Evolution of Loss Reserve Risk

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Abstract

Property and casualty insurers face risks in many key areas, such as operations, natural catastrophes and underwriting. Among the underwriting risks is the potential financial impact of adverse loss reserves development.

While multiple standard actuarial methods exist for evaluating the adequacy of reserves, little information exists on how deficiencies evolve over time. No risk models currently exist to make statements regarding the probability of a level of deficiency over a fixed time horizon. For example, the probability that current reserves will become 20 percent deficient over the next two years is difficult to determine. Current models only make estimates over the “lifetime of liability” or run-off period.

The ability to analyze reserve risk over fixed time horizons is important from several perspectives. First, from a risk management perspective, the time horizon over which a risk will likely emerge is crucial. Understanding the time horizon allows for the creation of appropriate mitigation strategies and an understanding of interrelations with other risks. Second, most other financial risks (e.g., credit and market) are measured over short fixed time horizons. A comparable measure of reserve/underwriting risk is important and required for many emerging capital measuring applications, such as Solvency II.

This paper illustrates a model of loss reserve risk that will incorporate how risk evolves over time at annual time horizons. The paper will illustrate how to build and parameterize the model using multiple years of financial statement data. The model produces results for a sample line of business for time horizons from one to 10 years.

1. Introduction

The appropriate measurement of required capital by modeling economic capital (EC) levels has become an important issue for property and casualty (P&C) insurers. Regulatory paradigms have emerged in the United Kingdom and continental Europe, allowing companies to build their own EC models that can then interface with those of regulators. In the United States, lacking a regulatory initiative, rating agencies view internal EC models as a necessity. Company use of EC models is considered a key element of effective risk and capital management, both of which are considered in the rating process.

While insurance companies seek to manage risk using EC tools, no universal methodology exists. The insurance standard that has emerged in all of Europe, driven by Solvency II, is derived from banking risk management and capital analysis paradigms. Most P&C insurers in the United States have relied on factor-based methodologies borrowed from regulatory or rating agency formulae and dynamic financial analysis (DFA) models.

Under the Solvency II framework, insurers will have to establish technical provisions to cover future claims expected from policyholders. Technical provisions will be equivalent to the amount another insurer would be expected to pay to assume and meet the original insurer's policyholder obligations. Insurers must also have available financial resources sufficient to cover both a minimum capital requirement and a solvency capital requirement (SCR).

The SCR is based on a value-at-risk (VaR) measure calibrated to a 99.5 percent confidence level over a one-year time horizon. The SCR is meant to cover all risks that an insurer faces, including insurance, market, credit and operational risks. Loss reserve risk is usually considered a component of insurance risk, along with underwriting and catastrophe risks. Loss reserve risk is the risk that the amount held in reserve to pay for current policyholder obligations will prove inadequate.

The VaR measure is commonly used in financial services to assess the risk associated with a portfolio of assets and liabilities. VaR attempts to answer the question of how much money could be lost if events develop in an adverse and unexpected way. VaR measures the worst expected loss over a specific time interval at a given confidence level. For example, if VaR is measured over a one-year period at a confidence level of 99.5 percent, then this corresponds to the worst loss expected to occur in a single year over the next 200 years.

2. Background on Value-at-Risk Methods

Traditional U.S. actuarial approaches take a much different view of insurance risk than the VaR approach used in Solvency II. VaR methodologies have their roots in financial risk management tools that were originally used on a daily basis to monitor the potential fluctuations of trading portfolios. VaR originally viewed risk as the fluctuation in the market values of risky trading positions. Over time, VaR methods have evolved into a broader set of applications, utilizing longer time horizons with which to analyze potential fluctuations in the market value of a firm.

In *Economic Capital: A Practitioner Guide*, Ashish Dev describes the background, rationale and elements of the VaR view of EC used in a banking environment:

Market Value Definition of Risk

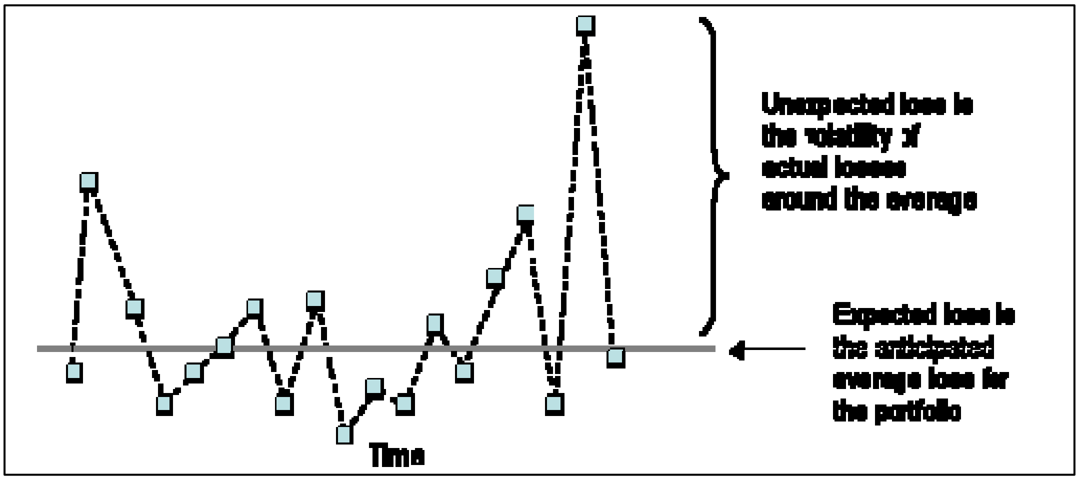
"Over the past decade, economic capital has steadily progressed toward market value models. Most commercial portfolio frameworks have by now discarded first-generation economic capital models based only on default risk, although these models persist in some cases for consumer portfolios. Given the goal of ensuring capital adequacy for a certain level of solvency, the volatility of market value is the best measure of a bank's risk and, therefore, its capital requirement.

"Ultimately, shareholders are interested in the total return on their investment in the bank's stock and its risk in market value terms. They compare the return earned on their investment to a required return based on its risk. Bondholders also care about market values. The value of their fixed-income investment is a function of the credit spread of the bank, the level of interest rates and the expected cash flows of the debt. Since both stockholders and bondholders evaluate their investments based on market values, management should evaluate its opportunities with the same market value discipline. Defining risk in market value terms reinforces this discipline by aligning the interests of business managers with those of shareholders and bondholders.

Capitalization and Confidence Levels

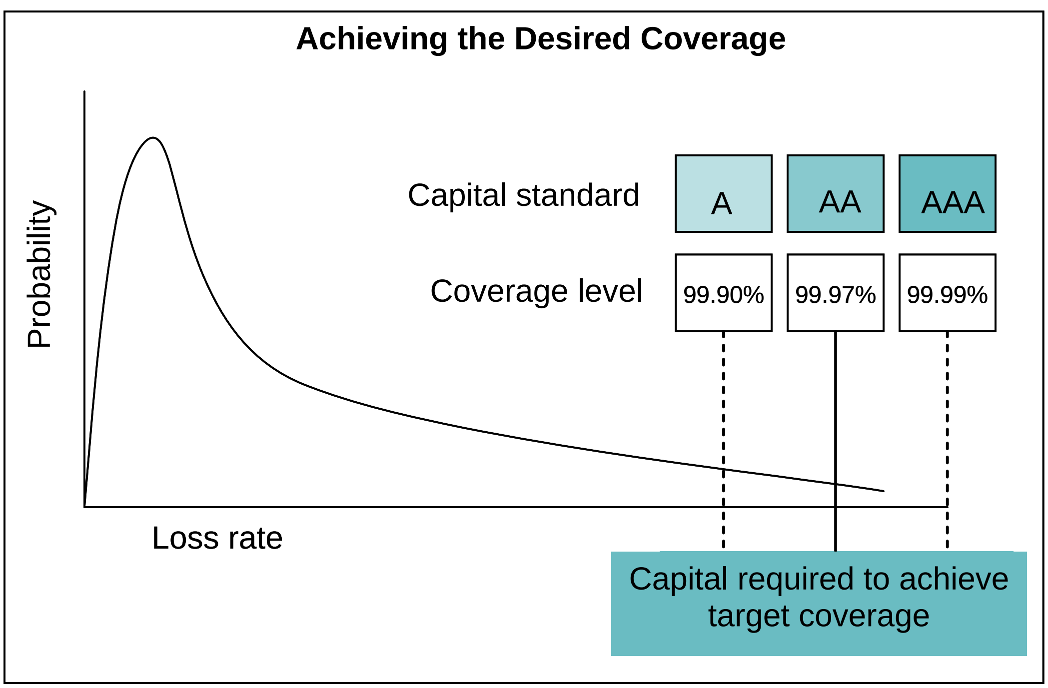
"Two estimates describe a bank's risk profile: expected loss and unexpected loss. As illustrated in Figure 1, expected loss is the average rate of loss expected from a portfolio. If losses equaled their expected levels, there would be no need for capital. Unexpected loss is the volatility of losses around their expected levels. Unexpected loss determines the economic capital requirement.

Figure 1
Expected Loss Versus Unexpected Loss



"To prevent insolvency, economic capital must cover unexpected losses to a high degree of confidence. Banks often link their choice of confidence level to a standard of solvency implied by a credit rating of A or AA for their senior debt. The historical one-year default rates for A firms and AA firms are approximately 10 basis points and 3 basis points, respectively. These target ratings therefore require that the institution have sufficient equity to buffer losses over a one-year period with confidence levels of 99.90 percent and 99.97 percent (see Figure 2)."

Figure 2



While VaR models are viewed as risk models using a percentile or probability of ruin risk measure, the differentiating characteristic is at a more basic level. Differentiating requires answering the basic questions of what is considered an adverse event in the model and over what time horizon an adverse event can emerge. A VaR can be described as measuring an adverse change in market value over a one-year time horizon. Other risk models, such as DFA models, view risk as an adverse change in accounting values over longer time horizons.

3. Traditional View of Reserve Risk for Property and Casualty Companies

While much of the methodology discussed relating to VaR and EC models could be applied to an insurance enterprise, construction of a similar model requires information on how the prices of assets or liabilities change. An active market, in which a large volume of assets and liabilities is bought and sold, is needed to develop a historical profile of changes in value under multiple market conditions. Absent such a market, a proxy for how the market value of the asset or liability would change under stress conditions could also be developed.

The concept of market valuation of both assets and liabilities is an emerging issue for insurers due to discussions regarding fair value and IFRS. Historically, insurers in the United States have operated through statutory accounting in which the majority of investments—fixed income assets—were held at amortized value and loss reserves were held at an undiscounted nominal value. This accounting view is a significant deviation from the "mark-to-market" perspective that drove the development of VaR-based EC models for other financial service institutions.

The differences between the traditional actuarial and financial view of risk are also driven by the concept of time horizon. Standard actuarial models do not produce results over a discrete time horizon, but rather results at "ultimate" or "life of liability" basis. The actuarial methods are focused on the magnitude of the final value, not on how an estimate may move to its final value.

The reason for ignoring the time step in actuarial methods is driven by its lack of relevance to its intended use. Current actuarial triangulation or chain ladder methods are used to produce best estimates of loss reserves for financial statement purposes. The focus is to set an adequate reserve value and reasonable range of potential incurred losses.

The point of the actuarial loss reserve estimation methods is to set a best estimate that will not change, while VaR focuses on how much the estimate could change over a time horizon. Hybrid methods, such as the Mack method and the bootstrapping method, have been developed to produce estimates of reserve variability using development triangle data. Unfortunately, the best estimate and distributions produced by these methods are not right for the VaR calculations since they ignore the time horizon.

Most types of assets held by insurers can be analyzed in various historic market conditions due to the existence of long-term, active markets. A wealth of standardized and consistent financial market data also exists to create a needed proxy for market values of most other asset types.

Insurance liabilities on the other end of the spectrum pose some unique challenges. No active market exists for insurance company liabilities. In a limited way, market prices can be observed through sales of companies, reinsurance transactions or securitizations. The numbers of transactions are small and information is not always public, so even this information is of limited value.

Given all of the issues mentioned above, attempts have been made to extend VaR and EC methodology into the P&C insurance world. Some notable examples are the paper published by Nakada, Shah, Koyluoglu and Collignon, "P&C RAROC: A Catalyst for Improved Capital Management in the Property and Casualty Insurance Industry," in *The Journal of Risk and Finance* (Fall 1999); and "White Paper of the Swiss Solvency Test" (Swiss Federal Office of Private Insurance, November 2004). While these papers lay out the principles of an EC framework, they do not define a methodology that could encompass loss reserve risk in a consistent manner.

4. Reserve Risk Incorporating Time Horizon for Property and Casualty Companies

To fully specify a VaR model for P&C insurance reserve applications, we can rely on the banking concepts discussed above. Reserve risk VaR (RRVaR), as in banking applications, focuses on the unexpected loss in net loss reserves (NLR) at some percentile, 1–α, where NLR equals the recorded value of NLRs as of a financial statement date.

The following equations and Figure 3 specify the model.

$$F_{NR}(x) = \Pr(NR \leq x)$$

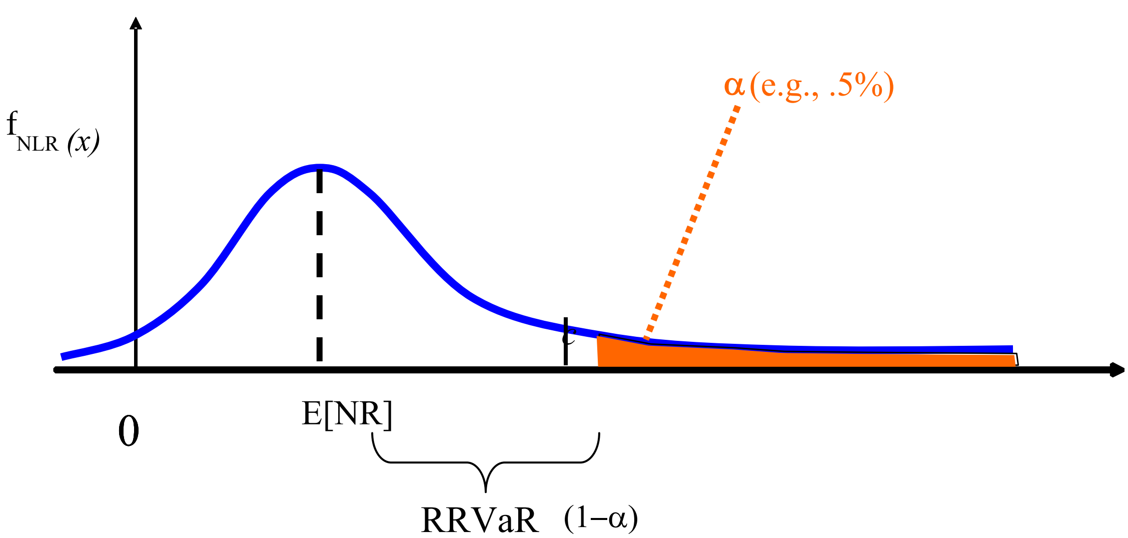
$$F_{NR}(c) = \Pr(NR \leq c) = 1 - \alpha$$

$$RRVaR_{(1-\alpha)} = \left| c - E[NR] \right|$$

$$RRVaR_{(1-\alpha)} = EC_{(1-\alpha)}$$

$$\text{Over a fixed annual interval,} \geq \text{one - year where } c \geq E[NR]$$

Figure 3



The appropriate model to satisfy the required VaR structure will have several key criteria:

- Produce a distribution of potential changes in loss reserve estimates;
- Provide a proxy for market value reserve estimates; and
- Produce results over appropriate time horizons or time steps.

The first criterion requires that the model produce a distribution of results, along with an expected value. Since a VaR model requires distribution percentiles, multiple model forms could satisfy this criterion. Those that have been used in practice include closed form distribution models, simulations and bootstrap sampling models; however, they will not work because they do not incorporate a time horizon.

The second criterion requires that a market value proxy be calculated for each outcome that comprises the adverse reserve distribution. Extensive research has been done about stating fair value of insurance liabilities for the purposes of implementing International Financial Reporting Standards. The basic techniques involve discounting cash flows at an appropriate interest rate and then increasing the discounted value with a market value margin (MVM). The MVM is an adjustment that is meant to approximate a purchaser's risk premium or cost of committed capital required in a transfer situation.

The concept behind an MVM creates intriguing issues that are beyond the scope of this paper. One such issue is how the MVM would react in a stress situation in which a large adjustment to reserves is made. Most methods currently contemplated assume the MVM is a fixed proportion of the expected value of the reserve liability. This is obviously a simplification that may be acceptable in a typical situation, but is not acceptable in the extreme tail situations that drive the need for EC.

For purposes of this paper, an adequate calculation of the fair value of reserve estimates will be the discounted value of the expected reserve payout cash flows at a risk-free rate. While this approach ignores some theoretical issues, its simplicity will aid in the discussion and development of a VaR model for loss reserves.

5. Specifying a Value-at-Risk Model for Loss Reserve Risk

To satisfy the criteria discussed above, a new type of loss reserve variability model will need to be specified. Prior to developing the model, it is useful to discuss and clarify some basic concepts regarding the composition and estimation of loss reserves and how they impact reserve risk.

P&C loss reserves can be viewed as a portfolio of reserves that are composed of separate sub-portfolios from each accident year (AY). An AY is the underlying subgrouping of reserves used for statistical and financial statement purposes in insurance. AY contributions to a company's current reserve position can be viewed as different components of reported and open (RO) and incurred but not reported (IBNR) claims which, when aggregated, drive the company's required reserve position as of an accounting date.

As of any accounting date, the total reserve contribution is derived from AYs with different levels of maturity or seasoning. Typically, the current AY (corresponding with the accounting year) is seasoned by 12 months at a year-end accounting date. The first and second prior AYs are seasoned by 24 months and 36 months, respectively. This pattern increments by an additional 12 months for each older AY and continues for as many years as an insurance company has been in business and claims are still open. In general, the variability of an AY's ultimate value should decrease as it matures since more claims are closed and more information is known about the RO claims the longer they have been reported to the insurer.

The actuarial reserve estimation process involves analyzing AY development patterns from older, more mature AYs and imputing the same level of growth to less mature years. Development patterns are typically represented by the percentage growth observed in paid or case-incurred amounts by AYs as they mature. Multiple actuarial methods utilizing different development patterns are typically used to produce loss reserve estimates.

Deciding on a reserve level to establish in company financial statements necessarily involves a set of judgments about the appropriate value for each AY in the face of uncertainty. Companies strive to reduce uncertainty by using multiple actuarial estimation methods, tracking price level changes, understanding operational or data issues and using expert judgment. The financial statement reserves are set at a point in time, therefore, are not the direct result of a mathematical calculation or single actuarial method; rather, they are a combination of judgments which weigh many factors. Along with the results of the actuarial calculations, factors such as future economic conditions, jury attitudes and the state of the insurance market are considered. The reserving process has some similarities to how prices are set in an active market for financial instruments; the process is not always completely rational.

A true model of the volatility of reserve estimates cannot be reproduced by a simple mathematical method because it combines so many company business processes, judgments and sources of risk. Most models, like the Mack method, assume that variability in reserves can be estimated by using the historic variability in loss reserve triangles. While the variability in the data may capture some sources of risk, it ignores many others. Risk, such as process and parameter risk, may be captured, but model risk and operational risk are ignored. All of these risks can manifest themselves as adverse loss development and should be captured in a reserve risk model. An appropriate solution requires observation of actual changes in estimates over time, similar to the way a study of the market price volatility of a financial instrument requires observing actual price changes over time.

As previously mentioned, because no open active market exists for loss reserve liabilities, other sources of information must be used. In the case of U.S. P&C insurers, the available data takes the form of the detailed information included in Schedule P of the Annual Statement. This schedule tracks AY reserve run-off for the prior 10 years and is available electronically from several sources.

5.1 Data Exploration and Analysis

The data used to develop the VaR model were drawn from Annual Statement data from 1995–2006 filings, as provided by Highline Media's P&C Insurance Data product. We limited our analysis to the Private Passenger Auto Liability line of business, utilizing the information available from Schedule P, Parts 2B, Ultimate Loss and Allocated Loss Adjustment Expenses (ALAE) and 3B, Cumulative Paid Loss and ALAE. This line of business provided over 17,000 data points to calculate ratios and metrics for changes between 12 and 24 months of development. There were over 10,000 data points available for analysis for changes between 108 months and 120 months of development.

These data were sufficient to calculate various metrics used in our data exploration, notably the incremental and cumulative changes in ultimate, the initial reserve, current reserve and reserve at interim points in time. Ratios of changes in ultimate relative to a selected reserve base, e.g., the cumulative change in ultimate relative to the initial reserve at the end of 12 months of development, were calculated.

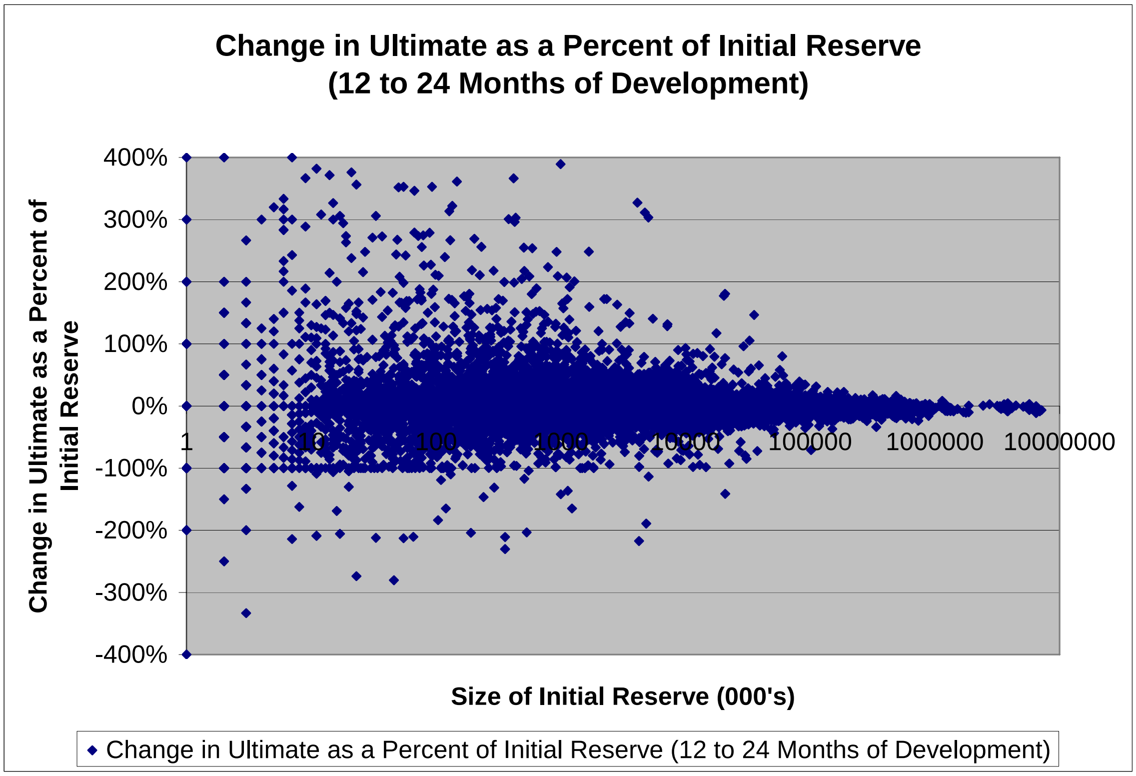
Univariate and bivariate analyses were performed on the selected metrics to identify suspected outlier data points and relationships between the metrics. The following two observations were noted:

1. The spread of the metrics, related to changes in an AY ultimate, decreased with the size of the initial reserve, i.e., as initial reserve increases in size, the amount of change experienced relative to the reserve balance drops proportionally.

2. The metrics for cumulative changes in an AY ultimate were strongly correlated, and the correlation between two consecutive periods increased the more developed the periods are, i.e., the incremental changes in ultimate are inversely related to time.

We applied these observations in developing a model to estimate the future one-year change in ultimate. The first observation poses a problem in that any model using this data would have to account for this size-related variability, or the model would exhibit heteroscedasticity, or non-constant variance. This is illustrated in Figure 4.

Figure 4



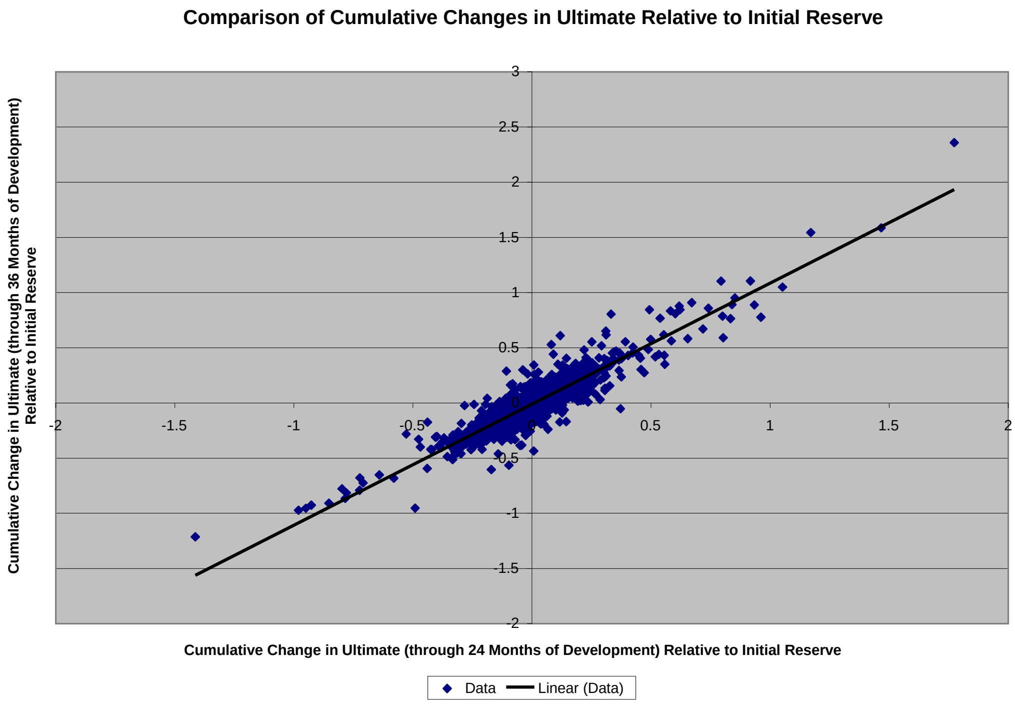
Heteroscedasticity violates an assumption underlying linear regression that errors have a constant variance. We accounted for this by segmenting the data into two size categories and utilizing a reserve size variable in model development. The size categories were segmented in terms of whether or not the AY's initial reserve was greater than or equal to $10 million. For data in the "greater than $10 million" category, the size of the initial reserve variable was not significant and was dropped from the model. For data in the smaller reserve category, the size of the initial reserve variable was significant for nearly every model and successfully mitigated the heteroscedasticity issues.

The second observation is what helped formulate the model. If the cumulative changes in ultimates are strongly correlated, one can use the cumulative change at one point in time to estimate the subsequent cumulative change, and thereby estimate the total change. This thought process is developed further in the next section.

5.2 Derivation of Model

Our assumption is that for an accident year at any given point of development, the subsequent change in ultimate for the accident year depends on information known at the end of the development period, notably the initial reserve and the cumulative change in ultimate. From our analysis, we assume that a linear relationship exists between the cumulative change in ultimate through a period (k-1) relative to the initial reserve and the cumulative change in ultimate through a period (k) relative to the initial reserve. The typical relationship we observed is displayed in Figure 5 below, relating AY changes at 36 months to the 24-month values.

Figure 5



Algebra helps us arrive at a simple formula for the model:

$$
\begin{aligned}
\left(ChgUlt_k\right) &= \left(\frac{CChgUlt_k}{\mathrm{Re}\,s_0} - \frac{CChgUlt_{k-1}}{\mathrm{Re}\,s_0}\right)\cdot \mathrm{Re}\,s_0 \\
&= \left(\left[\beta_{k0} + \beta_{k1}\cdot\left(\frac{CChgUlt_{k-1}}{\mathrm{Re}\,s_0}\right) + \varepsilon_k\right] - \left(\frac{CChgUlt_{k-1}}{\mathrm{Re}\,s_0}\right)\right)\cdot \mathrm{Re}\,s_0 \\
&= \left(\left[\beta_{k0} + \left(\beta_{k1} - 1\right)\cdot\left(\frac{CChgUlt_{k-1}}{\mathrm{Re}\,s_0}\right) + \varepsilon_k\right]\right)\cdot \mathrm{Re}\,s_0 \\
&= \left(\beta_{k0} + \varepsilon_k\right)\cdot \mathrm{Re}\,s_0 + \left(\beta_{k1} - 1\right)\cdot\left(CChgUlt_{k-1}\right)
\end{aligned}
$$

Explanation of terms:

$\mathrm{Re}\,s_0$	Initial reserve, i.e., reserve at the end of the first period of development
$ChgUlt_k$	Change in ultimates from period (k-1) to period (k)
$CChgUlt_k$	Cumulative change in ultimates to period (k)
$\beta_{k0}, \beta_{k1}, \varepsilon_k$	Regression parameters to go from period (k-1) to period (k), noting the constant coefficient, the independent variable coefficient and the random term, respectively

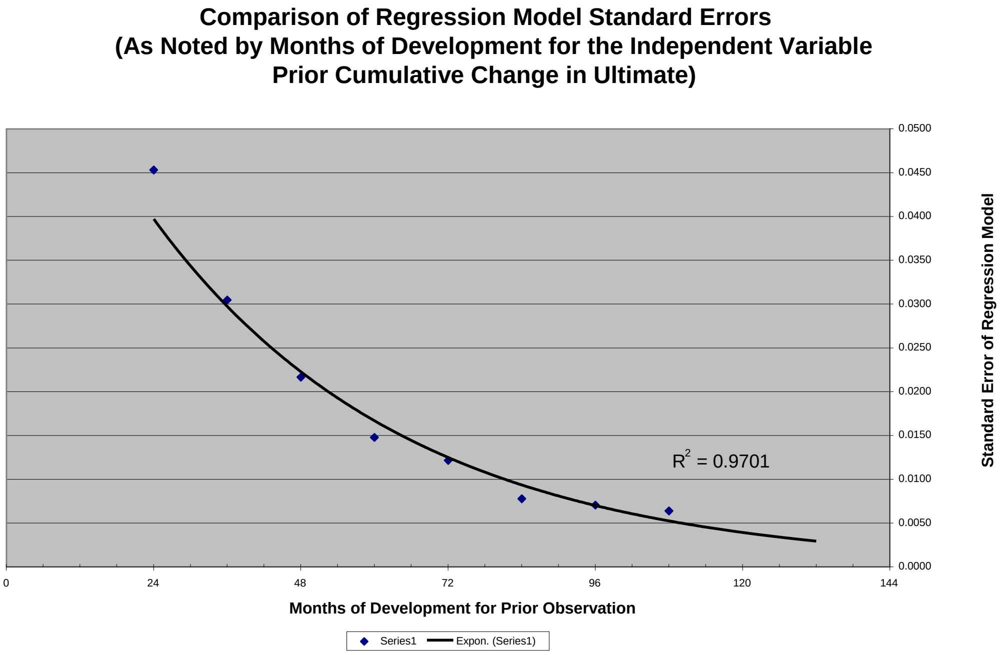
The random error term is scaled by the size of the initial reserve and is independent of the cumulative change in ultimate to period k-1. This allows the formula to be applied successively to estimate a series of future changes in ultimate.

A significant item to address is points in development for which there are no data to model. For instance, given Annual Statement data, there is no means to estimate change in ultimate beyond 120 months of development. To estimate change from 120 months to 132 months of development, we need to consider all parameter sets and use these to make assumptions for these periods. Consider the following set of parameter estimates:

Time period (k)	β_{k0}	β_{k1}	SE
108-120	-0.0008	1.0000	0.0064
96-108	-0.0010	1.0000	0.0071
84-96	-0.0017	1.0000	0.0078
72-84	-0.0035	0.9865	0.0122
60-72	-0.0032	1.0164	0.0148
48-60	-0.0038	1.0558	0.0216
36-48	-0.0043	1.0986	0.0305
24-36	-0.0100	1.1803	0.0453

To simulate results, an estimate of the error is necessary. The table above shows the model standard error (SE) decreases as the development period increases. This does not appear to be a linear relationship; in fact, the relationship appears exponential when plotted, as shown in Figure 6.

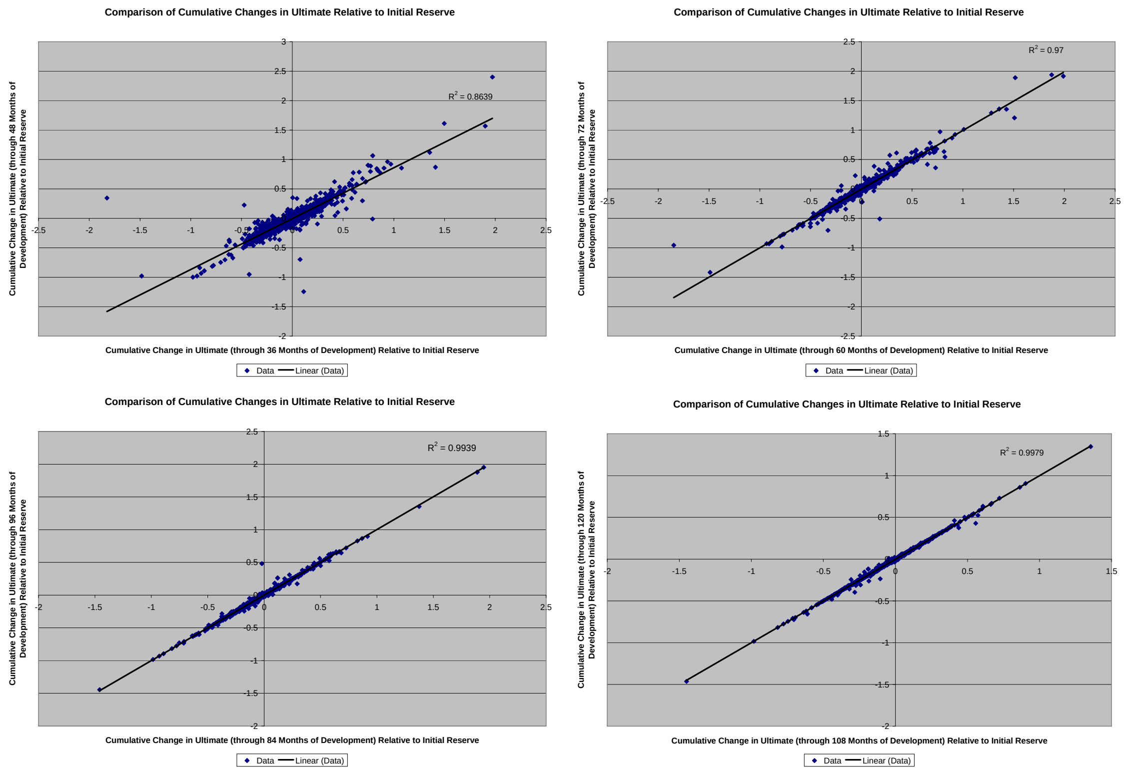
Figure 6



Given the R-squared value, a fitted exponential curve to the SE estimates could provide a reasonable estimate for periods that we cannot model. After review of the plots depicting the cumulative change in ultimate for a given period plotted against the cumulative change in ultimate from the prior period, exponential decay of the standard error seems reasonable.

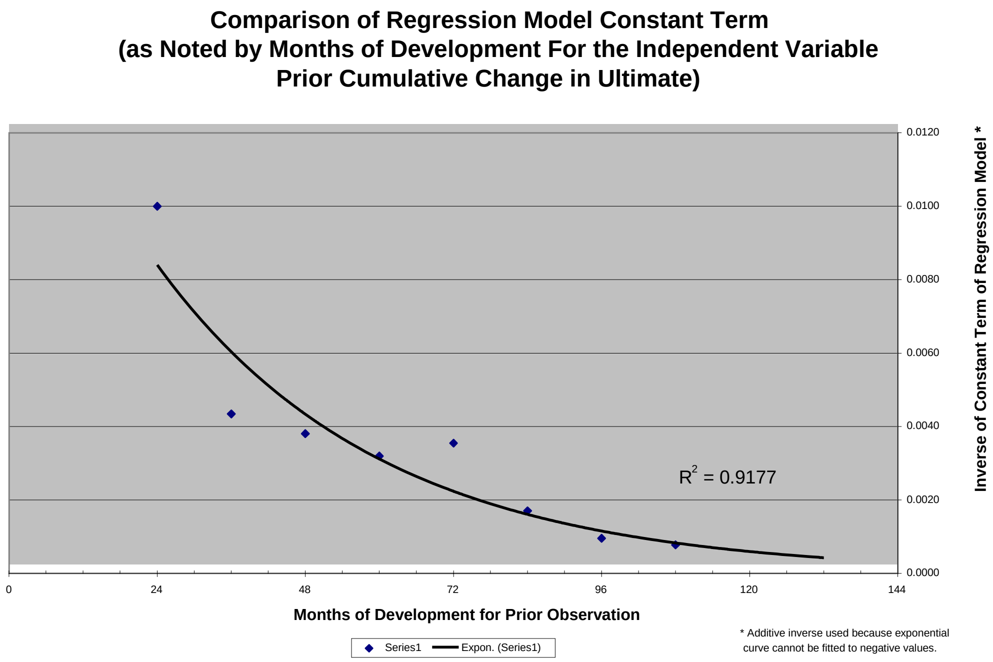
In Figure 7 are the plots of cumulative change in ultimate relative to initial reserve, comparing 36 months to 48 months, 60 months to 72 months, 84 months to 96 months and 108 months to 120 months of development.

Figure 7



We may also assume a constant value of 1 for the linear parameter (β_{k1}), as it appears to asymptotically approach this value, and doing so implies that, beyond 120 months of development, the expectation for changes in ultimate should approach zero. Finally, the value for the constant term (β_{k0}) appears to approach zero, so an exponential trend could be applied to estimate this parameter as well, as seen in the following plot in Figure 8:

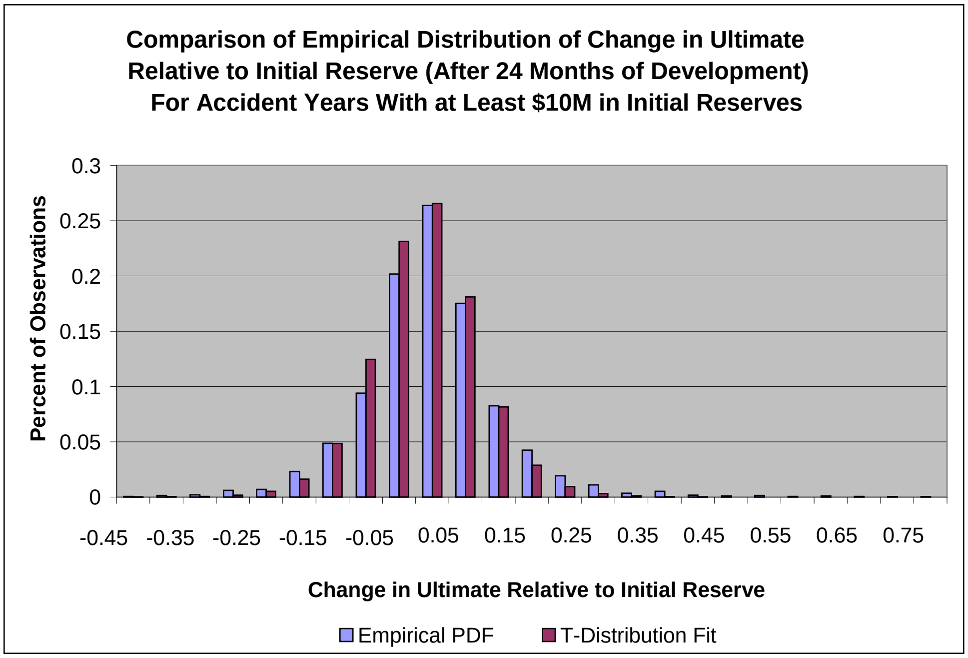
Figure 8



5.3 Estimating the Initial Change in Ultimate between 12 to 24 Months of Development

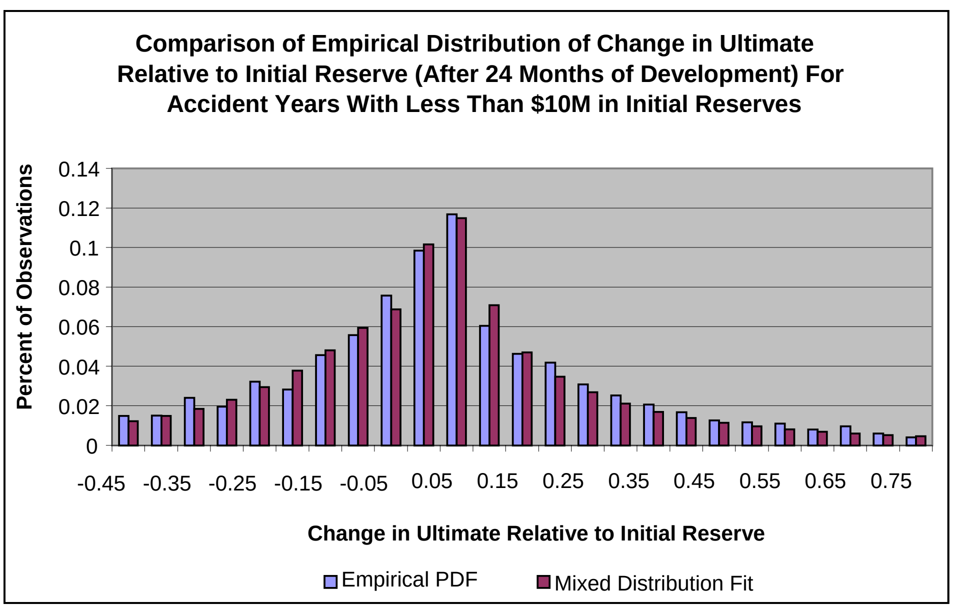
We cannot model the first change using a linear regression model because there is no independent variable. However, we can fit a distribution to the data which would provide percentiles for use in the model. For the accident years with initial reserves greater than or equal to $10 million, we found the change in ultimate from 12 to 24 months of development as a percent of initial reserve is approximately t-distributed, with an allowance for shift and scaling parameters. This can be seen in Figure 9.

Figure 9



For the accident years with initial reserves less than $10 million, we used a mixture of two distributions: a t-distribution, with shift and scaling parameters, and a Fisher-Tippett distribution, also with shift and scaling parameters. The addition of the Fisher-Tippett distribution allows for the data to be skewed. This is illustrated in Figure 10.

Figure 10



5.4 Applying Model to Subsequent Periods

By using the model output along with the original inputs, one can estimate the accident year's change in ultimate in the subsequent period or a series of subsequent periods. This is illustrated with the following model formula, extending the process another step:

$$\begin{aligned}\left(ChgUlt_{(k+1)}\right) &= \left(\beta_{(k+1)0} + \varepsilon_{(k+1)}\right)\cdot \mathrm{Re}\,s_0 + \left(\beta_{(k+1)1} - 1\right)\cdot\left(CChgUlt_k\right)\\ &= \left(\beta_{(k+1)0} + \varepsilon_{(k+1)}\right)\cdot \mathrm{Re}\,s_0 + \left(\beta_{(k+1)1} - 1\right)\cdot\left(CChgUlt_{(k-1)} + ChgUlt_k\right)\end{aligned}$$

So, to estimate the subsequent step, the result from the prior step is incorporated and the (k+1) set of parameters is applied. Carrying these formulae forward (j) periods would result in the following:

$$\begin{aligned}\left(ChgUlt_{(k+j)}\right) &= \left(\beta_{(k+j)0} + \varepsilon_{(k+j)}\right)\cdot \mathrm{Re}\,s_0 + \left(\beta_{(k+j)1} - 1\right)\cdot\left(CChgUlt_{(k+j)-1}\right)\\ &= \left(\beta_{(k+j)0} + \varepsilon_{(k+j)}\right)\cdot \mathrm{Re}\,s_0 + \left(\beta_{(k+j)1} - 1\right)\cdot\left(CChgUlt_{(k-1)} + \sum_{i=0}^{j-1} ChgUlt_{(k+i)}\right)\end{aligned}$$

5.5 Applying Correlation between Accident Years

A model must provide some means of correlating development of accident years for a given calendar year. In the model, this was done by using a normal copula and a correlation matrix. The correlation matrix is assumed to exhibit the following two behaviors.

Accident years that are close together are correlated more strongly than accident years that are further apart, e.g., the experience for accident years 1999 and 2000 would be more correlated than for 1999 and 2005.

The correlation between accident years close together diminishes over time, e.g., the correlation between accident years 1999 and 2000 is strong during the first few periods of development, but begins to diminish during subsequent development periods.

This correlation matrix is calibrated using incremental change in ultimate, rather than cumulative change in ultimate, relative to the initial reserve. The random component of the model is applied to the initial reserve, rather than to the cumulative change in ultimate, so the correlation of the errors should be on an incremental basis. The correlation matrix also required some adjustment, as some empirical estimates of correlation between periods were negative, and it is common practice to use only non-negative correlations within a model.

The correlation is measured using the Annual Statement data, which only provides the first 120 months of development for accident years. To estimate the correlation between accident years beyond 120 months of development, we trended the empirical estimates using a regression model, where the accident year's age and lag were used as inputs. Prior to modeling, the correlations were transformed using the Logit transformation, so that the dependent variable's domain was all real numbers, not just the interval [0,1]; the predicted values were transformed back into correlation estimates using the inverse-Logit transform. Applying the regression model also smoothed the correlation estimates for the first 120 months of development, so that the matrix did not exhibit any irregularities.

The regression approach allows the creation of correlation matrices of any size. We created a matrix that is 21x21, which allows us to take the 11 rows of the Annual Statement Schedule P data and age them 10 years. For this application, we only need the 10 11x11 matrices falling along the diagonal. The matrix corresponding to the incremental development during the first future calendar period for accident years noted in the Annual Statement Schedule P is below:

Correlation between Accident Years of Different Annual Ages
Age of Accident Year (in Years)

Age of Accident Year	11	10	9	8	7	6	5	4	3	2	1
11	1.00	0.21	0.15	0.11	0.07	0.05	0.03	0.02	0.02	0.01	0.01
10	0.21	1.00	0.22	0.16	0.11	0.08	0.05	0.04	0.03	0.02	0.01
9	0.15	0.22	1.00	0.24	0.17	0.12	0.09	0.06	0.04	0.03	0.02
8	0.11	0.16	0.24	1.00	0.25	0.19	0.13	0.09	0.06	0.04	0.03
7	0.07	0.11	0.17	0.25	1.00	0.27	0.20	0.14	0.10	0.07	0.05
6	0.05	0.08	0.12	0.19	0.27	1.00	0.29	0.21	0.15	0.11	0.07
5	0.03	0.05	0.09	0.13	0.20	0.29	1.00	0.31	0.23	0.16	0.12
4	0.02	0.04	0.06	0.09	0.14	0.21	0.31	1.00	0.32	0.24	0.18
3	0.02	0.03	0.04	0.06	0.10	0.15	0.23	0.32	1.00	0.34	0.26
2	0.01	0.02	0.03	0.04	0.07	0.11	0.16	0.24	0.34	1.00	0.36
1	0.01	0.01	0.02	0.03	0.05	0.07	0.12	0.18	0.26	0.36	1.00

It should be noted that this correlation matrix may not be positive-definite, i.e., it has no Choleski Decomposition to use in a copula. Rather than using Choleski Decomposition, we used Spectral Decomposition as described by Rebonato and Jäckel. This entails finding a matrix that is arbitrarily close to the original which can be decomposed and used in a copula.

5.6 Applying Correlation between Time-Steps of the Repeated Model

If the model is applied successively, the results from successive iterations must be correlated; otherwise, the calendar year development would be independent. The calendar year correlation was estimated using accident year development aggregated by calendar year and measured over a series of years.

Correlation for calendar year development is implemented in the model using the Iman-Conover method. A normal copula was used to generate correlated random variables representing the calendar year aggregate. The rank of the variables in their respective columns represents the rank of the aggregate accident year change in ultimate, i.e., if the first row of the calendar year variable rank matrix is {500, 2401, 37, …}, then the 500th worst iteration from the first time-step is used, then the 2401st worst iteration from the second time-step is used, then the 37th worst iteration from the third time-step is used, etc.

Another form of correlation between years, the serial correlation of random errors, was considered and implemented in the model. The data suggest that errors between time-steps are not independent, and that prior errors should be incorporated into the model as well. This was accomplished by making the error in the successive terms a mixture of the random error from the prior step and the random error generated for the current step. Analysis of the serial correlation of the data suggests a mixture parameter of 30 percent should be used, i.e., 70 percent random error generated for current step, 30 percent random error from prior step. The addition of errors from steps earlier in the process was considered, but the sample serial correlation from analysis decreased rapidly and did not seem material enough to warrant inclusion in the model.

Results of the repeated model are as illustrated in the next section.

5.7 Illustration of Model Results

Part B - Private Passenger Auto Liability
Industry Aggregate

Schedule P Part 2

Calendar Year

Accident Year	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	(1) Current Reserve	(2) Initial Reserve	(3) Cumulative Chg in Ult
Prior	34,752,578	31,346,443	30,167,738	29,907,574	29,927,309	30,067,107	30,069,405	30,470,625	31,038,755	31,244,832	3,272,851	34,752,578	-3,507,746
1997	50,200,041	48,290,567	47,535,635	47,271,589	47,127,111	47,077,980	47,039,351	47,034,045	47,039,455	47,029,752	189,520	31,688,173	-3,170,289
1998		49,434,552	48,460,577	48,284,985	48,131,836	48,135,650	48,069,855	48,082,718	48,050,243	48,059,599	274,561	30,755,018	-1,374,953
1999			51,723,573	51,624,048	51,564,759	51,666,795	51,566,364	51,566,370	51,611,145	51,624,968	457,175	31,190,704	-98,605
2000				54,595,508	54,925,540	55,082,187	55,161,479	55,166,228	55,216,804	55,226,484	727,614	32,372,910	630,976
2001					57,013,900	56,780,128	56,680,946	56,719,452	56,860,754	56,819,386	1,211,797	33,946,336	-194,514
2002						60,401,184	59,704,434	59,392,363	59,454,492	59,388,076	2,408,578	36,244,781	-1,013,108
2003							61,641,577	59,671,262	58,945,904	58,712,992	5,005,421	37,495,349	-2,928,585
2004								62,261,624	59,968,158	59,065,620	9,787,887	37,851,774	-3,196,004
2005									63,194,894	61,049,536	18,288,344	38,047,147	-2,145,358
2006										62,784,420	37,101,198	37,101,198	0
											78,724,946		

Schedule P Part 3
Calendar Year

Accident Year	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Prior	0	12,254,730	19,167,111	23,138,007	25,317,110	26,355,590	26,948,074	27,412,486	27,726,891	27,971,981
1997	18,511,868	32,741,805	39,395,546	43,154,607	45,166,895	46,091,431	46,417,086	46,654,814	46,784,768	46,840,232
1998		18,679,534	33,396,138	40,252,877	44,102,374	46,117,821	46,990,271	47,445,784	47,674,543	47,785,038
1999			20,532,869	36,312,876	43,511,959	47,436,687	49,497,795	50,453,888	50,921,028	51,167,793
2000				22,222,598	39,156,911	46,605,247	50,800,756	52,976,433	53,987,662	54,498,870
2001					23,067,564	40,240,041	47,946,837	52,270,348	54,565,025	55,607,589
2002						24,156,403	41,941,707	50,033,846	54,541,463	56,979,498
2003							24,146,228	41,478,517	49,192,971	53,707,571
2004								24,409,850	41,554,907	49,277,733
2005									25,147,747	42,761,192
2006										25,683,222

These data are drawn from the 2006 Industry Aggregate for Private Passenger Auto Liability, as provided by Highline Media. The initial and current reserve and the cumulative change in ultimate are shown.

Part B - Private Passenger Auto Liability
Industry Aggregate

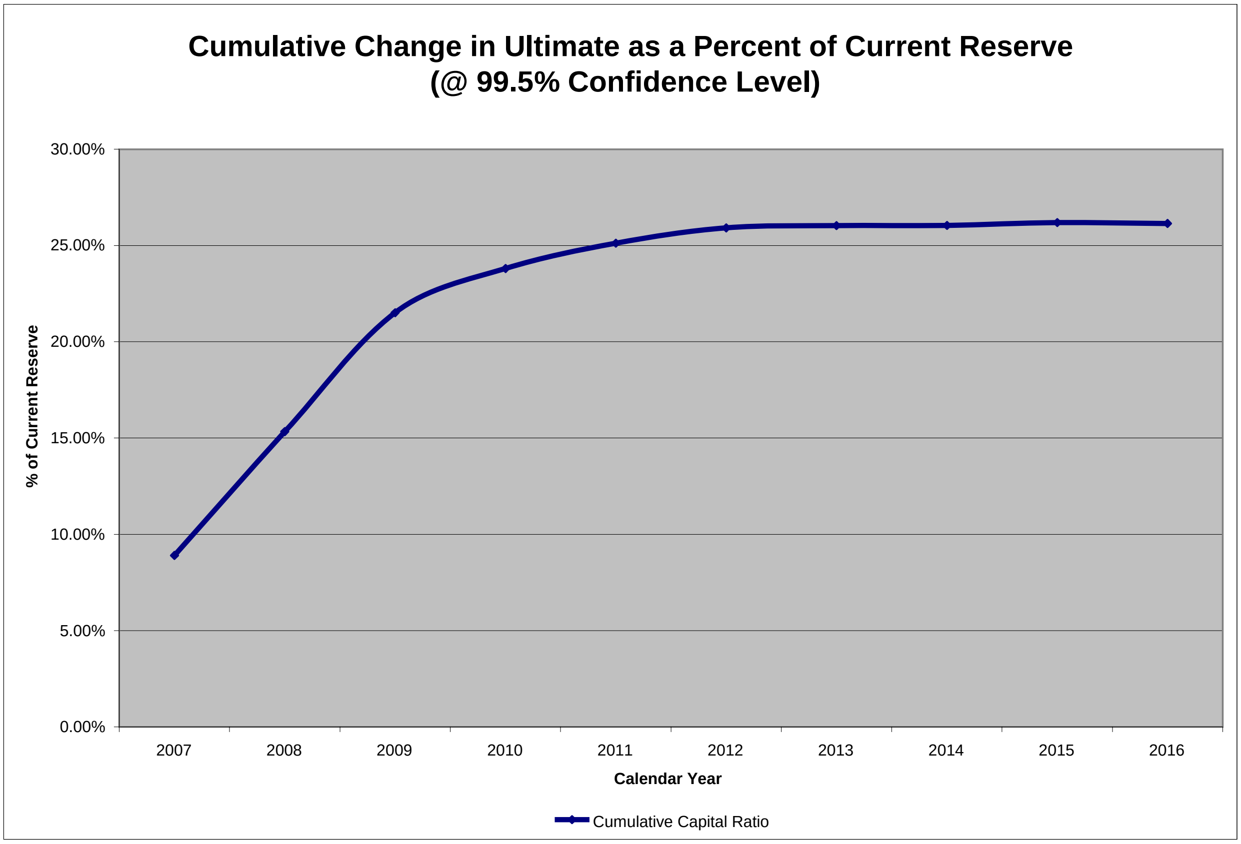
Alpha
0.005

Accident Year	2007 Chg in Ultimate	2008 Chg in Ultimate	2009 Chg in Ultimate	2010 Chg in Ultimate	2011 Chg in Ultimate	2012 Chg in Ultimate	2013 Chg in Ultimate	2014 Chg in Ultimate	2015 Chg in Ultimate	2016 Chg in Ultimate
Prior	51,772	-808	21,527	57,032	-21,068	-7,784	8,878	5,285	-807	-2,453
1997	158,841	10,899	-13,997	-12,068	-4,317	4,009	-11,686	-12,573	110	751
1998	72,604	-6,936	7,900	-13,041	-8,697	21,101	-9,734	2,636	7,414	4,195
1999	-84,472	311,512	155,620	80,751	26,043	16,716	7,262	12,968	2,508	-6,987
2000	3,006	180,302	48,752	-129,117	-20,920	10,282	7,876	-38,291	-7,602	10,762
2001	455,931	-179,218	227,359	37,678	53,731	66,416	44,558	-15,545	-21,137	17,047
2002	-96,574	121,397	43,373	-80,037	-32,140	176,510	70,899	59,831	13,547	-1,871
2003	724,378	-26,197	184,642	122,917	76,437	-185,855	-59,537	56,221	-5,723	-11,541
2004	295,435	416,195	156,384	721,975	270,770	229,618	217,179	117,723	-22,502	-73,964
2005	949,268	935,922	1,535,525	998,618	319,757	81,603	-86,428	-5,594	-2,553	71,523
2006	4,479,450	3,299,583	2,502,204	19,847	374,711	216,057	-98,661	-176,983	149,042	-35,251
Incremental	7,009,638	5,062,651	4,869,288	1,804,555	1,034,308	628,674	90,606	5,677	112,298	-27,787
Cumulative	7,009,638	12,072,290	16,941,577	18,746,132	19,780,440	20,409,114	20,499,720	20,505,398	20,617,696	20,589,908
Capital Ratio *	8.90%	15.33%	21.52%	23.81%	25.13%	25.92%	26.04%	26.05%	26.19%	26.15%

* *Capital Ratio is applied to reserve as of 12/31/2006*

The estimates for 2007 through 2016 are determined by applying the model for initial reserves with values greater than $10 million successively, as noted above. The cumulative charge, as a percentage of current reserves, increases steeply, then levels off around 26 percent. The development of this metric is shown in Figure 11.

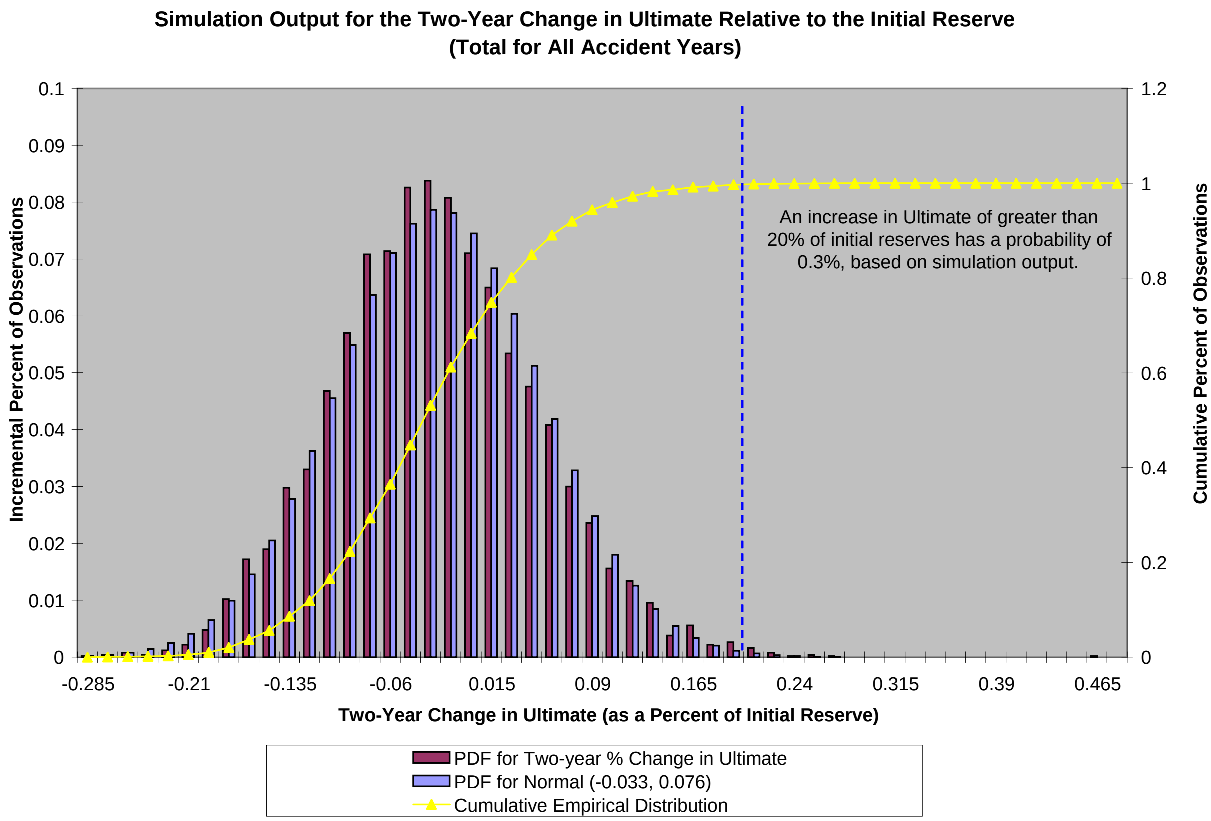
Figure 11



5.8 Applying Model to Intervening Periods

As the model can produce results along annual time steps, we can generate results to make probability statements at different annual intervals. As suggested in the introduction, a relevant example is: What is the probability that current reserves will become 20 percent deficient over the next two years? As illustrated in Figure 12 below, the probability is around 0.3 percent.

Figure 12



6. Conclusion

By incorporating time horizon, we have developed a model for P&C insurer loss reserve risk that conforms to the structure of a VaR model. This step is necessary for insurers to be able to understand how loss reserve risk evolves over time and integrate risk models into existing EC modeling paradigms. Full integration requires that all risk sources, whether from assets or liabilities, are expressed in common time horizons. The common horizon, typically selected as one year, allows the risk distribution to be aggregated.

In addition, VaR models are usually calibrated at very high percentiles that have proven difficult to reasonably match in insurance applications. With less available data, the tails of distributions are hard to parameterize with confidence at the extremes. The RRVaR method utilizes the data available in a way to maximize the sample size and also seems to produce plausible results at the extreme percentiles.

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