Session 5B: Mortality Measurement
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Paper Presented:
"Mortality Experience Data"
By Paul Sweeting

In his paper Paul Sweeting used credibility techniques to measure mortality experience in a portfolio that is part of a larger group. This looks like a very good idea and can be applied in several cases. While the idea is good, I think further studies are needed, particularly in the case of experience based on insured populations. The model does not take into account the measurement of mortality in amounts. It is based on measurement in numbers. Particularly for products where the sum assured depends on income measuring in amounts and results in significant lower mortality rates than by mortality measurement based on numbers. Based on my own experience, I have seen 15 to 20 percent lower mortality rates utilizing measuring in amounts. I don't know yet how to include this measuring in amounts in the model. Does the model work with compound distributions? Also, grouping by age can lead to mistakes in case the amounts are age dependent. Therefore I advise a further study. This can be very useful for both actuarial science as well as actuarial practice.

Modeling the High Ages

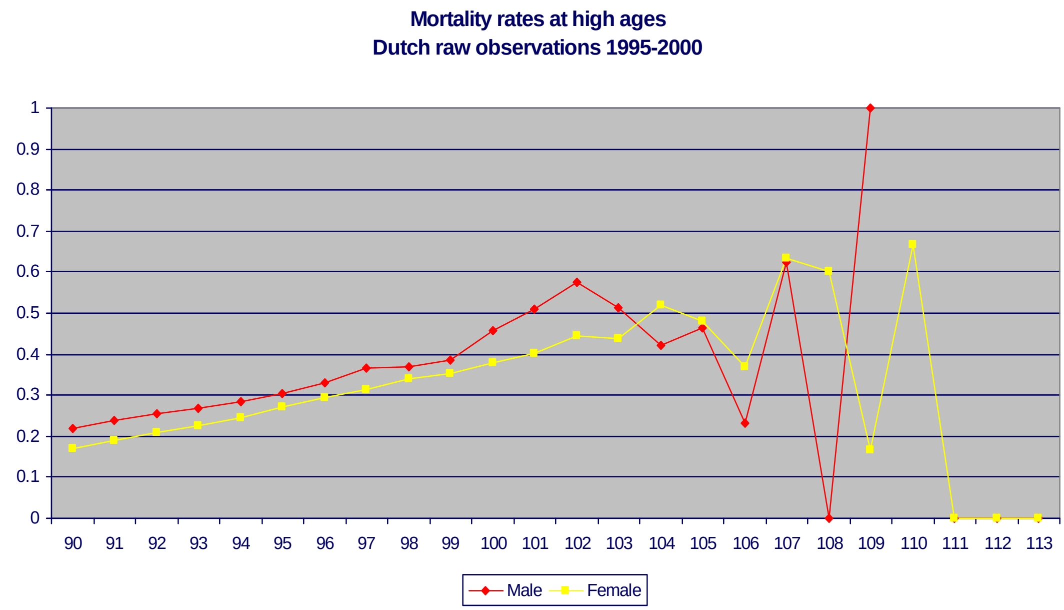
On the other two papers—one by Bob Howard and the other by Natalia Gavrilova and Leonid Gavrilov—I have no comments.

Modeling the high ages of mortality tables is getting more and more important now as more people reach ages above 100. I would like to show a model I made for the Netherlands to extend the population mortality table 1995-2000 and also the mortality tables that came after that. It is based on the Gompertz law, an old technique that looks to be in a revival. Further, I used ideas learned at earlier Living to 100 and Beyond conferences such as:

- Unclear if there is an increase of omega
- At least no exponential increase of the q(x) over the high ages
- Kind of maximum q(x) at the very high age of 0.5-0.6.
- Minimal difference between male and female high age mortality
- Minimal difference between countries for high age mortality

Why do we need a separate model for the high ages?

First, the raw results at the high ages are very volatile.



In the graph above, it can be seen that the raw mortality rate at high ages are sometimes based on only one or two observations. Therefore, most smoothing models cannot be used. Still, information of the life expectancy around age 100 is still rather stable.

This is an overview of the e(100) for the Netherlands.

Period	Male	Female
1971-1975	1.78	1.92
1976-1980	1.91	2.18
1981-1985	1.96	2.09
1986-1990	1.96	2.12
1991-1995	1.79	1.92
1996-2000	1.55	1.93

In this table, we can also see there is not a clear development over time: no increase. Also there is hardly any difference between male and female.

The model I developed for the Netherlands uses the life expectancy and the last reliable q(x) to model high ages. Gomperz law was applied from a certain age x_0 and above.

Define: $fn(x) = -\ln\{1 - q(x)\}$
Then: $fn(x+1) = fn(x) \times \alpha$
So: $fn(x_0 + t) = fn(x_0) \times \alpha^t$

With x_0 being the age where the smoothed table is still reliable and with some algebra, the formula above can be translated into (with $x>x_0$):

$$q(x) = 1 - e^{e^{\alpha(x-x_0)\ln\{1-q(x_0)\}}}$$

The life expectancy at x_0 follows:

$$\bar{e}(x_0) = 0.5 + \sum_{x_t = x_0} \prod_{x=x_0}^{x_t} e^{e^{\alpha(x-x_0)\ln\{1-q(x_0)\}}}$$

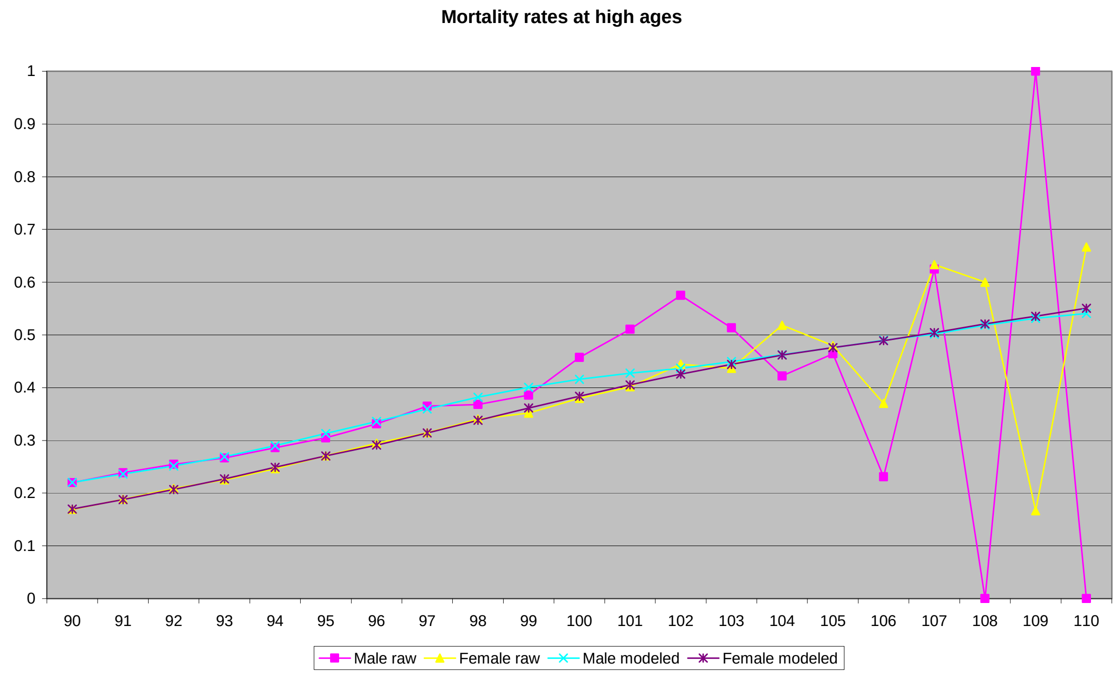
In this formula, only the α needs to be estimated for some x_0; the rest is known.

The x_0 can be found by optimizing (when the smoothed table is close to the raw one, using square differences).

In case the life expectancy is not known, it is safe to use the life expectancy of another similar country where the e(x) is reliable.

The good thing in this model is that only objective data are used and no subjective assumptions and the observed life expectancy is not changed.

The results for the Netherlands look like:



This result also fits the conclusions of earlier Living to 100 and above conferences.

In all tables after the 1995-2000 table, this model is used in combination with the smoothing method "Van Broekhoven algorithm," as published in the North American Actuarial Journal (NAAJ) of 2002.