# Discussant Comments
# Concurrent Session 3B:
# Comparing Mortality of Different Groups

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**KAI KAUFHOLD:** We have just heard three interesting presentations on thought-provoking and useful work. I would like to give you my thoughts on the papers which were submitted and open the discussion with some questions which I had while reviewing the papers.

Zhou and Li's paper on longevity Greeks is motivated by the fact that there have been only a few capital market transactions using longevity-based financial instruments, and there is hope that a more liquid market would facilitate more efficiently spreading this risk.

Yue and Wang address an issue which is indirectly related. Modeling longevity risk for small populations is one of the prerequisites for understanding longevity basis risk, which is perceived to be one of the obstacles to having more standardized longevity risk transfer in the capital markets. Yue and Wang propose an intuitive and straightforward method to implement small population longevity models by borrowing information from larger populations with similar characteristics.

Martin Genz's paper on mortality evolution ties in with this proposal, because he implements a framework for describing the mortality dynamics of any given population in terms of the normalized curve of deaths, i.e., the probability density for the age at death $d(x)$.

## Longevity Greeks: Definitions

One thing I especially liked about Zhou and Li's paper was that it gives a crisp definition of the concept of Greeks, which someone who hasn't studied advanced financial mathematics can still grasp. The key concept of hedging is to understand how your financial position will react to changes in the environment, or rather, the markets, and how to use that knowledge of the sensitivities to design a hedge. So we need two things: We need to model the financial position—say, a pension liability which is the present value of benefits paid out to survivors of a given portfolio. The changes in the environment are trickier to nail down. Basically, it is the chance in mortality or survivorship experienced by the reference population. But since mortality or survivorship is very much a function of age and time, as well as potentially lots of other risk factors, Zhou and Li propose measuring the sensitivities in terms of a model instead of observed actual deaths.

The good old Lee-Carter model lends itself to this purpose wonderfully, because it boils the mortality dynamics down to one time series, $\kappa_t$. So, we can define delta, gamma and vega in terms of the dynamics, or time dependence, of $\kappa_t$. Here vega is the new kid in town. It describes how the system reacts to changes in volatility. For that purpose, Zhou and Li had to extend the Lee-Carter model to include conditional heteroskedasticity, which a layman interprets as the changes in volatility of $\kappa_t$. This will be one of my questions in a minute.

## Longevity Greeks: Hedging Strategies

An interesting point which Zhou and Li raise is that you can conceptually distinguish between risk-minimizing hedges and a sensitivity-matching approach. In my view, one could see this as a way to distinguish between the insurance and banking mind-sets. In insurance, we either hold the risk or we get rid of it. This is like either building a fortress to withstand attack from the evil environment or to send someone else into battle for us. I visualize the sensitivity-matching banking approach to risk management as rolling with the punch, i.e., taking a position but staying light on the feet to dodge the hook and jab—much more Zen-like, no?

As Kenneth showed us, they calculate delta, gamma and vega for pension liabilities and a *q*-forward hedging instrument and then run simulations to test the different hedging strategies. To me, the key takeaway from their paper is that the optimal hedge depends on the composition of the portfolio in respect of age and time to maturity. Sounds simple, but it's worth looking at closely.

## Longevity Greeks: Questions

My first question to Kenneth is, Do we really need a time-dependent volatility in our mortality model? Or perhaps more accurately, what do we gain in comparison to limiting the analysis to the more recent history with much less volatility?

I see the advantage of a model-based hedging approach. But doesn't that make us even more exposed to model risk than we already are? You mention Cairns-Blake-Dowd as an alternative, as well as higher-order ARIMA [autoregressive integrated moving average] processes as potential generalizations of the GARCH [generalized autoregressive conditional heteroskedasticity] process you have described. Won't that turn out to be very difficult to model?

## Small Populations: Basic Idea

Turning to Yue and Wang's paper on life tables techniques, I would like to say that I am very intrigued with the concept of using a simple approach. The simplest possible model that does the job is always the best. And life table techniques have the added bonus that we practitioners are very familiar with them.

What Yue and Wang propose is to borrow the slope of mortality from a larger population in order to smooth out the volatility of the small population and also to remove bias. They also put forward an alternative approach which is even more traditional, i.e., using Whittaker-Henderson to smooth the small-population mortality before then applying a Lee-Carter projection model. For illustration, they simulate a set of different small-population data sets which differ in systematic ways from the reference population.

## Small Populations: Questions

From reading the draft paper, I was not able to understand exactly where the bias comes from when they fit a Lee-Carter model to a small population. It would be helpful to have that explained in more detail.

I would also have liked to see a comparison of Yue and Wang's life table techniques with other more modern means of graduating mortality tables, such as the graduation by formula method used by the U.K. actuarial profession in its table derivation, the method of parametric survival models proposed by Richards (2008), and *p*-spline smoothing techniques (Richards and Currie 2011), which can be applied without reference portfolio and which may not suffer from the same difficulties as the Lee-Carter model.

Does the Cairns-Blake-Dowd model show the same problems—i.e., biased parameter estimates—as the Lee-Carter model?

## Mortality Evolution: Basic Concept

Martin studies the dynamics of four components which describe the curve of deaths: $M$ [modal age at death], UB [upper bound], DOI [degree of inequality], $d(M)$ [number of deaths at $M$]. The beauty of this paper is that once you see the pictures, it is self-explanatory. The whole premise is to put forward a simple framework within which we can easily discuss and compare mortality trends.

## Mortality Evolution: Time Series for Evolution

Once the four components are defined, we have to systematically characterize the time trend for each component. That's a grand way of saying that we check whether the component decreases, increases or stays the same. Genz does this within a maximum likelihood polygonal fit and sets a limit to what is a significant trend. Otherwise, the component is judged to be neutral or stationary.

I believe it may turn out to be especially useful that Genz proposes a measure for relative similarity between time series of the mortality trend for different populations. For instance, this measure could be applied when using Yue and Wang's life table techniques for small populations, where one has to choose which larger population to use as a benchmark.

## Mortality Evolution: Question

Referring to the presentation of Kiril Andreev earlier in the conference, I would suggest that it makes sense to include a discussion of data quality and its impact on the observed apparent mortality trends within the mortality evolution framework.