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## **A New Approach to Assess Model Risk in High Dimensions**

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# A New Approach to Assess Model Risk in High Dimensions

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## Abstract

A central problem for regulators and risk managers concerns the evaluation of the risk of an aggregate portfolio  $S = \sum_{i=1}^d X_i$ . This problem is mainly a numerical issue once the joint distribution of  $(X_1, X_2, \dots, X_d)$  is completely specified. Unfortunately, while the marginal distributions of the risks  $X_i$  are often known, their interaction (dependence) is usually either unknown or only partially known, implying that any computed risk measure of  $S$  is subject to model uncertainty.

Previous academic research has mainly focused on the maximum and the minimum possible values of a given risk measure of  $S$ , in the case that only the marginal distributions are known. This approach leads to wide bounds as all information on the dependence is ignored.

In this paper we integrate in a natural way available information on the multivariate dependence and we provide easy-to-compute bounds on the risk measure at hand. We observe that incorporating the information of a well fitted multivariate model may, or may not, lead to much tighter bounds, a feature that also depends on the risk measure used. We illustrate this point by showing that the Value-at-Risk at a very high confidence level (as used in Basel II) is typically prone to very high model risk, even if one knows the multivariate distribution almost everywhere.

Our results allow to determine which risk measures can benefit from adding dependence information (i.e., leading to more narrow bounds), and thus for which the development of accurate multivariate models is a meaningful task.

*Key-words:* Model Risk, VaR, TVaR, variance, tail dependence, tail correlation.

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