Projection Of Mortality Rates At Advanced Ages In Canada With A New Lee-Carter Type Model

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Abstract

Proper mortality forecast at advanced ages is an important challenge for demographers and actuaries. For this particular population, it has been shown that logistic models for the force of mortality (such as Perks' and Kannisto's models) usually provide very good modeling and forecasts (Thatcher, Kannisto and Vaupel, 1998). However, these models are not frequently used in actuarial practice. In demography, the Lee-Carter model, which offers a simple methodology, has been preferred (Lee and Carter, 1992) for forecasting.

In this paper, we will show that the modeling and forecasting of advanced age population can be improved by combining features of the logistic model for the force of mortality, and the Lee-Carter model. This combination has been inspired by a linear reparametrization of the logistic models taken in Doray (2008).

In particular, our model will be applied to the Canadian male population aged 70 to 99 years old and to the Canadian female population aged 80 to 105 years old. The results of our model will be compared to those of two well-known models, the original Lee-Carter model for each sex, and the logistic model of Lee-Carter (Lee, 2000) for each sex. For both sexes, we found that the use of our model presents values closer to the observations and that the forecasts are quite realistic.

Keywords:

Logistic models, Lee-Carter model, advanced age population, mortality rate, central rate of mortality, force of mortality, forecast methods, singular value decomposition, time series, Box-Jenkins method.

0. Introduction

Abilities of older people vary greatly according to the age group in which they are. Given that the Canadian population is getting older and that the needs of older people will be increasing, the elaboration of administrative and budgetary plans based on the most accurate demographic projections is essential.

This paper has been written with this objective in mind, projecting mortality at advanced ages for the Canadian population, males aged 70 to 99 and females aged 80 to 105.

On 19 July 2002, in an article which appeared in *Le Devoir* newspaper, F. Nault, a Statistics Canada demographer, was analyzing the 2001 census. The over age 80 population had increased by 41% in 10 years in Canada, while the 45-64 age group had increased by 36%, in spite of the massive arrival of baby-boomers in this age group. Centenarians were more numerous than ever in Canada: they were 3795 in 2001 and 3125 in 1996, an increase of 21% in 5 years.

Population ageing is not a phenomenon particular to Canada. Throughout the world, this situation is more and more pronounced since the end of the fifties, especially in developed countries. For example, in 2006, among the G8 countries, Canada was the third oldest country, with a median population age of 36.8 years, after the United States and Russia.

The principal causes related to this ageing phenomenon are well known: the fall in the birth rate and the increase in longevity. But beyond these reasons, this situation takes on great importance with the public decision makers. They must avoid deficits in budgets and guarantee an adequate financing of various government programs such as recruitment of manpower, job creation, social housing, old-age pensions and health care. All this requires exhaustive studies and accurate projections to quantify and qualify the ageing population.

The paper is organized as follows.

In section 1, we review some logistic models for the force of mortality for the old-age population, models which have been successfully used in many countries.

In section 2, we present the Lee-Carter model and its modified forms. Lee and Carter (1992) used extrapolation to project mortality, a model first applied in the US. They modeled the central rate of mortality at age x for a given time period t, with three series.

In section 3, Doray and Tang propose a new modified Lee-Carter model, which combines features of Kannisto's model and the Lee-Carter model.

Section 4 will present the data available in Canada at advanced ages, for males and females (1976-2005), taken from the Canadian Longevity Database (males aged 70 to 99 years and females aged 80 to 105 years).

In section 5, using these two populations, we will compare the results of three models, the original Lee-Carter model, the logistic model of Lee-Carter and Doray and Tang's model, for the values of the parameters, the modeling errors, and the projected mortality rates for the period 2006-2035. Doray and Tang's model will be seen to have the smallest mean absolute relative error among the three models.

1. Logistic models for the force of mortality

The 1970's witnessed a clear fall in mortality as well as an increase in the number of deaths at advanced ages (Kannisto, 1996). In this context, appropriate modeling of this population group is essential.

Many mathematical models have studied the evolution of the force of mortality for the advanced age population, for example exponential or logistic models.

In 1998, Thatcher, Kannisto and Vaupel wrote a monograph studying the population aged 80 to 120 years in 13 countries (Austria, Denmark, England and Wales, Finland, France, West Germany, Iceland, Italy, Japan, Netherlands, Norway, Sweden and Switzerland). Those countries were selected according to strict quality criteria of the data, described in their work (Thatcher et al., 1998).

Their monograph revealed many interesting conclusions:

1- Gompertz model ($\mu_x = \mathrm{B}e^{\mu x}$) produced the largest errors;

2- Weibull's model ($\mu_x = \mathrm{a}x^b$) was only slightly better;

3- Heligman and Pollard (1980) model ($q_x / p_x = GH^x$), was the third worst model.

These three models were overestimating mortality beyond age 100.

In all the countries, for the periods 1960-70, 1970-80, 1980-90, and for the cohort born in 1871-1880, the best fitting models for the old-age population were the model of Perks' (1932)

$$\mu_x = \frac{A + Be^{\mu x}}{1 + Ce^{\mu x}},$$

and its special case with the parameters $A=0$ and $B=C$, Kannisto's model (Kannisto, 1992),

$$\mu_x = \frac{Be^{\mu x}}{1 + Be^{\mu x}}.$$

Perks' and Kannisto's models are both logistic-type models. As x tends to infinity, μ_x tends to a constant, equal to 1 for Kannisto's and B/C for Perks' model, contrarily to exponential models such as Gompertz or Makeham, where the force of mortality tends to infinity.

Doray (2008) has shown for Kannisto's model, that the logit of μ_x, defined as

$$\text{logit } \mu_x = \log\left(\frac{\mu_x}{1 - \mu_x}\right),$$

could be expressed, after a reparametrization, as a linear function of the parameters a and μ,

$$\text{logit } \mu_x = a + \mu x,$$

where $a = \log B$. Note that since μ_x is less than 1 for Kannisto's model, $\mu_x / (1 - \mu_x)$ is positive and its logarithm can always be taken.

Introducing random errors, a linear model was then defined. Its parameters were estimated by weighted least-squares, and their properties (consistency, asymptotic unbiasedness and normality) developed.

2. The Lee-Carter model and its modifications

Lee and Carter introduced their model in 1992 to project mortality in the US for both sexes combined, with extrapolation methods. They modeled the central rate of mortality at age *x* for a given time period *t*, denoted $m_{x,t}$, with three series,

a_x, the average level of mortality at age *x*

b_x, the rate of improvement of the level of mortality at age *x*, and

k_t, the level of mortality at time *t*, as

$$\log m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t} ,$$

where $\varepsilon_{x,t}$ is a random error for age *x* at time *t.*

The central rate of mortality, defined as the number of deaths at age *x* divided by the average number of people of age *x* who lived during period *t*, is seen to be an exponential function of parameters depending on age *x* and time *t.*

During the projection, only the parameter k_t is extrapolated with the Box-Jenkins time series method.

Lee and Carter have also suggested that each sex be modeled separately. Since its development, this model has been used in many countries, including Japan and Italy. It has also been used to model morbidity and fertility, besides mortality.

Many desirable features have made it popular among demographers and actuaries. It gives a good fit to data and it requires the extrapolation of only one parameter. Moreover, uncertainties can be considered during projections. Finally, the method is simple to use.

However, the Lee-Carter model has also aroused many criticisms. Its principal problems are a temporal invariability of parameters a_x and b_x, unverified in reality, as well as the divergence created between sexes and regions. The case of Canada is a good example, where we observe that the differences between life expectancies for males and females seem to diminish over time. Moreover, the differences in life expectancies between the provinces remain constant over time (Paquette, 2006). The Lee-Carter model however, only projects divergence situations in the long term.

Lee and Nault have submitted proposals to correct this problem (Lee and Nault, 1993).
Furthermore, the Lee-Carter method shares the disadvantages of other methods of projection based on extrapolation. The historical tendency of mortality is certainly not a guarantee for the future. Also, advances in medicine, changes in lifestyle and the appearance of new diseases are not considered. In spite of all this, experimental studies have shown that the the model gives good results when it is used over the period 1900 to 1990 (Lee, 2000).

Various modifications to the basic model have been proposed in the literature, for example the augmented common factor Lee-Carter method (Li and Lee, 2005). This method solves the long term divergence problems between sexes, ethnic groups and regions in the original Lee-Carter model. It requires partitioning of the population to be projected, by similar characteristics, which may be the sex or the socio-economic status of certain groups of the population studied.
Statistics Canada has compared the augmented common factor Lee-Carter method with the Lee-Carter model for the period 1971-2002 and concluded that the method of Li and Lee produced better modeling and projections for Canada. The problems of divergence between sexes and regions were greatly eliminated (Paquette, 2006).

Another modification to the Lee-Carter model which has been considered in the literature is the Lee-Carter logistic model (Lee, 2000)

$$\text{logit}\, m_{x,t} = \log\left(\frac{m_{x,t}}{1-m_{x,t}}\right) = a_x + b_x k_t + \varepsilon_{x,t}.$$

3. A new modified Lee-Carter model

Tuljapurkar, Li and Boe (2000) examined mortality in the G7 countries for the years 1950 to 1994. They noticed that for all the countries in this period, mortality at each age was decreasing exponentially at a more or less constant rate. In this section, we will try to improve the performance of the Lee-Carter model for modeling and projecting the old-age population.

As written in Section 1, logistic models for the force of mortality, such as Perks' or Kannisto's, were the best ones to model mortality of advanced age population in industrialized countries. Also, after reparametrizing Kannisto's model, the logit of μ_x , could be expressed as a linear function of two parameters.

Combining features of Kannisto's model and the Lee-Carter model, Doray and Tang propose a new model for the old-age population, where the logit of μ_x is modeled as a function analogous to the one in the Lee-Carter model,

$$\text{logit } \mu_{x,t} = \log\left(\frac{\mu_{x,t}}{1-\mu_{x,t}}\right) = a_x + b_x\, k_t + \varepsilon_{x,t}\,.$$

To estimate the parameters of the model, we will first review the method of Lee and Carter for the model

$$\log\,(q_{x,t}) = a_x + b_x\, k_t + \varepsilon_{x,t} \qquad (1)$$

where

$q_{x,t}$ is the rate of mortality at age x and time t

a_x is the average level of mortality at age x

b_x is the rate of change of the level of mortality at age x

k_t is the level of mortality at time t and

$\varepsilon_{x,t}$ is a random error with mean 0 and variance σ^2.

Note that the original Lee-Carter model was defined with the natural logarithm of the central rate of mortality; relations and approximations relating the central rate of mortality $m_{x,t}$ to the probability of death $q_{x,t}$ yield negligible approximation errors. However, the difference between $q_{x,t}$ and the force of mortality which is very small when $q_{x,t}$ is small, grows larger as $q_{x,t}$ increases.

The uniqueness of solutions imposes two constraints:

1) $\sum b_x = 1$;

2) $\sum k_t = 0$.

Let us adopt the following notation. The right upper index will indicate the number of the estimation (since each parameter is estimated, then reestimated). So, separately for males and females, the Lee-Carter method goes through the following steps:

1- A first estimate of the average level of mortality according to age is calculated with the average over time for the model,

$$\hat{a}_x^1 = \frac{\sum_t \ln(q_{x,t})}{T}$$

where T is the total number of reference years.

2- Singular value decomposition of matrix $ln(q_{x,t})-\hat{a}_x^1$ gives preliminary estimates for parameters b_x and k_t, which we denote b_x^{SVD} and k_t^{SVD}.

3- With the uniqueness conditions, we obtain

$$\hat{b}_x = \frac{\hat{b}_x^{SVD}}{\sum_x \hat{b}_x^{SVD}} \quad ; \quad \hat{k}_t^1 = \hat{k}_t^{SVD} \sum_x \hat{b}_x^{SVD}$$

4- We obtain the second estimate of k_t, that is $\hat{k}^2_t$, by requiring equality between the expected and observed numbers of deaths,

$$D_t = \sum N_{x,t}\, q_{x,t},$$

where $N_{x,t}$ is the population aged x at time t and D_t is the total number of deaths in year t.

5- Finally, we obtain the parameters

$$\hat{k}_t = \hat{k}^2_t - \text{Average}(\hat{k}^2_t) \text{ and } \hat{a}_x = \hat{a}_x^1 + b_x \text{ Average}(\hat{k}^2_t).$$

6- After this, with the Box-Jenkins time series method, we project the series of the level of mortality at time *t*. The extent of the projection period was chosen as 30 years.

7- We have projected the q_x values for the years 2015, 2025 and 2035 and we have compared our modeling and projections with the observed values for years 1976, 1986, 1996 and 2005.

This method is applied analogously for the other two models, the logistic model of Lee-Carter (Lee, 2000)

$$\text{logit}(q_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}, \qquad (2)$$

and the model we propose

$$\text{logit}(\mu_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \qquad (3)$$

where we used the approximation $\mu_{x+0.5} = -\ln p_x$.

4. Canadian data available at advanced ages

Analyzing the quality of data provided by Statistics Canada at old ages, Bourbeau and Lebel (2000) have concluded that between ages 80 and 99, the quality of data of Statistics Canada is good. Over age 100, they found many cases of overestimation of age at death or age declared during the census.

The *Human Mortality Database* (HMD) is a database created by the Max Planck Institute for Demographic Research in Rostock and the Department of Demography of the University of California at Berkeley. It contains comparable data from 34 countries including Canada (*Canadian Longevity Database* or *Base de données sur la longévité canadienne*, BDLC). Statistics Canada and BDLC present similar, but not totally identical data. For example, Statistics Canada presents data for ages 0 to 99 by single year and then for the open interval 100+, while BDLC presents data for ages 0 to 109 and then the age group 110+. We have used the BDLC data for Canada, which are more easily accessible, specifically the national tables entitled Deaths, Population, Tables - Males and Tables – Females.

In Canada, a new mortality tendency has been observed since the beginning of the seventies (Lee and Nault, 1993), when lower mortality rates have been observed among men and women. Technological and medical advances of this period as well as the establishment of an universal system of medical and hospital insurance are responsible for this. With the Box-Jenkins time series method, a reference period of less than 30 years is not very reliable. For these reasons, we have kept 30 years of data as reference and only data of years 1976 to 2005 were used in the models. After many trials, the following ages were selected for our models: males aged 70 to 99 and females aged 80 to 105.

So, for the three models used in Section 5, the data come from the BDLC, as corrected by Bourbeau et al. (2003). The last check of our data goes back to February 2008.

5. Projected mortality rates with three models

In this section, we will present and analyze the results of our modeling and projections until 2035 for males and females. To avoid any ambiguity, the three models compared, the Lee-Carter model, the logistic model of Lee-Carter and Doray and Tang's model, appear in Table 1. We will first look at the parameters of the models, then the modeling errors, and finally the projection results.

5.1 Modeling results

The average level of mortality at age x, a_x, does not change with time and represents the basic level of observed mortality. This parameter is valid over the modeling and projection periods, that is between 1976 and 2035 (Table 2).

Figure 3.1 shows that for men up to age 85, the average levels of mortality with age are more or less similar for the 3 models. Beyond age 85, these levels diverge from each other: they become lower for Model 1 and higher for Model 3. For Model 2, at these high ages, the average levels of mortality with age are between those of Models 1 and 3. For women, the curves of the basic level of mortality look like those of men (Figure 3.2). However, the differentiation mentioned previously appears a bit earlier, around age 80.

This parameter alone does not reveal much. However, as expected, for the same ages, the basic level of mortality is lower among women than men.

The rate of change of mortality at age x, b_x, also does not change with time. It indicates which rates decrease more slowly with respect to others, in response to changes in the level of mortality at time t (k_t). It is valid during the modeling period (1976-2005) and the projection period (2006-2035). It is defined as

$$\frac{d\ln(q_{x,t})}{dt} = b_x \frac{dk_t}{dt}.$$

Table 3 contains the results. For men, Figure 3.3 shows that the rate of change in mortality at each age decreases in a constant way. Moreover, the 3 models yield similar numbers. Figure 3.4 shows this rate of change for women. The decrease is more erratic than the one observed in men. At high ages, according to the model considered, this rate differs. Beyond age 90, b_x becomes negative and seems higher for Model 3 and lower for Model 1. Model 2 presents a rate between these other two models.

In summary, for both sexes, there is no constancy of the rate of change of mortality across ages, in conformity with the observations. However, for the same ages, let us mention that this parameter differs a lot with sex: it is higher for women and lower for men.

The level of mortality at time t, k_t, (Table 4) does not change with age. This parameter represents the change in mortality with time. It is the only parameter that will be projected. For both men and women, it does not vary much with the models: Figures 3.5 and 3.6 present decreasing and superimposed curves. However, by comparing this parameter between sexes, we observe that at advances ages, the change in mortality over time is lower among women, in accordance with observations.

5.2 Modeling errors

Table 5 gives the average modeling errors for each model. It reveals, for both men and women, that Model 3, which we have proposed, produce the smallest errors for the period 1976-2005. Indeed, for men, the average absolute relative errors vary between 11% and 23% for Model 1, between 5% and 7% for Model 2 and between 2% and 5% for Model 3. For women, these errors vary between 14% and 16% for Model 1, between 5% and 8% for Model 2 and between 1% and 6% for Model 3.

Finally, let us discuss briefly the error terms of the 3 modelings. For a given year (across the ages), the error terms are not randomly distributed: the errors decrease as ages increase. This is however not true when the ages are fixed and we study the error terms across time, as shown in Table 5.

5.3 Projection Results

The projection of the level of mortality at time *t* is done with the Box-Jenkins time series method (Table 6). For the three models, for males, we concluded that an *ARIMA(1, 2, 1)* model was the best possible choice, while for women, an *ARIMA(1, 1, 1)* seemed to be the choice.

Figure 3.7 shows, both for men and women, that projections over the years 2006-2035 follow the decreasing trendency observed in the modeling period 1976-2005.. However, by comparing both sexes, we observe that, at advanced ages, the level of mortality at time *t,* is much lower in women.

The following Tables and Figures show the modelings and projections of the rate of mortality for the various models for men (Table 7 and Figures 3.8 to 3.11) and women (Table 8 and Figures 3.12 to 3.14). The projections follow the tendency of the last 30 years, in accordance with the modeling. However, Figures 3.11 and 3.14 show an increasing tendency for the rates of mortality, a phenomenon which does not appear in other Figures. As explained by one of the reviewers, one reason for this might be the fact that, because of medical developments, more persons reach the age of 100. In the past, you needed to be much stronger yhan nowadays to reach that age. Compared to the past, the effect of medical developments results in an on average less healthy population at higher ages.

Consistent with the modelings, we observe, both for men and women, that Model 3 predicts the lowest rates of mortality, for all ages and all years. Moreover, we note that the phenomenon of convergence between the rates of mortality of men and women does appear in our numbers.

6- Conclusion

In this paper, we have explained the importance of a good projection of mortality at advanced ages. Also, we have presented the principal models for projecting mortality at advanced ages (Gompertz, Heligman-Pollard, Lee-Carter, Weibull, and Perks and Kannisto's logistic models).

Our study was concentrated on mortality at advanced ages in Canada for separate sexes. Consequently, we used the BDLC data (Canadian Longevity Database). For historical and practical reasons, we restricted our modeling to data of years 1976 to 2005. Ages were also restricted, 80 to 105 for women and 70 to 99 for men.

Our principal objective was to compare three models: the original Lee-Carter model, the logistic model of Lee-Carter and a logistic model we have proposed for the force of mortality, inspired by a linear reparametrisation of Kannisto's model (Doray, 2008).

The results we have obtained are rather interesting. By following the Lee-Carter method, we obtained the estimates of the parameters of the 3 models, in order to establish their performance in modeling mortality at advanced ages. Since they all have the same number of parameters, the best fitting model should be the one which best projects mortality. Our results were conclusive: it was Model 3. This model yielded the smallest mean relative absolute errors, between 1% and 6% for women and between 2% and 5% for men.

The next step would be to compare the 3 models by modeling and projecting on a provincial and territorial basis and try to find a way, if possible, to correct the divergence problems, similarly to what Lee and Li (2005) have done.

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Table 1 : Model Identification

Model 1 : Lee-Carter 1992	$ln(q_{x,t})$ =	$a_x + b_x\, k_t + \varepsilon'_{x,t}$
Model 2 : Lee-Carter 2000	$logit(q_{x,t})$ =	$a_x + b_x\, k_t + \varepsilon'_{x,t}$
Model 3 : Doray and Tang	$logit(\mu_{x,t})$ =	$a_x + b_x\, k_t + \varepsilon'_{x,t}$

Table 2 : Average level of mortality with age, sex and model, Canada, 1976-2035

Men				Women			
age	Model 1	Model 2	Model 3	age	Model 1	Model 2	Model 3
70	-3.18204352	-3.28621335	-3.34798480	*70*	-	-	-
71	-3.09994952	-3.20401163	-3.26619025	*71*	-	-	-
72	-3.01211363	-3.10639781	-3.16349007	*72*	-	-	-
73	-2.92795170	-3.01400630	-3.06695287	*73*	-	-	-
74	-2.84909954	-2.92368320	-2.97058693	*74*	-	-	-
75	-2.76511384	-2.83188699	-2.87488945	*75*	-	-	-
76	-2.69251090	-2.74427428	-2.77904223	*76*	-	-	-
77	-2.60984435	-2.65359010	-2.68432985	*77*	-	-	-
78	-2.53237834	-2.56187247	-2.58486689	*78*	-	-	-
79	-2.43764964	-2.45946466	-2.47867963	*79*	-	-	-
80	-2.34984465	-2.35620163	-2.36689588	80	-2.68285313	-2.82212896	-2.90808643
81	-2.28004820	-2.27114133	-2.27320069	81	-2.61013183	-2.72632202	-2.79980805
82	-2.20448766	-2.17979448	-2.17294290	82	-2.49898175	-2.60596237	-2.67531693
83	-2.12992218	-2.08659285	-2.06883178	83	-2.42451699	-2.50764020	-2.56376644
84	-2.04441816	-1.98726338	-1.96158795	84	-2.30064416	-2.37581928	-2.42895382
85	-1.98786549	-1.90549822	-1.86407107	85	-2.23057711	-2.28030206	-2.31915593
86	-1.91010244	-1.81131377	-1.75981165	86	-2.10688355	-2.15100462	-2.18836502
87	-1.83997317	-1.72004886	-1.65499098	87	-2.07258864	-2.06616628	-2.07289442
88	-1.76856245	-1.62881939	-1.55060379	88	-2.00765884	-1.97088585	-1.95929172
89	-1.70781454	-1.54677561	-1.45398422	89	-1.91531557	-1.85578625	-1.83080868
90	-1.62823928	-1.44926762	-1.34394308	90	-1.81411188	-1.73755991	-1.70263810
91	-1.56338634	-1.35568830	-1.22907071	91	-1.74328285	-1.63447816	-1.57875691
92	-1.49464389	-1.26142562	-1.11510407	92	-1.67944975	-1.54007716	-1.46344834
93	-1.42293499	-1.16851871	-1.00499108	93	-1.58783294	-1.42574068	-1.33380891
94	-1.36893652	-1.08795424	-0.90197701	94	-1.52458925	-1.33474938	-1.22257004
95	-1.31904734	-1.01131981	-0.80141663	95	-1.47090713	-1.24346100	-1.10218756
96	-1.26406472	-0.92970785	-0.69459324	96	-1.40569526	-1.14690428	-0.98008726
97	-1.21119492	-0.84970276	-0.58734459	97	-1.34281080	-1.05208705	-0.85763837
98	-1.16048632	-0.77143503	-0.47964220	98	-1.28229674	-0.95913704	-0.73476038
99	-1.11193811	-0.69497966	-0.37137763	99	-1.22418125	-0.86819017	-0.61137828
100	-	-	-	100	-1.16850076	-0.77938682	-0.48739310
101	-	-	-	101	-1.11528396	-0.69286555	-0.36268582
102	-	-	-	102	-1.06452870	-0.60875775	-0.23713266
103	-	-	-	103	-1.01626053	-0.52720540	-0.11057787
104	-	-	-	104	-0.97048443	-0.44835016	0.01713162
105	-	-	-	105	-0.92716795	-0.37228938	0.14622521

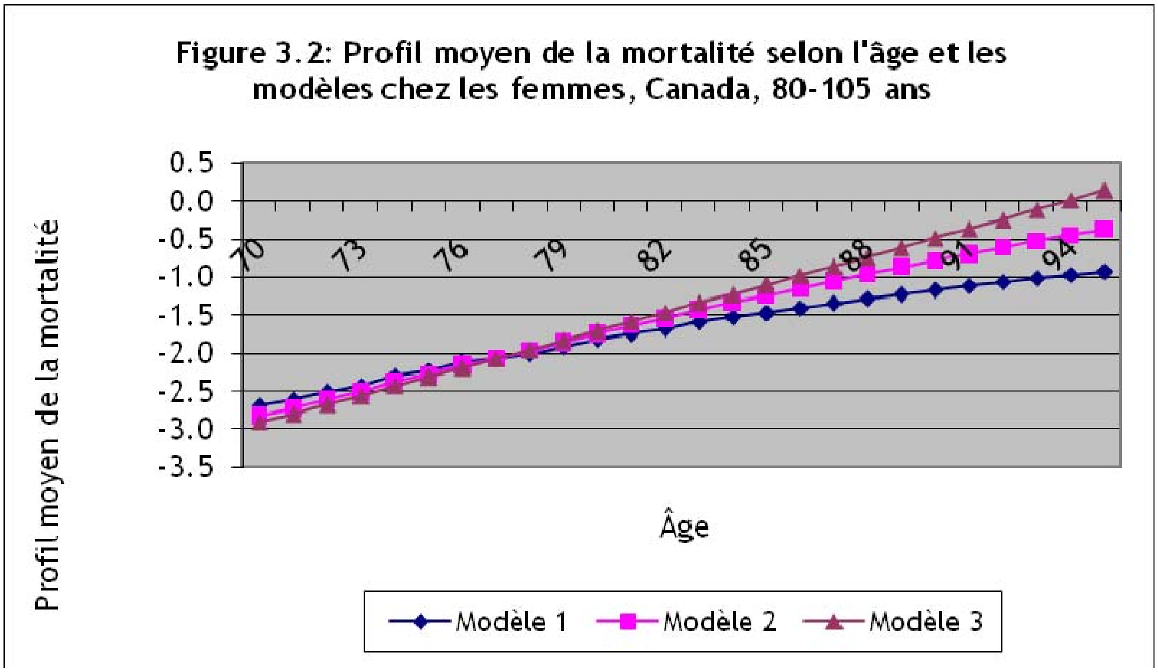


Table 3 : Rate of change of the mortality rate with age, sex and model, Canada, 1976-2035

Men				Women			
age	*Model1*	*Model 2*	*Model 3*	*age*	*Model 1*	*Model 2*	*Model 3*
70	0.07210403	0.06890087	0.06708072	70	-	-	-
71	0.07364519	0.07057112	0.06881078	71	-	-	-
72	0.07067324	0.06794568	0.06637011	72	-	-	-
73	0.06859812	0.06621208	0.06481716	73	-	-	-
74	0.06498966	0.06299572	0.06181034	74	-	-	-
75	0.06351102	0.06184477	0.06083741	75	-	-	-
76	0.05839836	0.05711693	0.05632533	76	-	-	-
77	0.05724180	0.05629042	0.05568006	77	-	-	-
78	0.05317639	0.05262208	0.05223995	78	-	-	-
79	0.05318571	0.05298184	0.05280129	79	-	-	-
80	0.04962112	0.04981081	0.04986796	80	0.11679365	0.11609824	0.12962301
81	0.04571715	0.04622303	0.04647334	81	0.10646365	0.10643680	0.11905998
82	0.04222711	0.04310457	0.04359209	82	0.10560140	0.10626795	0.11920961
83	0.03773650	0.03890163	0.03958120	83	0.09558087	0.09675447	0.10877402
84	0.03654737	0.03809266	0.03903154	84	0.09708688	0.09915176	0.11139111
85	0.02884471	0.03039679	0.03136613	85	0.08700481	0.08954897	0.10063427
86	0.02695992	0.02878421	0.02996127	86	0.09116394	0.09481379	0.10677859
87	0.02299737	0.02492884	0.02621587	87	0.06675536	0.07025309	0.07941956
88	0.02021345	0.02220168	0.02356413	88	0.05534959	0.05881441	0.06595362
89	0.01646125	0.01838041	0.01973760	89	0.05068624	0.05470418	0.06191195
90	0.01640892	0.01862241	0.02024864	90	0.05053625	0.05536356	0.06350186
91	0.01121704	0.01301513	0.01438885	91	0.04100080	0.04564489	0.05192641
92	0.00840677	0.00998806	0.01125019	92	0.03234847	0.03653749	0.04202905
93	0.00840130	0.01012837	0.01156463	93	0.03142079	0.03610946	0.04093458
94	0.00504955	0.00634316	0.00748746	94	0.02574822	0.02999839	0.03346290
95	0.00141127	0.00185783	0.00225268	95	0.01517380	0.01793892	0.01896466
96	-0.00071640	-0.00082957	-0.00095761	96	0.00989592	0.01180430	0.01114627
97	-0.00265628	-0.00340344	-0.00419014	97	0.00501254	0.00587297	0.00328339
98	-0.00440631	-0.00585483	-0.00745348	98	0.00054480	0.00017253	-0.00463990
99	-0.00596533	-0.00817327	-0.01075550	99	-0.00348456	-0.00525870	-0.01262770
100	-	-	-	100	-0.00706764	-0.01039369	-0.02070626
101	-	-	-	101	-0.01020391	-0.01520644	-0.02890661
102	-	-	-	102	-0.01288864	-0.01965663	-0.03724242
103	-	-	-	103	-0.01513995	-0.02372898	-0.04577019
104	-	-	-	104	-0.01697398	-0.02739921	-0.05453228
105	-	-	-	105	-0.01840931	-0.03064254	-0.06357948

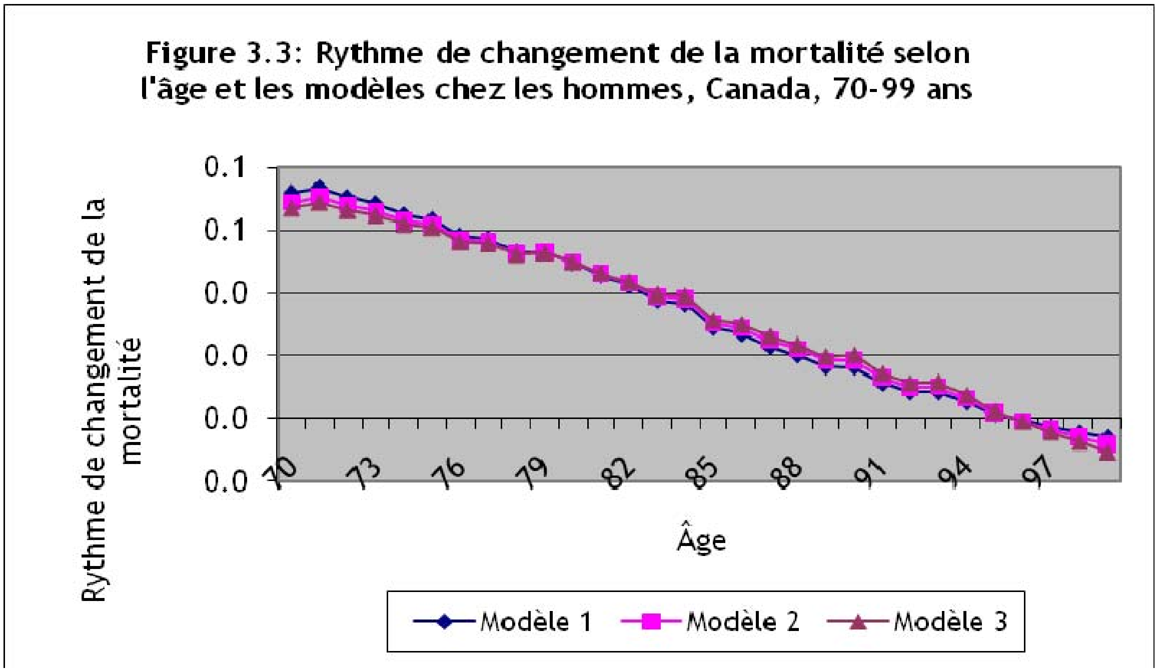


Figure 3.3: Rythme de changement de la mortalité selon l'âge et les modèles chez les hommes, Canada, 70-99 ans
Rythme de changement de la mortalité
0.1
0.1
0.0
0.0
0.0
0.0
70
73
76
79
82
85
88
91
94
97
Âge
Modèle 1
Modèle 2
Modèle 3

Table 4 : Level of mortality at time *t* with sex and model, Canada
Men aged 70-99 and Women aged 80-105

Men				*Women*			
year	*Model 1*	*Model 2*	*Model 3*	*year*	*Model 1*	*Model 2*	*Model 3*
1976	3,88639709	4,05277458	4,15219965	1976	2,10101073	2,29260980	2,20443316
1977	3,08214516	3,22147394	3,30342473	1977	1,46895410	1,63963291	1,60009796
1978	2,80790686	2,93454718	3,00643774	1978	1,17242414	1,31618924	1,29093220
1979	2,24891058	2,35564595	2,41435598	1979	0,71867233	0,83745826	0,84062370
1980	2,56230167	2,68890120	2,75841973	1980	0,92727254	1,01451811	0,98217089
1981	2,02793548	2,14111265	2,20283933	1981	0,48842658	0,56032392	0,55866325
1982	2,26937817	2,40061245	2,47308538	1982	0,77010512	0,81841024	0,77918204
1983	1,83121644	1,95326041	2,02134751	1983	0,36962653	0,39709061	0,38203760
1984	1,56637357	1,68386555	1,75002045	1984	0,23191346	0,24180937	0,22876217
1985	1,83844729	1,97325656	2,05079021	1985	0,43498194	0,44002995	0,40488683
1986	1,50489732	1,63267488	1,70811268	1986	0,61428450	0,61608550	0,56198960
1987	1,08563939	1,18901836	1,25043770	1987	0,29317147	0,28475714	0,25297404
1988	1,45111270	1,56394465	1,63316645	1988	0,39201985	0,38289173	0,34163717
1989	0,84408954	0,92535190	0,97745927	1989	0,03367894	0,01908828	0,00580633
1990	0,31581745	0,36828882	0,40296912	1990	-0,07561404	-0,09771536	-0,10555413
1991	0,16402720	0,21198563	0,24497182	1991	-0,20801061	-0,23286518	-0,23100251
1992	-0,24428977	-0,21319935	-0,19029964	1992	-0,47671243	-0,51150982	-0,49099919
1993	0,00519922	0,04329495	0,07086017	1993	-0,09032436	-0,11570677	-0,12410362
1994	-0,38461008	-0,37114273	-0,35846283	1994	-0,14715241	-0,17147978	-0,17371049
1995	-0,46839225	-0,46776572	-0,46288146	1995	-0,16793118	-0,18673184	-0,18376207
1996	-0,75461969	-0,77850844	-0,78884034	1996	-0,24636651	-0,27327605	-0,26702294
1997	-0,96374853	-1,00947297	-1,03326328	1997	-0,12746842	-0,16027277	-0,16642327
1998	-1,08356291	-1,14527854	-1,17876150	1998	-0,27700924	-0,32465452	-0,32406915
1999	-1,49641299	-1,58953097	-1,64276708	1999	-0,42775314	-0,48618341	-0,47653579
2000	-2,60456586	-2,75725974	-2,84602562	2000	-0,78810859	-0,85524029	-0,81821522
2001	-3,25054125	-3,45025643	-3,56785999	2001	-0,98315477	-1,05609564	-1,00593089
2002	-3,68891079	-3,92999761	-4,07344491	2002	-1,06952751	-1,14634806	-1,09135864
2003	-4,07137234	-4,35641314	-4,52733531	2003	-1,38078668	-1,46826901	-1,39233983
2004	-5,00146743	-5,36410487	-5,58326232	2004	-1,66692270	-1,77000254	-1,67851602
2005	-5,47930124	-5,90707918	-6,16769367	2005	-1,88369964	-2,00454403	-1,90465318

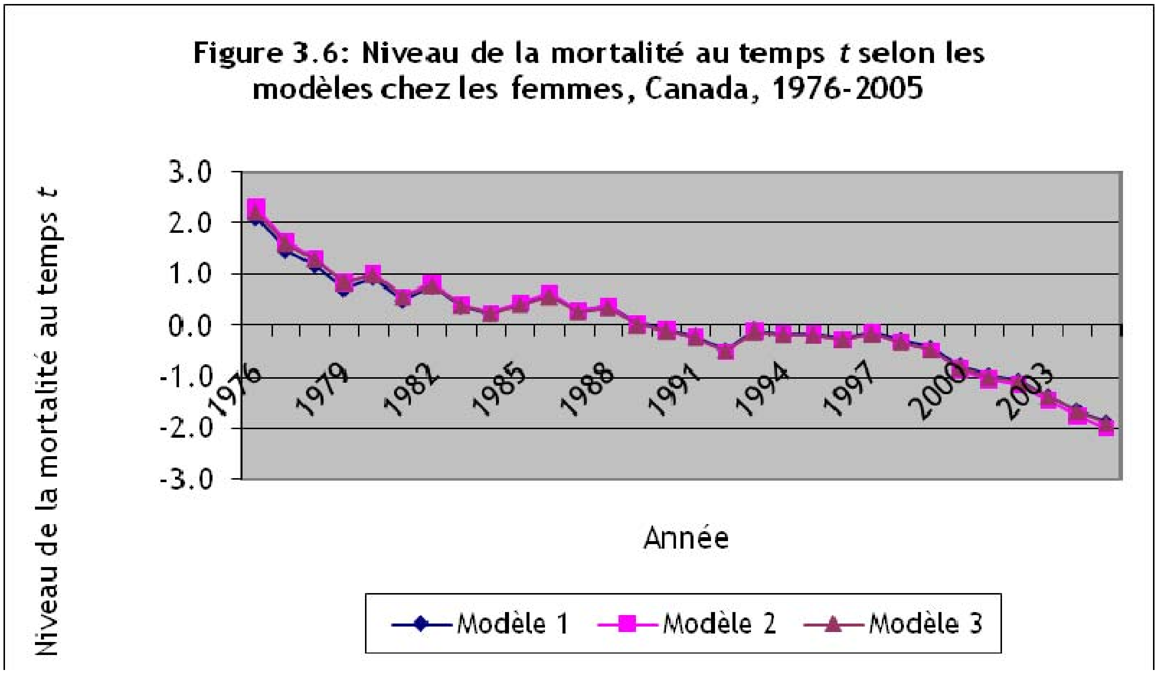


Table 5 : Modeling errors with sex and model, Canada, 1976-2005

Men	Women

Type of error	*Year*	*Model 1*	*Model 2*	*Model 3*	*Model 1*	*Model 2*	*Model 3*
Mean relative Error (%)	1976	-11.5	-3.3	0.7	-8.9	-1.1	2.7
	1986	-22.4	-5.2	-1.5	-13.2	-5.2	-1.4
	1996	-11.0	-3.8	-0.1	-12.1	-4.2	-0.4
	2005	-6.4	0.7	4.3	-13.5	-5.7	-2.0
Mean absolute Relative error (%)	1976	14.2	6.2	3.1	16.0	8.2	6.1
	1986	22.4	5.2	2.1	14.3	5.4	1.6
	1996	12.7	6.0	2.8	14.8	6.0	1.9
	2005	11.0	4.8	4.4	14.3	5.7	2.2
Mean error	1976	0.006433	-0.000562	-0.003984	-0.000704	-0.008033	-0.012249
	1986	0.011127	0.005374	0.002495	0.012033	0.005613	0.002405
	1996	0.005073	-0.000001	-0.002569	0.006861	0.001122	-0.001492
	2005	-0.002967	-0.007187	-0.009409	0.010683	0.005939	0.004318
Mean absolute Error	1976	0.014923	0.008080	0.005978	0.024829	0.016194	0.014863
	1986	0.011127	0.005502	0.003217	0.016062	0.006116	0.002662
	1996	0.010191	0.005911	0.004187	0.017073	0.007595	0.003353
	2005	0.012170	0.009245	0.009471	0.014031	0.006002	0.004503
Mean Quadratic Error	1976	0.000255	0.000105	0.000083	0.000783	0.000407	0.000371
	1986	0.000143	0.000037	0.000020	0.000322	0.000049	0.000008
	1996	0.000124	0.000045	0.000034	0.000349	0.000073	0.000017
	2005	0.000329	0.000281	0.000276	0.000240	0.000046	0.000031

Table 6 : Level of mortality at time *t* (modeled 1976 to 2005 and projected 2006 à 2035) with sex and model, Canada

	Men			*Women*		
year	*Model 1*	*Model 2*	*Model 3*	*Model 1*	*Model 2*	*Model 3*
1976	3.8864	4.0528	4.1522	2.1010	2.2926	2.2044
1977	3.0821	3.2215	3.3034	1.4690	1.6396	1.6001
1978	2.8079	2.9345	3.0064	1.1724	1.3162	1.2909
1979	2.2489	2.3556	2.4144	0.7187	0.8375	0.8406

1980	2.5623	2.6889	2.7584	0.9273	1.0145	0.9822
1981	2.0279	2.1411	2.2028	0.4884	0.5603	0.5587
1982	2.2694	2.4006	2.4731	0.7701	0.8184	0.7792
1983	1.8312	1.9533	2.0213	0.3696	0.3971	0.3820
1984	1.5664	1.6839	1.7500	0.2319	0.2418	0.2288
1985	1.8384	1.9733	2.0508	0.4350	0.4400	0.4049
1986	1.5049	1.6327	1.7081	0.6143	0.6161	0.5620
1987	1.0856	1.1890	1.2504	0.2932	0.2848	0.2530
1988	1.4511	1.5639	1.6332	0.3920	0.3829	0.3416
1989	0.8441	0.9254	0.9775	0.0337	0.0191	0.0058
1990	0.3158	0.3683	0.4030	-0.0756	-0.0977	-0.1056
1991	0.1640	0.2120	0.2450	-0.2080	-0.2329	-0.2310
1992	-0.2443	-0.2132	-0.1903	-0.4767	-0.5115	-0.4910
1993	0.0052	0.0433	0.0709	-0.0903	-0.1157	-0.1241
1994	-0.3846	-0.3711	-0.3585	-0.1472	-0.1715	-0.1737
1995	-0.4684	-0.4678	-0.4629	-0.1679	-0.1867	-0.1838
1996	-0.7546	-0.7785	-0.7888	-0.2464	-0.2733	-0.2670
1997	-0.9637	-1.0095	-1.0333	-0.1275	-0.1603	-0.1664
1998	-1.0836	-1.1453	-1.1788	-0.2770	-0.3247	-0.3241
1999	-1.4964	-1.5895	-1.6428	-0.4278	-0.4862	-0.4765
2000	-2.6046	-2.7573	-2.8460	-0.7881	-0.8552	-0.8182
2001	-3.2505	-3.4503	-3.5679	-0.9832	-1.0561	-1.0059
2002	-3.6889	-3.9300	-4.0734	-1.0695	-1.1463	-1.0914
2003	-4.0714	-4.3564	-4.5273	-1.3808	-1.4683	-1.3923
2004	-5.0015	-5.3641	-5.5833	-1.6669	-1.7700	-1.6785
2005	-5.4793	-5.9071	-6.1677	-1.8837	-2.0045	-1.9047
2006	-6.0996	-6.5885	-6.8876	-2.0061	-2.1384	-2.0336
2007	-6.6725	-7.2233	-7.5617	-2.1284	-2.2723	-2.1623
2008	-7.2611	-7.8738	-8.2513	-2.2507	-2.4061	-2.2910
2009	-7.8445	-8.5191	-8.9357	-2.3730	-2.5399	-2.4194
2010	-8.4297	-9.1661	-9.6218	-2.4952	-2.6737	-2.5478
2011	-9.0142	-9.8125	-10.3074	-2.6175	-2.8074	-2.6760
2012	-9.5990	-10.4591	-10.9931	-2.7397	-2.9412	-2.8040
2013	-10.1837	-11.1056	-11.6788	-2.8618	-3.0748	-2.9319
2014	-10.7684	-11.7522	-12.3645	-2.9840	-3.2085	-3.0597
2015	-11.3531	-12.3988	-13.0502	-3.1061	-3.3421	-3.1873
2016	-11.9378	-13.0453	-13.7359	-3.2282	-3.4758	-3.3148
2017	-12.5225	-13.6919	-14.4215	-3.3503	-3.6093	-3.4421
2018	-13.1072	-14.3385	-15.1072	-3.4723	-3.7429	-3.5693
2019	-13.6919	-14.9850	-15.7929	-3.5944	-3.8764	-3.6964
2020	-14.2767	-15.6316	-16.4786	-3.7164	-4.0099	-3.8233
2021	-14.8614	-16.2781	-17.1643	-3.8383	-4.1434	-3.9501
2022	-15.4461	-16.9247	-17.8500	-3.9603	-4.2768	-4.0767
2023	-16.0308	-17.5713	-18.5357	-4.0822	-4.4102	-4.2032
2024	-16.6155	-18.2178	-19.2214	-4.2041	-4.5436	-4.3295
2025	-17.2002	-18.8644	-19.9071	-4.3260	-4.6770	-4.4557
2026	-17.7849	-19.5109	-20.5927	-4.4478	-4.8103	-4.5818
2027	-18.3696	-20.1575	-21.2784	-4.5696	-4.9436	-4.7077
2028	-18.9543	-20.8041	-21.9641	-4.6914	-5.0769	-4.8335
2029	-19.5390	-21.4506	-22.6498	-4.8132	-5.2101	-4.9591

2030	-20.1238	-22.0972	-23.3355	-4.9349	-5.3434	-5.0846
2031	-20.7085	-22.7437	-24.0212	-5.0567	-5.4765	-5.2100
2032	-21.2932	-23.3903	-24.7069	-5.1783	-5.6097	-5.3352
2033	-21.8779	-24.0369	-25.3926	-5.3000	-5.7428	-5.4603
2034	-22.4626	-24.6834	-26.0783	-5.4217	-5.8760	-5.5852
2035	-23.0473	-25.3300	-26.7639	-5.5433	-6.0090	-5.7100

Table 7: Rates of mortality with age, year and model

Men Canada

		Model 1	*Model 2*	*Model 3*
age	*year*	$q_{x,t}$	$q_{x,t}$	$q_{x,t}$
70	1976	0.0549	0.0471	0.0434
	1986	0.0463	0.0402	0.0372
	1996	0.0393	0.0342	0.0318
	2005	0.0280	0.0243	0.0225
	2015	0.0183	0.0157	0.0143
	2025	0.0120	0.0101	0.0091
	2035	0.0079	0.0065	0.0058
80	1976	0.1157	0.1039	0.0982
	1986	0.1028	0.0932	0.0885
	1996	0.0919	0.0836	0.0794
	2005	0.0727	0.0660	0.0625
	2015	0.0543	0.0486	0.0456
	2025	0.0406	0.0357	0.0330
	2035	0.0304	0.0261	0.0238
90	1976	0.2092	0.2020	0.1983
	1986	0.2012	0.1948	0.1915
	1996	0.1939	0.1879	0.1847
	2005	0.1794	0.1738	0.1707
	2015	0.1629	0.1571	0.1537
	2025	0.1480	0.1418	0.1379
	2035	0.1345	0.1278	0.1234
99	1976	0.3214	0.3256	0.3280
	1986	0.3260	0.3300	0.3322
	1996	0.3304	0.3343	0.3365
	2005	0.3398	0.3437	0.3458
	2015	0.3520	0.3558	0.3576
	2025	0.3645	0.3680	0.3692
	2035	0.3774	0.3804	0.3807

Table 8: Rates of mortality with age, year and model

Women, Canada

		Model 1	*Model 2*	*Model 3*
age	t	$q_{x,t}$	$q_{x,t}$	$q_{x,t}$
80	1976	0.0874	0.0720	0.0655
	1986	0.0735	0.0601	0.0539
	1996	0.0664	0.0545	0.0488
	2005	0.0549	0.0450	0.0401
	2015	0.0476	0.0388	0.0342
	2025	0.0413	0.0334	0.0293
	2035	0.0358	0.0288	0.0251
90	1976	0.1812	0.1665	0.1591
	1986	0.1681	0.1540	0.1469
	1996	0.1610	0.1477	0.1409
	2005	0.1482	0.1360	0.1298
	2015	0.1393	0.1276	0.1215
	2025	0.1310	0.1196	0.1137
	2035	0.1232	0.1120	0.1064
100	1976	0.3063	0.3093	0.3091
	1986	0.3095	0.3131	0.3146
	1996	0.3114	0.3151	0.3174
	2005	0.3150	0.3190	0.3228
	2015	0.3177	0.3220	0.3271
	2025	0.3205	0.3250	0.3313
	2035	0.3233	0.3281	0.3355

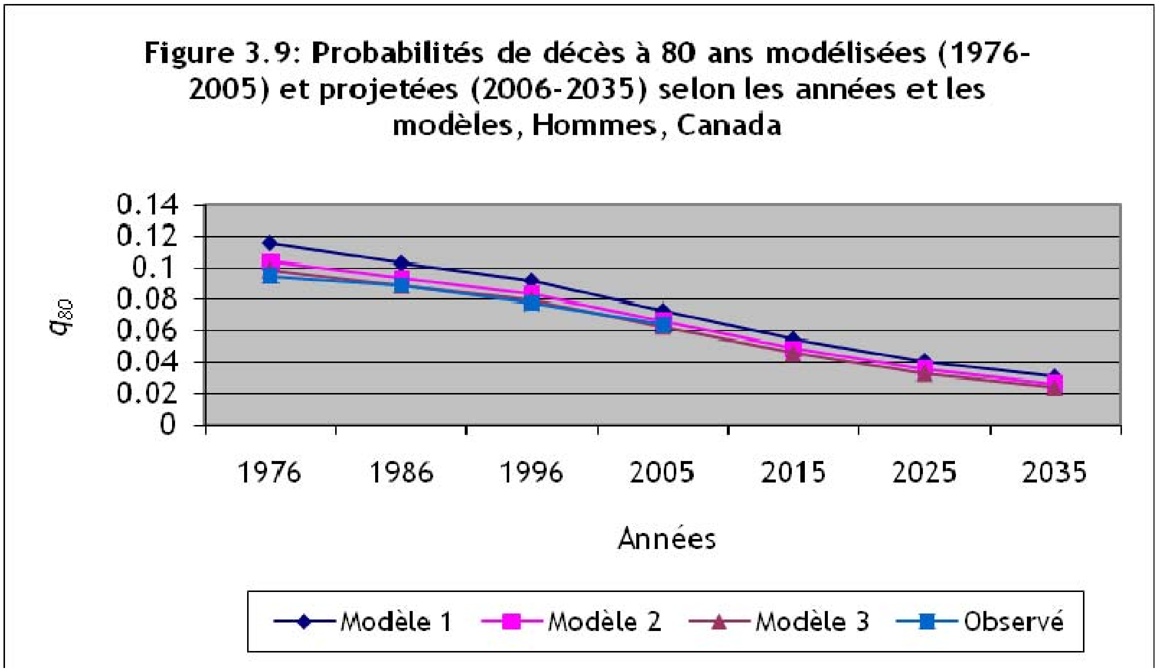
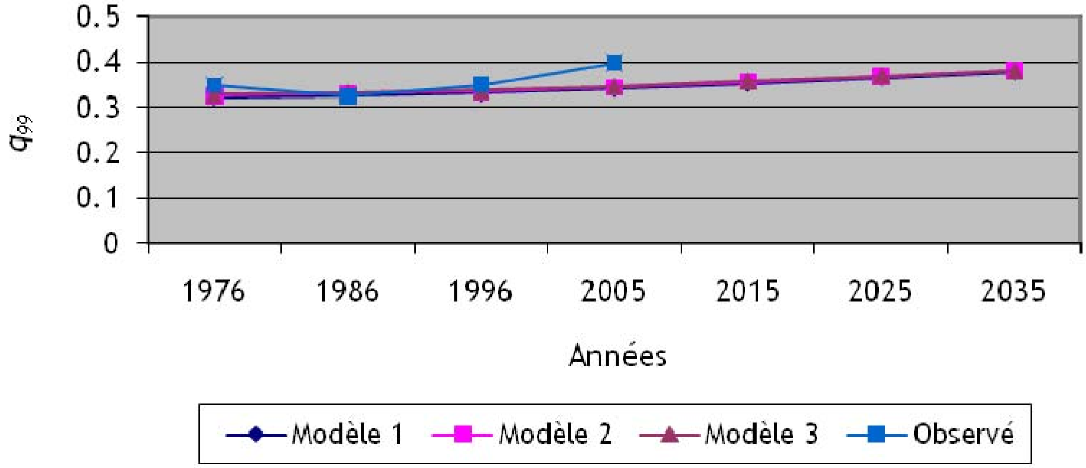


Figure 3.9: Probabilités de décès à 80 ans modélisées (1976-2005) et projetées (2006-2035) selon les années et les modèles, Hommes, Canada
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
q_{80}
1976
1986
1996
2005
2015
2025
2035
Années
Modèle 1
Modèle 2
Modèle 3
Observé

Figure 3.11: Probabilités de décès à 99 ans modélisées (1976-2005) et projetées (2006-2035) selon les années et les modèles, Hommes, Canada



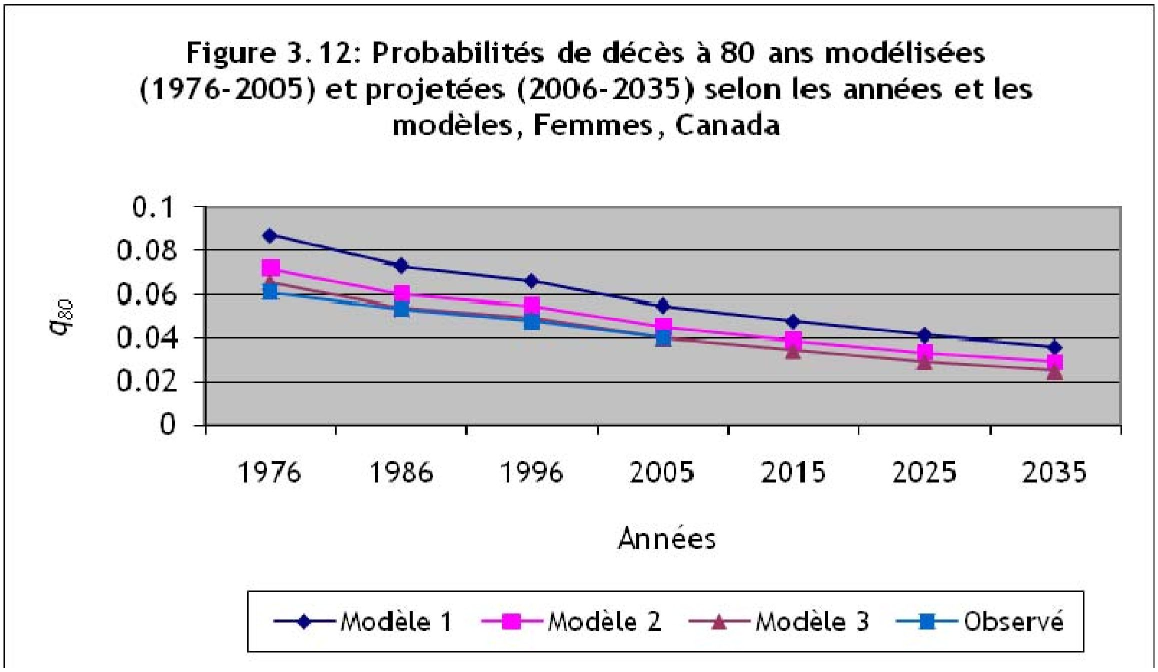


Figure 3.12: Probabilités de décès à 80 ans modélisées (1976-2005) et projetées (2006-2035) selon les années et les modèles, Femmes, Canada
0.1
0.08
0.06
0.04
0.02
0
q_{80}
1976
1986
1996
2005
2015
2025
2035
Années
Modèle 1
Modèle 2
Modèle 3
Observé

Figure 3.14: Probabilités de décès à 100 ans modélisées (1976-2005) et projetées (2006-2035) selon les années et les modèles, Femmes, Canada