OPTIMAL RETIREMENT TONTINES FOR THE 21ST CENTURY: WITH REFERENCE TO MORTALITY DERIVATIVES IN 1693

MOSHE A. MILEVSKY AND THOMAS S. SALISBURY

Presented at the Living to 100 Symposium

Orlando, Fla.

January 8–10, 2014

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Abstract. Historical tontines promised enormous rewards to the last few survivors at the expense of those died early. And, while this design *appealed to the gambling instinct*, it is a suboptimal way to manage and generate retirement income. This is why fair life annuities making constant payments – where the insurance company is exposed to longevity risk – induces greater lifetime utility. But, tontines do not have to be structured as a fixed cash-flow shared among a shrinking number of survivors and insurance companies do not actually sell fair life annuities, partially due to aggregate longevity risk.

In this paper we derive the tontine structure that maximizes lifetime utility, but doesn't expose the sponsor to any longevity risk. Technically speaking we solve the Euler Lagrange equation and examine its sensitivity to (i.) the size of the tontine pool, (ii.) individual longevity risk aversion, and (iii.) subjective health status. The optimal tontine varies with the individual's longevity risk aversion γ, the expected path of the mortality hazard rate λ_t, and the number of participants n. And, the historical (flat, constant) tontine structure is only optimal in the limit as longevity risk aversion $\gamma \to \infty$. We then introduce a structure called a *natural tontine* whose payout declines in exact proportion to the (expected) survival probabilities, which is near-optimal for all γ and n. We compare the utility of optimal tontines to the utility of loaded life annuities under reasonable demographic and economic conditions and find that the life annuity's advantage over tontines, is minimal. And, while similar insights were previously obtained (and confirmed) by Stamos (2008) within the context of *pooled annuity funds*, we employ a different framework to analyze optimal tontines.

We also take the opportunity to review the first-ever mortality-derivative issued by the British government, known as *King Williams's tontine of 1693*. Although it is widely acknowledged that mortality-derivatives were mis-priced in their early years, it is worth noting that both life annuities and tontines co-existed during that period. We conclude that tontines should be re-introduced and allowed to co-exist with life annuities. Individuals would likely select a portfolio of tontines and annuities that suit their personal preferences for consumption and longevity risk, as they did over 320 years ago.

Date: 25 February 2013 (Draft version 2.15).

This paper is for the conference proceedings of the *Society of Actuaries 2014 Living to 100 Symposium* and is a summary of research developed in Milevsky and Salisbury (2013a) and Milevsky (2013). The contact author Milevsky (milevsky@yorku.ca) is an Associate Professor of Finance at the Schulich School of Business, York University, and Executive Director of the IFID Centre. Salisbury is a Professor in the Department of Mathematics and Statistics at York University. The authors acknowledge funding from a Schulich Research Fellowship (Milevsky) and from NSERC (Salisbury), and wish to thank Rejo Peter, Dajena Collaku, Simon Dabrowski and Branislav Nikolic for research assistance.

"...Upon the same revenue more money can always be raised by tontines than by annuities for separate lives. An annuity, with a right of survivorship, is really worth more than an equal annuity for a separate life, and from the confidence which every man naturally has in his own good fortune, the principle upon which is founded the success of all lotteries, such an annuity generally sells for something more than it is worth. In countries where it is usual for government to raise money by granting annuities, tontines are upon this account generally preferred to annuities for separate lives..."

Adam Smith, The Wealth of Nations, 1776