A Study of Measuring the Mortality Compression

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Abstract

Mortality compression, a popular longevity risk research issue, means that age-at-death would concentrate on a narrower range; it is also related to the concept of rectangularization of the survival curve. In terms of statistical distribution, the mortality compression indicates age-at-death would degenerate to a certain age, or life expectancy has a limit and the variance of age-at-death distribution converges to zero. The convergence of life expectancy and the variance would shed light on longevity risk. Life expectancy is predicted to grow for the near future, but there is still no consensus on the convergence of the variance of age-at-death distribution.

In this study, we use statistical methods to evaluate mortality compression (or the convergence of variance) while considering data quality. Instead of applying the nonparametric methods, such as the shortest confidence interval for the distribution of age-at-death and of the modal age used in previous studies, we propose optimization methods for estimating the standard deviation of age-at-death distribution. Specifically, we compare the standard deviation of age-at-death above the mode, SD(M+), proposed by Kannisto (2000) and the method of nonlinear maximization (NM), and check which method has a smaller MSE (mean squared error). For the issue of data quality, we compare the estimation results of mortality rates from life table data and raw data.

We first use a computer simulation to evaluate the method. Then, based on data from the Human Mortality Database (HMD), we apply the NM method to both life table data (i.e., graduated mortality rates) and raw data, and check if there are significant differences. We found the proposed method can provide reliable estimates of life expectancy and its variance, even when age-at-death is recorded in integer.

Also, the estimates from the proposed method and raw data are smoother. However, there is not enough evidence to conclude if there is mortality compression based on the proposed NM method.

1. Introduction

Mortality improvement has become common in most countries since the end of World War II, and, according to the United Nations, the annual average increment of life expectancy is about 0.25 year. Human longevity is at its highest level in history and is likely to continue its pace in the near future. Together with the variance of age-at-death distribution, the mean change in life expectancy increases the difficulty of pricing annuity and health insurance products. This is known as longevity risk. Many countries, as well as private sectors companies, are introducing policies to reduce the burden of longevity risk. For example, many European countries are switching their pension systems from defined benefit to defined contribution (Whitehouse 2007), and individuals are asked to bear the longevity risk.

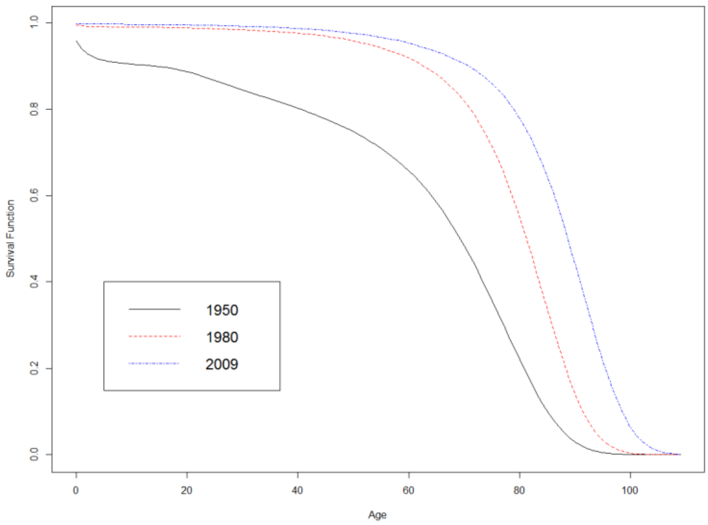

Figure 1. The survival curves of Japanese females

No matter how much individuals bear the burden, predicting life expectancy is still a key factor in dealing with longevity risk. Stochastic mortality models are a popular tool in making the prediction, and among these models, the Lee-Carter model (Lee and Carter 1992) is a popular choice. Most mortality models assume mortality improvement will continue, which means the age-specific mortality rates for all ages would converge to 0. This idea is very similar to mortality compression or rectangularization, indicating that the shape of the survival curve looks like a rectangle and almost all people die at a certain age. The survival curves of the Japanese female can be used to demonstrate mortality compression (figure 1) based on the data from the Human Mortality Database (HMD). The survival curve looks like a rectangle in 2009, compared to the 1950 curve.

If mortality compression is true, the risk of pricing annuity and health insurance products and designing the national pension system would be reduced. However, there is still not enough evidence to confirm mortality compression (Li, Hardy and Tan 2008; Yue 2012), and the data quality is crucial in the mortality study. For example, even for the HMD, the official population counts are not available for age 90 and older in all countries (Jdanov et al. 2008). Thus, adjustments (i.e., graduation) need to be made in the HMD for mortality rates over age 80 by the method of extinct generations, and over age 90 by the survivor ratio method (Thatcher, Kannisto and Andreev 2002).

In addition to the issue of data quality, the method of estimation also plays an important role in studying mortality compression. There are quite a few methods for measuring mortality compression; calculating the variance of age-at-death is one of the popular methods. Kannisto (2000) proposed the standard deviation of age-at-death above the mode, or SD(M+), which is to compute the variance of death age over the modal age (i.e., the age with the largest number of deaths). Like the calculation of sample variance, also noted by Kannisto, the result of SD(M+) would be affected by many factors, such as if the modal age is not unique and if there are outliers.

Most other measures of mortality compression, such as the interquartile of age distribution (Wilmoth and Horiuchi 1999), the shortest age interval covering 25 percent, 50 percent and 75 percent of deaths, and verticalization (Cheung et al. 2005), also rely on the ages of observed deaths. Because the death records are usually in the format of integer age, these measures can be very discrete and oscillate a lot between two consecutive years. This is also one of the reasons there is no concrete evidence supporting mortality compression.

In this study, we propose a statistical approach for evaluating mortality compression to reduce the discreteness of mortality measures. Specifically, we consider optimization methods, e.g., nonlinear maximization and weighted least squares, for estimating the modal age and the standard deviation of age-at-death distribution. We shall use computer simulation and empirical studies to evaluate the proposed methods, and compare them to the SD(M+) by Kannisto. We introduce the proposed methods in the next section, followed by the simulation and empirical studies. The limitation and study notes will be given in section 5.

2. Methodology

Mortality compression was first proposed by Fries (1980) and is related to the idea of morbidity compression. There are quite a few ways to measure mortality compression. For example, Wilmoth and Horiuchi (1999) proposed 10 measurements and they recommended the interquartile (IQR). In addition to the IQR, Kannisto (2000 and 2001) also calculated the standard deviation, or SD(M+), and percentiles of age-of-death distribution, as well as the shortest age interval of deaths from 22 countries. The SD(M+) can be defined as

$$SD(M+) = \sqrt{\frac{\sum_x f(x)(x-M)^2}{\sum_x f(x)}}\ , \qquad (1)$$

where M is the modal age with the largest number of deaths and the summation is calculated for all ages beyond the modal age. Cheung et al. (2005) proposed three-dimensional measures for the survival curve, including horizontalization, verticalization and longevity extension.

These measurements basically are obtained from the observed information, via some simple calculations. For example, the value of SD(M+) is derived from the formula of sample variance. However, age-of-death is usually recorded as the format of integer age, via the definition age of last birthday or age of nearest birthday. Thus, the measurements would have a lot of fluctuations and often behave like a step function. For example, as shown in Yue (2012), no matter the standard deviation of age-of-death or the shortest age intervals of 25 percent, 50 percent and 75 percent, they all jump up and down between consecutive years.

There are several possible approaches to reduce the fluctuations of the observed measures. The numerical methods are a frequent choice since they can often provide good results. For example, Yue (2002) considered the optimization methods, including maximal likelihood estimation (MLE), nonlinear maximization (NM) and weighted least squares (WLS), to estimate the parameters of Gompertz’s law. He found that all three methods can obtain reliable estimates and the MLE has the smallest mean squared error (MSE). Coale and Kisker (1990) also used the WLS method to estimate the parameters of their proposed mortality model for the oldest-old group. Wilmoth (1993) proposed the WLS and MLE modifications to the parameter estimate of the Lee-Carter model when the mortality data of some age groups are missing.

In this study, we apply the methods of numerical optimization to measure mortality compression to reduce the influence of data format for recording age-of-death. To simplify the discussions, we will only use the NM method as a demonstration. The NM and MLE methods are expected to produce similar results in parameter estimation, and usually produce more accurate estimations than the WLS.

However, in practice, not all data are plugged into the MLE, and the NM method would produce more accurate estimates. This is why we choose the NM method in this study. Also, it should be noted that, in general, the age-of-death distribution is required to apply the optimization methods. In addition to the normal distribution, as suggested by Kannisto (2000), we also consider the logistic distribution, where the force of mortality satisfies $\mu_x = \dfrac{a e^{bx}}{1 + a e^{bx}}$ (Thatcher et al. 2010).

If the number of deaths at age *x*, or d_x, satisfies the function *f(x)*, then the NM estimate is to solve the following equation

$$\arg\min \sum_{x=M}^{M+2k} w_x \left(f(x) - d_x \right)^2 , \tag{2}$$

where w_x is the weight, M is the modal age, and *k* is the data range. If age-of-death follows the normal distribution, equation (2) can be written as

$$\arg\min_{M,\sigma} \sum_{x=M}^{M+2k} w_x \left(\frac{1}{\sqrt{2\pi}\sigma} \exp[\frac{-1}{2\sigma^2}(x-M)^2] - d_x \right)^2 . \tag{3}$$

For the logistic distribution, equation (2) can be expressed as

$$\arg\min_{M,\sigma} \sum_{x=M}^{M+2k} w_x \left\{ (\frac{1+be^{-bM}}{1+be^{b(x-M)}})^{1/b} \times \frac{be^{b(x-M)}}{1+be^{b(x-M)}} - d_x \right\}^2 . \tag{4}$$

Note that equation (2) is under the assumption of stationary population. However, in practice, the annual number of births may not be the same, or the radix l_0 is different every year. We can apply the idea of mortality rates and modify equations (2) and (3) as

$$\arg\min \sum_{x=M}^{M+2k} d_x \left(f(x) - \frac{d_x}{l_0} \right)^2 = \arg\min \sum_{x=M}^{M+2k} d_x \left(f(x) - {}_x p_0 \times q_x \right)^2 \tag{5}$$

and

$$\arg\min_{M,\sigma} \sum_{x=M}^{M+2k} d_x \left(\frac{1}{\sqrt{2\pi}\sigma} \exp[\frac{-1}{2\sigma^2}(x-M)^2] - {}_x p_0 \times q_x \right)^2 \tag{6}$$

where ${}_x p_0$ is the survival probability of age 0 to age *x*, and q_x is the death probability between age *x* and age *x + 1*. Similarly, we can also modify SD(M+) in equation (1) to

$$SD(M+) = \sqrt{\frac{\sum_{x=M}^{M+2k} d_x (x-M)^2}{\sum_{x=M}^{M+2k} d_x}}\ . \tag{7}$$

The modal age can be derived from the process of optimization for the NM methods, but it needs to be estimated separately for the SD(M+) method. The modal age can be determined by pure observation but it would have a lot of fluctuations and can be not unique if there are two or more modals. To deal with these problems, Kannisto (2000) suggested a modified estimate of the modal age,

$$M^* = x + \frac{f(x) - f(x-1)}{[f(x) - f(x-1)] + [f(x) - f(x+1)]}\ , \tag{8}$$

where *x* is the age with the largest number of deaths. We shall use equation (8) to obtain the modal age for the SD(M+) in this study.

3. Simulation

In this section, we will use computer simulation to evaluate if the proposed estimation method works. We first assume age-of-death follows normal distribution. Before showing the simulation results, we need to make a note about the estimation of the NM method. Since the death ages are usually recorded as the format of age at last birthday, the age of a person age *x* is between *x* and *x* + 1, or [*x*, *x* + 1). This means the average age of a person age x is $x + 1/2$. In other words, the estimate of modal age from the NM method needs to add another 1/2.

Example 1. Suppose age-of-death follows a normal distribution with modal (mean) M = 80 and standard deviation $\sigma = 10$. For each simulation run, we simulate 100,000 deaths whose age-at-death follows normal distribution. The idea behind this setting is similar to the radix ($l_0 = 100{,}000$) of life tables where there are 100,000 newborns. The simulation is repeated 1,000 times.

The data used to calculate the modal age and standard deviation are from age M – *k* to M + *k*, and the value of *k* is at least 5. We want to check if the observations used have a significant influence on the estimation result. Also, in practice, we can use the observed information or apply equation (8) to determine the M value for plugging into the NM method. It should be noted that, as mentioned by Kannisto (2000), the original setting for estimating the standard deviation for SD(M+) is to use the deaths beyond the modal age, or [M, ∞). However, according to our experiment on the proposed approach, we found that the estimation results for using the data age M – *k* ~ M + *k* are much better than those of age M ~ M + 2*k*. Still, we use the data age M ~ M + 2*k* for the NM method to compare with the SD(M+).

We use the bias and the variance of the estimate to evaluate the proposed method. Note that we also want to check if the death records up to the first decimal digit can provide more accurate estimates than using the records of integer age. As shown in figure 2, the NM method can obtain accurate estimates of the modal age, using the data of integer age or up to the first decimal digit. This is true for different *k* values. The estimate from equation (8) also seems unbiased, but it has a larger fluctuation for different *k* values.

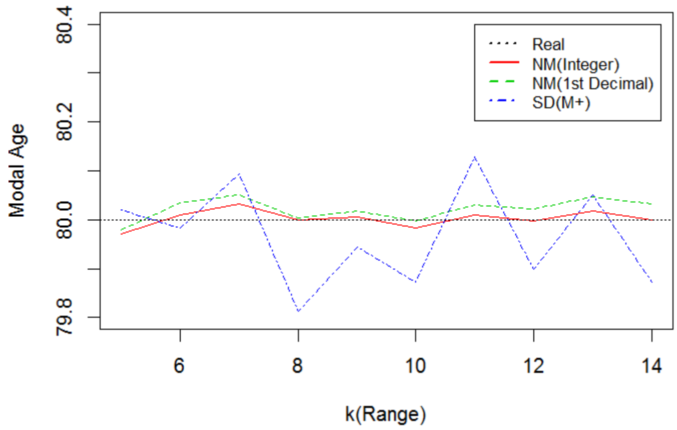

Figure 2. Bias of modal estimate (normal distribution)

We further use the MSE, or the sum of $(bias)^2$ and variance, of the estimate to evaluate the performance of estimation methods (figure 3). We use the logarithm of MSE since it is easy to distinguish the differences. It seems that the estimate of NM method has a smaller MSE and the MSE decreases as the value k increases. Interestingly, using the format of integer age is slightly better than using the data of first decimal. Also, the MSE of the estimate from equation (8) looks like a constant no matter how many data are used.

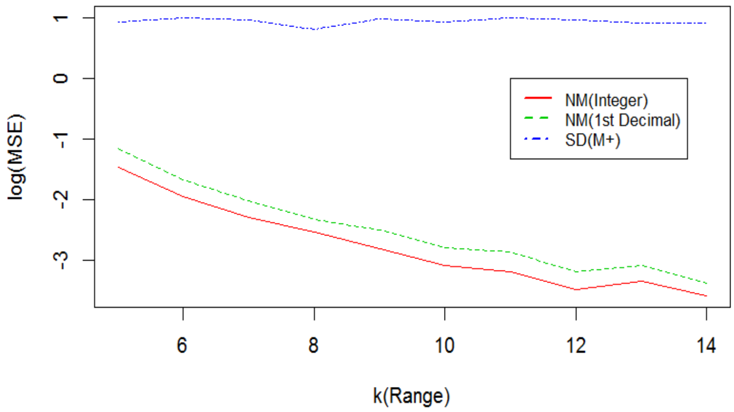

Figure 3. MSE of modal estimate (normal distribution)

The results of σ estimate are similar and show more obvious differences between the NM method and SD(M+). The NM method can obtain pretty accurate estimates for σ, for different *k* values (figure 4). On the contrary, the SD(M+) is underbiased and the bias becomes smaller as the data range *k* increases. However, the bias is still noticeable when more than 30 data points (different ages) are used, or *k* = 15. Using the criterion of MSE (figure 5), the NM method again has smaller MSE and is very consistent for all *k* values, similar to the result of M estimate. Based on these simulation results, it seems the NM method is reliable in providing the estimates of M and σ, and the data of integer age is preferred.

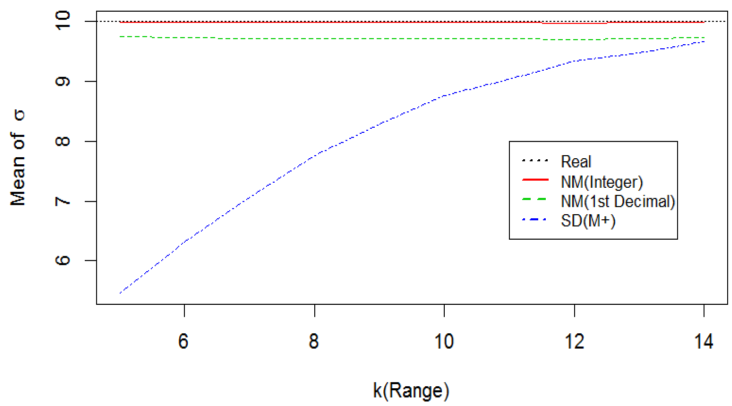

Figure 4. Bias of σ estimate (normal distribution)

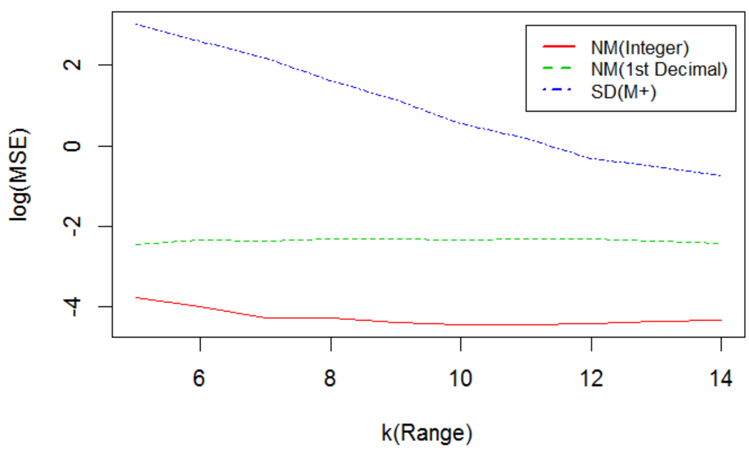

Figure 5. MSE of σ estimate (normal distribution)

Based on the simulation results, we found that the estimation result of SD(M+) can be influenced by the data used; this is critical in verifying mortality compression. The mortality data of the elderly were limited in the past and have only become available in recent years for many countries. For example, the highest age group of mortality rates in Taiwan is 89 before 1991 and is 99 since 1998. Of course, we can use extrapolation (like in the HMD) to let the highest age group be the same, but it might distort the original mortality rates.

We further use the coverage probability of the estimate of modal age to evaluate the estimation methods. The variances of the modal age estimate and standard deviation σ can be acquired from the replications of Monte Carlo simulation. Together with the estimate of modal age and σ for every simulation replication, we can use the variance to construct the confidence intervals. Among 1,000 simulation replications, we record the probability of confidence intervals constructed on which the true modal age is contained and this probability is called the coverage probability. Table 1 shows the coverage probabilities of the modal age and σ for the NM method and SD(M+). The NM method meets its expectation, when the data are recorded in integer age or accurate up to the first decimal. On the other hand, the SD(M+) is underbiased for estimating σ and cannot achieve its significance level. Note that, in practice, the variances and confidence intervals of the parameters M and σ can be acquired from bootstrap simulation.

Table 1. Coverage probability of normal distribution

	NM		SD(M+)
	Integer age	First decimal	
M	.951	.952	.969
σ^2	.956	.937	**.000**

Note: Coverage values larger than .964 or smaller than .936 are significantly different than 95 percent level, and are marked in red.

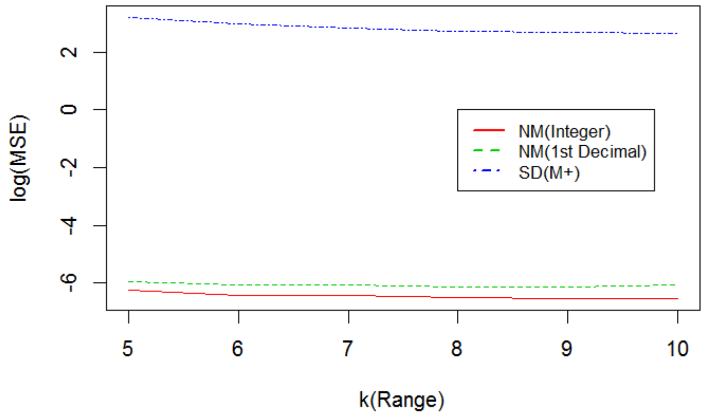

Figure 6. MSE of σ estimate (logistic distribution)

Example 2. In this example, we assume age-at-death follows logistic distribution. Also, to let the age-at-death have the same mean and standard deviation (80 and 10, respectively) as in example 1, we choose a = 80 and b = 0.1336837. Again, we simulate 100,000 deaths from the logistic distribution and repeat the simulation 1,000 times.

Since the estimation results are very similar to those in example 1, we will use the case of σ as a demonstration. In particular, figure 6 shows the MSE of σ estimate for different methods. Again, the NM method has smaller MSE, and using the integer age is still better. Interestingly, similar to the case of normal distribution, the MSE of

SD(M+) decreases as the *k* increases, but the MSE of the NM method is quite stable no matter how many data are involved.

We can also use the coverage probability of parameters' estimates to evaluate the estimation methods (table 2). Again, the NM method is better than SD(M+) in estimating both M and σ and using the mortality data of integer age provides reliable estimates. However, the coverage probabilities of the NM method with up-to-first decimal age fail to meet the nominal 0.95 level.

Table 2. Coverage probability of logistic distribution

	NM		SD(M+)
	Integer age	First decimal	
M	.948	**.924**	**.893**
σ^2	.956	**.927**	**.000**

Note: Coverage values larger than .964 or smaller than .936 are significantly different than 95 percent level, and are marked in red.

In this section, we use computer simulation to evaluate the proposed NM method and found that, when age-at-death follows normal or logistic distribution, it can give reliable and accurate estimates of the modal age and standard deviation σ. We also found that the death record of integer age is sufficient; this is good news since most mortality records are kept in this way. We shall apply the NM method to empirical data in the next section.

4. Empirical Analysis and Considerations

In addition to the empirical study of mortality data from the HMD, we want to explore if the graduation method influences the estimates of modal age and standard deviation σ. We shall study the effect of graduation first.

Example 3. Similar to example 1, we assume age-at-death follows normal distribution with modal age 80 and standard deviation 10. Also, there are 100,000 deaths in every simulation run. We choose the Whittaker-Henderson method (London 1985) to graduate the raw data, by minimizing the following objective function,

$$F = \sum_{x=1}^{n} w_x (v_x - u_x)^2 + h \sum_{x=1}^{n-z} (\Delta^z v_x)^2 \ , \tag{9}$$

where w_x is the weight of age, h is the weight of smoothness, and u_x and v_x are the raw and graduated data of age x, respectively. The value of z is usually set to 3, and w_x is the sample size of age x. In this study, we consider two values for h (W1 and W2): One is the average sample size of all ages (Yue 1997) and the other is 10,000 times the average sample size. The purpose of W2 is to let the graduated values be very close to a polynomial of degree z – 1.

Table 3 shows the averages of parameter estimates from the raw data and two types of graduated data for various data range k. Again, the NM method has a smaller bias for all types of data used. The estimates in the cases of raw data and Whittaker-Henderson graduation W1 are very close, but the W1 seems to have a larger fluctuation in the modal estimate. On the other hand, the Whittaker-Henderson graduation W2 has obvious impacts on both the NM method and SD(M+), and the estimates of modal age and standard deviation σ are distorted.

Table 3. Modal parameter estimates for various graduation methods

M	NM			SD(M+)		
K	Raw	W1	W2	Raw	W1	W2
5	79.98	80.07	80.38	79.45	79.37	75.02
6	80.01	79.87	80.37	79.37	79.51	75.02
7	80.00	79.98	80.50	79.43	79.46	75.02
8	79.99	79.82	80.60	79.48	79.66	75.02
9	79.99	80.02	80.58	79.41	79.40	75.04
10	79.99	80.01	80.57	79.37	79.35	75.04
σ	NM			SD(M+)		
K	Raw	W1	W2	Raw	W1	W2
5	9.99	10.00	9.73	5.47	5.47	5.50
6	10.00	10.00	9.70	6.34	6.34	6.39
7	9.99	10.00	9.67	7.10	7.09	7.19
8	10.00	10.00	9.67	7.71	7.71	7.89
9	10.00	10.00	9.66	8.31	8.31	8.46
10	10.01	10.01	9.66	8.69	8.69	8.91

Note: W1 and W2 are the results under Whittaker-Henderson graduation.

The results are similar when age-at-death follows logistic distribution (not shown here), and we would underestimate the σ estimate for the NM method if the W2 graduated data are used. It should be noted that, in addition to the graduation method and its settings, the estimation results depend on the sample size and the distribution of age-at-death. This finding is similar to that in Yue (2012). We suggest using the raw data to explore the mortality compression, although most past work on mortality compression is based on the graduated (or life table) data.

We continue exploring mortality compression using the real mortality data, and compare the estimation results based on raw data and life table data.

Example 4. We now analyze the mortality data from the HMD (year 1950–2009) and study if there is mortality compression via estimating the standard deviation σ, using the NM method and SD(M+). In addition, we want to explore the influence of using graduation data.

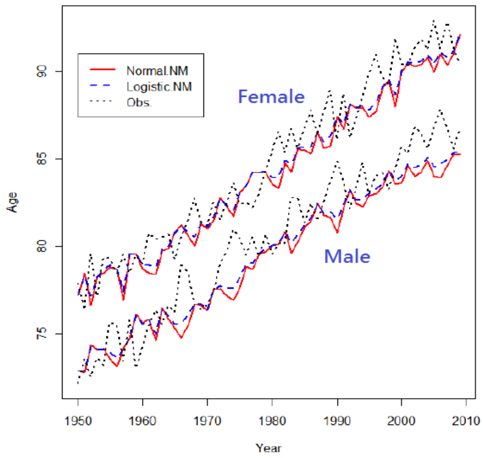

Figure 7. Japan modal estimates for NM and SD(M+)

Note the modal age increases annually, like life expectancy, and the results are quite the same in almost all countries. We can use the Japan mortality data to show the common pattern. Figure 7 shows the estimated modal ages for the NM method, under normal and logistic assumption and compares them with the observed information. The increasing rate of modal age is about the same for the three approaches, and the estimates of NM method are smoother for both genders. We found the estimates of normal and logistic assumption for the NM method are very similar, and we will only show the results for the normal assumption for the rest of this study.

There are 37 countries in the HMD and the patterns of σ estimates can roughly be separated into two groups. Basically, from 1950–80, the σ estimates decrease annually but they do not always decrease since 1980. We can use the female data from Australia, France, Italy, Japan and the United States to give a more detailed interpretation. (The results of male data are in the appendix.) As shown in figures 8 and 9, the σ estimates are smoother for the NM method and thus it is easy to distinguish if there is mortality compression via the NM method. The σ estimates of Australia, France and Italy continue the decreasing pattern and it seems the mortality compression is a reasonable assumption. However, for Japan and the United States, the σ estimates fluctuate and do not continue their pace of decrement before 1980. The σ estimates of SD(M+) do not have obvious sign of decreasing.

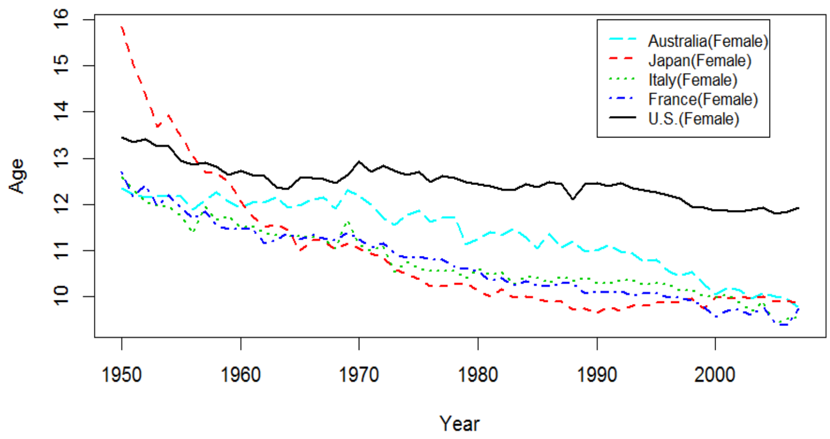

Figure 8. σ estimates for various countries using NM (female)

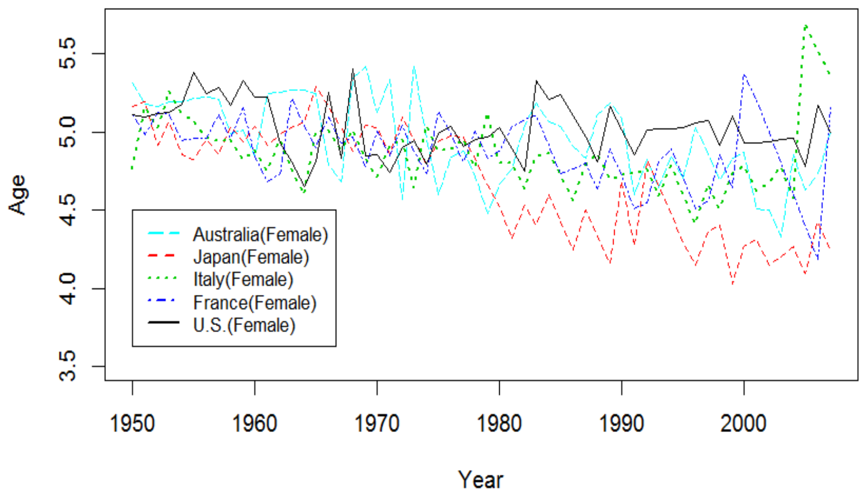

Figure 9. σ estimates for various countries using SD(M+) (female)

This is a very interesting finding. Intuitively, we would guess mortality compression is a more likely assumption for countries with higher life expectancies. Among these five countries, life expectancy is the highest in Japan and the lowest in the United States. It seems there are no strict connections between life expectancy (or modal age) and mortality compression. Still, although the results are not shown here, we found the shortest interval covering 50 percent of deaths continues its decreasing pattern for the HMD countries. There is still no decisive evidence to support or oppose mortality compression, depending on the measures used.

Similar to example 3, we can use the empirical data to verify the influence of using graduated data. Since the results of σ estimates are about the same for using the raw data, we should look at the estimate of the 95th percentile, i.e., 95 percent of deaths die at this age. Figure 9 shows the results of Japan data. Although the graduation is intended to reduce the fluctuation in mortality rates, the estimates of the 95th percentile show larger fluctuations for both genders. Again, this matches what we found in the previous section. If possible, we prefer using the raw data, rather than the graduated.

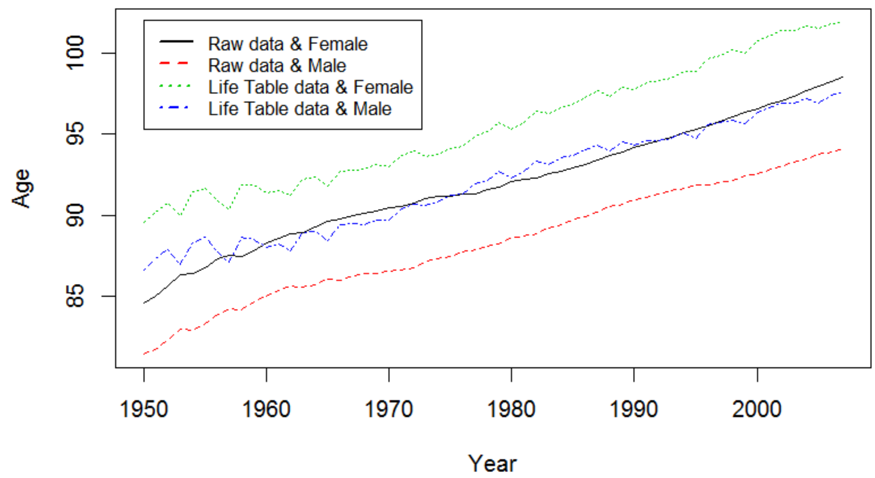

Figure 9. Estimates of the 95th percentile for Japan (NM method)

5. Discussions

Mortality compression is a famous conjecture. Although many past studies support the theory, there is still not enough evidence to make a conclusion. In fact, recently there have been studies that show quite the opposite result. In addition to the data quality (including insufficient data sizes), the compression measures and their estimation methods make the study of mortality compression difficult. In this paper, we propose applying the nonlinear maximization (NM) method to estimate the standard deviation σ for the age-at-death distribution, and using it to evaluate if there is mortality compression.

We first use computer simulation to evaluate the NM method. Comparing it to the estimation method used before, we found the NM method can produce more reliable estimates for the modal age and σ, with respect to the mean squared error and coverage probability, if age-at-death follows normal or logistic distribution. The estimates of NM method are very stable and apparently are not affected by the data range. On the contrary, the estimates of the observed method or SD(M+) can be influenced by the data range. The fewer data points there are, the larger bias σ estimate has. Some might argue the original idea of SD(M+) is to use all data beyond modal age, but the mortality data for the oldest-old are usually questionable and are often recorded in the format of five-age or 10-age groups. This means we might need to consider using data only at a certain range for estimation, as in this study.

We also found the graduation methods have an effect on the estimation result. Ironically, although the graduation process is intended to reduce the oscillations caused by insufficient sample size, it incurs the variations of estimates, as well as inducing bias in the NM method. It is likely the graduation process distorts the information carried by the data. However, if the parameters of graduation methods are chosen properly, as with the Whittaker-Henderson method W1 in table 3, the NM method can still produce accurate estimates. Nonetheless, unless dealing with small areas of mortality data, we recommend applying the raw data to the estimation methods.

We also applied the NM method to data from the countries in the Human Mortality Database (HMD), and checked if the standard deviation σ gradually converges. Unfortunately, discrepancy exists in estimates between countries and we cannot conclude whether there is mortality compression. For example, the σ estimates are converging for Australia, France and Italy, but they are not for Japan and the United States. This finding is similar to that in Li, Hardy and Tan (2008) and Yue (2012). Perhaps there are not enough elderly data and it is still too early to make a judgment. Another possibility is we shall define other compression measures that take all mortality data into the calculation, since only partial data are involved in applying the NM method or SD(M+). Possible candidates include indices for measuring unevenness, such as Gini's index, and a preliminary study shows it produces smoother estimates if the variance of age-at-death distribution converges.

It should be noted that the results of the NM method rely on the age-at-death distribution. If the distribution is misspecified, the NM method would have larger bias in the modal age and σ estimates, comparing to SD(M+). In practice, to avoid giving improper distribution assumptions, we can apply the goodness-of-fit test (e.g., Pearson chi-square and Kolmogorov-Smirnov tests) to verify the underlying distribution. We will also continue exploring other optimization methods, such as weighted least squares and maximum likelihood estimation (Yue 2002), and see if they can provide more stable estimates than the NM method.

One of our motivations for studying the mortality compression is to investigate what it means to price insurance products and how it will affect the insurance business. The prolonging of life expectancy increases the difficulty of pricing annuity and health insurance products. If mortality compression holds, it would somewhat reduce the difficulty. However, according to our empirical results of σ estimate, it is still too early to make any conclusions. Another evidence against compression is that the estimates of the 95^{th} percentile for the age-at-death distribution and the estimates show no signs of slowing down.

There is another indication that the estimates of the 95^{th} percentile keep increasing, also mentioned in Yue (2012). Although the distribution of age-at-death is well studied below the 95^{th} percentile, it does not mean the mortality rates of the oldest-old and beyond can be acquired via extrapolating the mortality trend. By using extrapolation, we would neglect the impact of longevity risk. This is one of the main reasons many developed countries consider modifying their national pension systems from defined benefit to defined contribution. Since we are not sure of the right tail of age-at-death distribution, maybe we need to be more conservative about designing insurance products and give up the idea of providing coverage for the whole life. After all, we think that living longer should be the individual's responsibility (at least most of it), and asking other people to share the burden is not fair.

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Appendix. The σ estimates for various countries using NM and SD(M+)

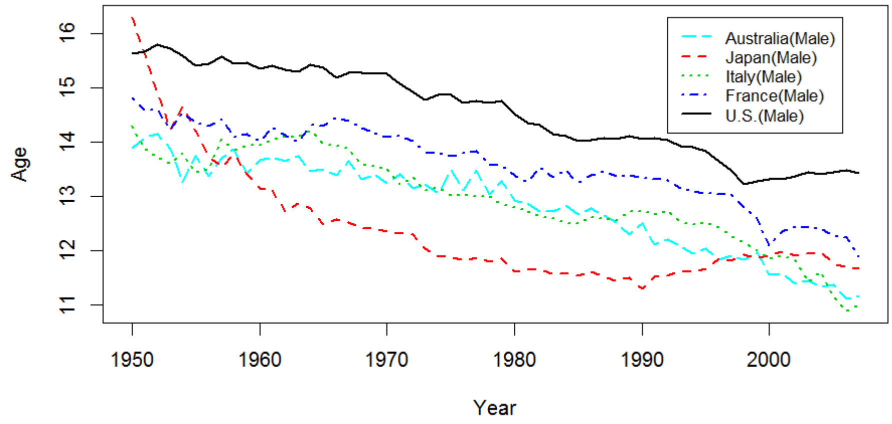

Figure A-1. σ estimates for various countries using NM (male)

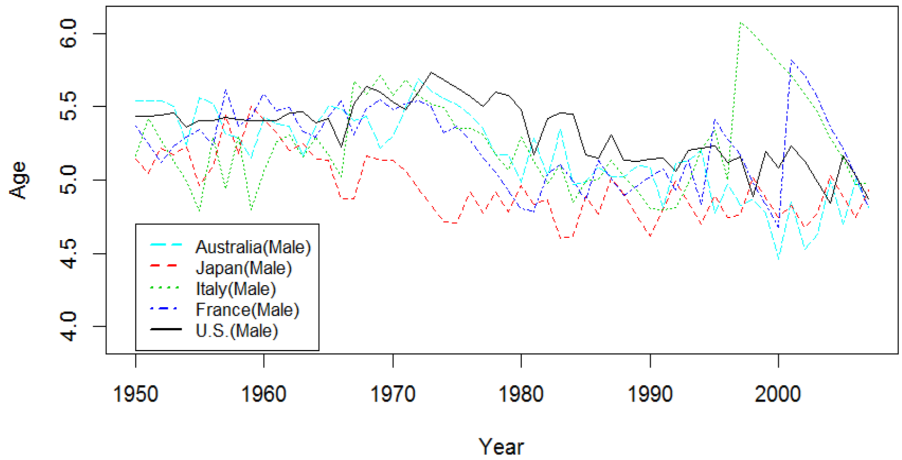

Figure A-2. σ estimates for various countries using SD(M+) (male)