covered are the essential features of LTC, LTC risk, the profile of an LTC user, LTC insurance, taxation, and LTC decision making.

7.10.1 Essential features of LTC

In contrast to most health services, LTC generally is not designed to treat an illness or condition. Although it can include skilled nursing care, it consists primarily of help with basic activities of daily living (ADL) (such as bathing, eating, dressing, and using the toilet) and with tasks necessary for independent living (such as shopping, cooking, and housework) [Johnson and Uccello (2005)].

³⁰ Readers interested in the range of securities and derivatives associated with inflation-linked products are referred to the books Benaben (Editor) (2005) and Benaben and Goldenberg (Editors) (2008).

Long-term care may be required due to physical limitations or disabilities resulting from injury, illness, or the normal aging process. It also can be due to a cognitive impairment resulting from such things as a stroke or Alzheimer's disease [Kiplinger (2009)].

Kaplan (2007) reviews the long-term care options currently available, including home care, assisted living facilities, and nursing homes and the extent of the coverage provided by Medicare, and related private insurance policies, Medicaid, and long-term care insurance.

Three-fifths of long-term care recipients are age 65 or older [Johnson and Uccello (2005)].

7.10.2 LTC risk

According to Munnell et al. (2009), even if households work to age 65 and annuitize all their financial assets, including their reverse mortgages receipts, 44 percent will be 'at risk,' in the sense that they will be unable to maintain their standard of living in retirement. When health care costs were included explicitly, the percentage of households 'at risk' increased to 61 percent and including the cost of long-term care insurance raises the number to 64 percent. Because the costs of long-term care insurance and other health expenditures are rising while the income system is contracting, a much larger percent of later cohorts will be 'at risk' than earlier ones.

As to projected LTC risk, Stallard (2000) discussed a methodology for developing estimates of LTC needs and expenditures and applying them to population projections of the elderly. The basis for his analysis were population projections of the Social Security Administration and National Long Term Care Surveys.

7.10.3 Profile of an LTC user

Cohen (2008) follows an admissions cohort of LTC claimants to document their characteristics. To this end, they: obtain their demographic and disability profile and monitor how it changes over time; learn how and why people use services and transitions between care settings; and evaluate the current and potential role of care management in the process.

Further insight into the characteristics of an LTC user was provided by Stum (2000), who examined the meaning attributed to later life financial security goals from the perspective of involved family members. The study focused on six goal patterns potentially important to later life financial security: self-sufficiency; spouse's financial security; control; bequests; qualifying for public assistance; and privacy. Not surprising, he concluded that the interpretation of these patterns, and their ranking, were individual specific.

7.10.4 LTC insurance

Long-term care insurance (LTCI) has evolved significantly since its inception, offering more flexibility and covering a broader range of benefits. Currently, the key provisions are: the de-

ductible (elimination period or benefit waiting period), which ranges from 30 to 90 days; the care options, which may include nursing home, assisted living, and adult day care; the benefit level (the daily maximum benefit), which can range from \$100 to \$350 and averaged \$204 per day for a nursing home in 2008; the benefit period (duration of payments), which can span from two to six years, or a lifetime and averaged 2.4 years in 2008; the lifetime maximum benefit, the benefit level x benefit period x 365; inflation protection coverage, where the CPI averaged 4.1 percent over the 30 year period ending December 31, 2008; and the non-forfeiture benefit, which provides for benefit payments equal to some minimum amount, or the premiums paid if greater, as long as the insured pays premiums for at least a stipulated duration [Kiplinger (2009)].

Lawrie and Stanton (1998) noted that at the turn of the century, individual policies dominate the market and, even among employer sponsored programs, most are employee-pay-all and attract relatively low participation. Nonforfeiture, investment and inflation provisions are among the most important variables employers need to examine in insured arrangements.

McSweeney and Gomori (2004) found evidence of a shift to the view of long-term care insurance as an important component of the PRFS.

AHIP (2007) reported on who buys LTCI and their motives for doing so. Its major findings were that policies purchased by consumers today provide broader coverage, the average age of policyholders is getting younger, and consumers weigh many options in their decision of whether or not to purchase LTC coverage.

7.10.5 LTC premiums

Insurance companies started pricing and marketing the first generation of LTC products in the 1980s [Zaker-Shahrak (2009: 26)]. From the start, concern was expressed that for many companies the LTC premiums were inadequate and resulted from a pricing policy based not on the lifetime of the product but on a five or ten year horizon, with the anticipation of a rate increase when the premium became insufficient [Schaeffer (1988: 28)].

The apprehension about rate increases is not unfounded. In spite of the rate stability model law of the NAIC, which has been enacted into law in many states, “most insurance companies selling LTC products have implemented significant rate increases (many repeatedly). There still seems to be no end in sight for how many more rate increases are to follow” [Zaker-Shahrak (2009)].

7.10.6 Taxation

The Health Insurance Portability and Accountability Act of 1996 (HIPAA) resolved a good deal of the uncertainty surrounding the federal tax treatment of insured long-term care benefits [Lawrie and Stanton (1998)].

Essentially, LTCI is treated like accident and health insurance.³¹ Premiums for qualified long-term care insurance and non-reimbursed out-of-pocket expenses are deductible if the costs exceed 7.5% of adjusted gross annual income, the base for medical expenses deductions in the tax code. The amount of annual premiums that can be deducted depends on age. Most long-term care policies purchased before 1997 are considered qualified for federal tax purposes.

Benefits from qualified LTCI policies are not taxed except for per diem policies that pay amounts that exceed limits set by the IRS (the higher of the cost of qualified long-term care or \$290 per day for any period in 2010).

Kaplan (2007) concluded that the federal government's major initiatives to encourage LTCI through the tax deduction of premiums has not been very successful because the publicized tax benefits are often illusory and thus incapable of motivating would-be insurance buyers.

Similarly, Stevenson et al. (2009) found that state tax incentives were responsible for only a small portion of the growth in long-term care insurance, and provided only a small piece of the long-term care financing puzzle.

7.10.7 LTC decision making

McSweeney and Gomori (2004) examined the risk associated with long-term care, as well as a variety of insurance-based vehicles that can help protect consumers from this risk.

Li (2004) developed a decision support framework for retirement planning by integrating it with LTC financing decisions. The focus was on an individual decision maker's behavioral preferences and entailed the optimization of investment, consumption, and LTC insurance purchase decisions for an individual planner.

There are a number of free access calculators on the web that are adequate to provide a ball-park figure for the LTC cost associated with an individual and/or spouse. An example is the AARP calculator located at:

<http://longtermcare.genworth.com/SimpleEngine/submitAARPLgin.do>

7.10.8 Comment

Kiplinger (2009) and NAIC (2009) provide good summaries of and insights into LTC and LTCI.

³¹ Summaries of LTCI tax-deductibility rules are provided at <http://www.aaltci.org/long-term-care-insurance/learning-center/tax-for-business.php#state> and <http://www.axa-equitable.com/insurance/long-term-care/tax-tips-long-term-care-insurance.html>.

7.11 Life-care annuity

Tomlinson (2008) discusses a potential bundled risk product called a life-care annuity, which he claims is not available because the tax and regulatory environment presents significant hurdles to the introduction of innovative products. The simplest version is a standard income annuity with a pop-up benefit that provides extra income if the policyholder's health deteriorates to the extent that standard long-term-care insurance claim criteria, such as the loss of two or more activities of daily living, are satisfied.

Tomlinson (2008) sees the following three advantages for the product: adding the long-term care benefit to an income annuity reduces the likelihood of adverse selection, which can lead to more attractive pricing; the pop-up benefit of the life-care annuity acts to diminish an individual's reluctance to tie up savings in an income annuity owing to concerns that extra funds may be needed to cover long-term care expenses; and there will be less need to underwrite long-term care policies and reject individuals who may have some health problems, because of the counterbalancing life annuity component of the life-care annuity.

After some product modeling and sensitivity analysis, he concluded that adding the long-term care benefit only modestly increased the cost.

7.12 Comments

Ambachtsheer and Bodie debated the responsiveness of the market insofar as meeting the needs of retirees. Ambachtsheer (2007) briefly summarized the essence of the debate and elaborated on his perspective of its key conclusions. He wrote:

Bodie believes that markets will eventually sort things out and offer workers the savings and annuity products they need to engineer smooth financial transitions from work to retirement. Further, markets will perform these services at 'fair value' prices. Ambachtsheer, on the other hand, is not so sanguine about markets eventually sorting things out [and has the view that] only pro-active third party intervention will change the current state of dysfunctional disequilibrium in the retirement provision 'industry'.

8 Closing Comments and Observations

The purpose of this report has been to provide a literature review on the topic of post-retirement financial strategies (PRFS) from the perspective of an individual who is approaching retirement age. To this end, the topics covered included longevity risk, inflation risk, portfolio risk, investment withdrawals during retirement, retirement annuities and annuitization, and PRFS involving products.

We saw that studies related to longevity risk investigated issues such as estimates of the upper limits to human longevity, measuring shortfall risk in retirement, and the optimal investment strategy to minimize the probability of lifetime ruin.

The inflation risk articles focused on the consumer price index, the impact of inflation on annuities, and products devised to mitigate the effect of inflation.

The focus of the portfolio risk articles was on individual versus institutional investing, life-cycle models, conserving portfolios during retirement, investing retirement wealth and optimal investment strategy.

The articles that involve investment withdrawals during retirement covered topics such as withdrawal goals, sustainable investment withdrawals, optimum withdrawals, the role of social security, the role of home equity (reverse mortgages), and adjusting withdrawal rates for taxes and expenses.

The retirement annuities and annuitization articles included studies of the life annuity, demand for annuities, annuitization decisions, timing of annuity purchases, systematic withdrawals and ruin in retirement, mandatory annuitization, and the insurer's perspective.

Finally, the articles related to PRFS involving products dealt with products such as longevity insurance, tiered annuities, variable annuities, equity-indexed annuities, market-value adjusted annuity, guaranteed minimum benefits, enhanced annuities, target-date (life-cycle) funds, inflation linked products, long-term care insurance, and life-care annuity.

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