2017 Enterprise Risk Management Symposium

April 20–21, 2017, New Orleans

Elements of After-Tax Risk Management

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Abstract

Most risk management is done on a pre-tax basis with tax issues treated as an afterthought. This paper will outline what an after-tax risk management process should look like and how it can differ from a pre-tax model. One of the paper's key conclusions is that a tax authority is often an implicit participant in many business transactions and this can have material implications for risk management. The paper starts by developing a simple three-step model of an income tax structure and then uses that model to understand a tax authority as a special class of equity investor. The paper then goes on to consider the impact of the tax structure on economic capital, the fair value of liabilities and after-tax asset/liability management (ALM). In many cases, the impact of adding tax into the ALM process is to lengthen liability durations while reducing convexities. Another impact is to make insurance liabilities sensitive to interest rate volatility in a way that tends to offset the interest rate volatility of interest rate options and guarantees.

Introduction and Three Risk Management Questions

What is real money? In the life insurance industry, this question can have many different answers. For some people, the answer could be pre-tax International Financial Reporting Standards (IFRS) or U.S. Generally Accepted Accounting Principles (GAAP) earnings. This would be an understandable answer coming from an executive whose incentive compensation was tied to one of those metrics.

Even if pre-tax earnings are the basis for day-to-day decision-making, there can be situations where tax issues can play a more significant role, including the following.

- Merger and acquisition valuations are almost always done on an after-tax basis using an approach consulting actuaries call embedded value analysis.[2] This can result in a value

[1] The authors are both directors at Moody's Analytics Co. The views and opinions expressed here are those of the authors and not Moody's Analytics.

[2] For more background on embedded value, see American Academy of Actuaries, "Market Consistent Embedded Values," *Public Policy Practice Note*, March 2011, *https://www.actuary.org/files/MCEV%20Practice%20Note%20Final%20WEB%20031611.4.pdf/MCEV%20Practice%20Note%20Final%20WEB%20031611.4.pdf.*

being assigned to a block of life insurance liabilities that is materially different from any accounting or regulatory value.

- New product pricing is almost always done using a variation of the after-tax embedded value model. Pricing actuaries discovered many decades ago that a conservative approach to setting tax reserves creates the financial equivalent of an interest-free loan from the tax authority to the insurance company. The value of that loan can be used to put a more competitive price on the insurance product.

Another area where pre-tax risk management is often used is the asset/liability management (ALM) process. In the authors' experience, this is often conducted on a regulatory accounting model basis that, again, usually ignores tax issues. If we take the interest-free loan issue into account, we often find liability durations go up, liability convexities go down and some asset values can change.

The upshot is that a risk manager who focuses solely on pre-tax GAAP earnings is bound to get a surprise from time to time. So, again, what is real money?

The position taken by the authors in this paper is that embedded value metrics are the only truly consistent way to think about these issues, that is, embedded value is "real money." This point of view is already consistent with product pricing and the buying and selling of blocks of business but should be extended to the ALM process and all other day-to-day risk management activities. This is not possible in the current IFRS or US GAAP accounting environments.

Given any accounting or regulatory valuation model, an insurance enterprise is usually valued, by external investors, as the sum of

- Assets backing free surplus (FS) valued at market
- The value of in-force business defined as a risk-adjusted present value of future distributable earnings (PVDE)
- The potential value of future new business or franchise value; this is often estimated as the PVDE of five to 10 years of recent new business

A stylized formula for the PVDE metric can be written as

$$PVDE(0) = \sum_{t=1} \frac{\{BP_t + RC_{t-1}[1 + r(1-\tau)] - RC_t\}}{(1 + r + \pi)^t}.$$

The notation is as follows:

BP_t represents the after-tax profits that emerge under the reporting model for the time period $(t-1, t)$. These are the profits that emerge under the reporting model if assets are equal to the reserves required by that model.

RC_t is the required capital defined by the reporting model at time t, that is, assets required over and above reserves.

r is an assumed interest rate earned on assets backing the required capital. In a market consistent model, this would be the relevant risk-free rate plus any appropriate liquidity adjustment.

$\tau = \tau(t)$ is the assumed tax-rate scenario. This parameter will also play a role in the calculation of BP_t.

π is a spread defining the after-tax target return to the investor who puts up the required capital. If the assumptions underlying the profit projection pan out, the investor will earn a return of $r + \pi$ on their investment. For vanilla insurance liabilities, the risk premium is often taken to be $\pi = .06$, but other values are possible.

The formula assumes distributable earnings are the sum of emerging after-tax profits plus the impact (up or down) of changing capital requirements. That this is a reasonable way to define shareholder value is fundamental to what follows in this paper.

There is purely algebraic reshuffle of the PVDE definition that allows us to write the PVDE as the sum of current required capital plus the mismatch between the emergence of book profits and the cost of capital. The formula is

$$PVDE(0) = RC_0 + \sum_{t=1} \frac{[BP_t - RC_{t-1}(\pi + r\tau)]}{(1 + r + \pi)^t}.$$

This is an important formula because it tells us how an accounting model has to be engineered to show us what the risk enterprise is actually worth, that is, $PVDE(0) = RC_0$. This requires

- Assets and liabilities marked to market
- Required capital defined in a reasonable way
- The release of risk margins (profits) engineered so that $BP_t = RC_{t-1}(\pi + r\tau)$; this is usually referred to as the cost of capital approach to risk margins, that is, reserves are equal to best estimate values adjusted by risk margins as defined here

There are at least three ways in which the evolving IFRS/US GAAP accounting models fail to meet the standard outlined earlier:

- Not taking into account the appropriate tax issues when calculating insurance policy reserves
- Not allowing gains at issue to be recognized at the point of sale

- Not requiring the use of the cost-of-capital method for risk margins at this time although it appears IFRS guidance would allow it

In short, accounting models need not be risk management models. This is unfortunate but nevertheless real. Accounting models that claim to be market consistent don't necessarily tell risk managers what they need to know.

Having laid down a position on the current accounting environment, we ask three questions that should be of interest to risk managers.

- **For a given insurance liability, how much asset do we need to have on the balance sheet to mature the obligation?** We will call the answer to this question the fulfillment value of the liability or FVL. A high-level formula for the FVL is that it is the risk-neutral present value of
  - Best estimate liability cash flows or BEL
  - "Appropriate" tax cash flows; we will go into more detail as to exactly what this means, but, in broad terms, appropriate tax cash flows are those marginal taxes any participant in the industry would have to deal with if they sold that product
  - Risk margins to compensate investors for putting up the necessary risk capital; again, there will be more detail in the next sections

  One of the technical results derived in the next section is that FVL can be calculated as the risk-neutral present value of after-tax cash flows using after-tax interest rates.
- **If someone offers to take the liability off our hands for an asset transfer of X, should we take the offer?** Given an answer to the first question, we will define the transfer price of the liability (TPL) as the value where we are indifferent between manufacturing the liability ourselves or paying someone else to do it for us. If the tax base of the liability[3] is a known quantity V^{Tax}, then the total assets required to actually transfer the liability to a third party are not TPL but the sum of TPL and the marginal tax consequences of executing the transfer, that is,

$$TPL + \tau(V^{Tax} - TPL).$$

  Here, τ is the assumed marginal tax rate. We are indifferent then if

$$TPL + \tau(V^{Tax} - TPL) = FVL, \text{ or } TPL = \frac{FVL - \tau V^{Tax}}{1-\tau}.$$

[3] In the U.S., the tax base would be the tax reserve net of any tax DAC (deferred acquisition cost).

This is the definition of transfer price that will be used in this paper. The key point is that FVL, TPL are the answers to two different questions and, due to tax issues, can be two different numbers. We will call the difference $\tau(V^{Tax} - TPL)$, the deferred tax on liabilities ($DToL$).

- **Given the different values available, which one should we use?** The answer depends on the application but it is usually the TPL. Here are some examples.
    - **Economic financial statements.** If the actuary gives an accountant preparing an economic balance sheet the value TPL as the statement reserve and the accountant then computes a traditional undiscounted deferred tax liability $\tau(V^{Tax} - TPL)$, the resulting total balance sheet liability will be the FVL. This is appropriate and is what Canadian actuaries have been doing since 2002 in the Canadian GAAP financial reporting model.[4]
    - **Risk management.** How much economic capital (EC) do we need to hold for a mortality experience shock ΔQ?
      If the death benefit for a life insurance policy is DB, then the gross loss resulting from an experience shock is $\Delta Q(DB - FVL)$ since FVL is the total asset we have on the balance sheet for the contract. However, if the loss can be tax effected, the net after-tax loss will be

      $$\Delta Q(DB - FVL) - \tau \Delta Q(DB - V^{Tax}) = (1 - \tau)\Delta Q(DB - TPL).$$

      This tells us that the after-tax economic capital required to cover a short-term mortality fluctuation is $(1 - \tau)\Delta Q(DB - TPL)$. A risk manager setting standards for risk retention should then use $DB - TPL$ as their key metric for net amount at risk exposure.
    - **Risk management.** How much economic capital do we need to hold for a plausible shock to an actuarial assumption, for example, $q \to q + \Delta q$? In most tax jurisdictions, changing an economic assumption does not change the liability tax base or have any other immediate tax effect. Changing an assumption can change the FVL so the change ΔFVL is the required economic capital. In terms of TPL, this is

      $$\Delta FVL = (1 - \tau)\Delta TPL.$$

[4] Much of this paper can be thought of as the authors' adaptation of existing Canadian GAAP ideas about tax to a market consistent economic model. Additional details on current Canadian actuarial practice can be found in the Canadian Institute of Actuaries' Consolidated Standards of Practice. Some of this guidance will need to change to reflect the new rules of IFRS.

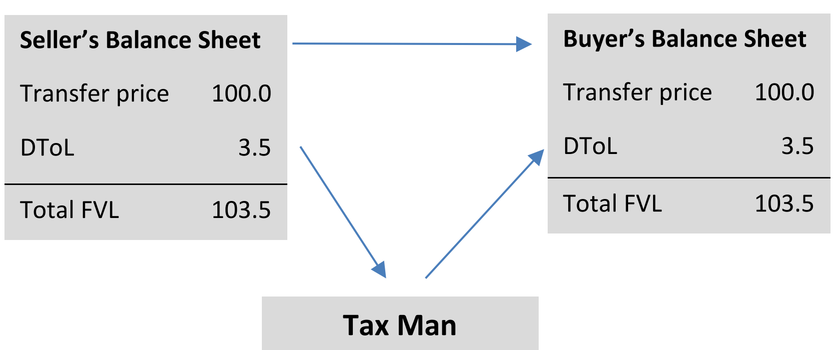
The only exception to this rule is when we actually change a tax-rate assumption $\tau \to \tau + \Delta\tau$, then it can be shown that

$$\Delta FVL = (1 - \tau)\Delta TPL + \Delta\tau(V^{Tax} - TPL).$$

A high-level summary of this discussion is that, for many day-to-day risk management purposes, it is useful to think of TPL as the "reserve." One risk management activity where this may not seem to apply is asset/liability management where FVL would appear to be central to the discussion. As we will see later, TPL turns out to be a very useful quantity for ALM as well.

One question that may be bothering the reader is as follows: If TPL is the price an insurer would be willing to pay to get rid of a liability, and that value is different from FVL, why would another insurer be willing to accept TPL to take on the liability? Figure 1 shows why this can make sense if the tax base V^{Tax} does not change when the block of liabilities is transferred from one insurer to another. This is what actually happens in the United States since tax reserves are defined by a combination of statute and regulation, not market prices.

Figure 1. Simple Liability Transfer



In Figure 1, we have $TPL = 100, V^{Tax} = 110, \tau = .35$ and so $FVL = 103.5$. The parties have agreed to a direct asset transfer of 100, which generates a taxable gain of 10 for the seller and a tax liability of 3.5. The buyer receives the 100 asset transfer and then sets up the same tax liability of 110, resulting in a tax loss of 10 and a tax benefit of 3.5. The net result is that the entire FVL has made its way from seller to buyer even though the *DToL* has gone indirectly via the tax man. It is therefore reasonable for the two parties to agree that 100 is an appropriate transfer price, assuming they agree on all other issues.

This is not what happens when two U.S. insurers trade a bond since the tax base of the bond would reset to its transfer price when traded from one legal entity to another.

There is an important case where the tax base of a bond does not reset to market and that is when we are in going-concern mode and a company is effectively selling the bond to itself. This has after-tax ALM consequences in that an observed market price is not necessarily the right value to use when truing up the asset side of an economic balance sheet to a calculated FVL.

Following the introduction, this paper has five sections:

- **A three-step model of an income tax structure, the foundation for what follows.** A key conclusion is that it is useful to think of a tax authority as a shareholder with some complicated financial options we refer to as the "tax man's put."
- **Calculating *FVL/TPL*.** We show how to compute *FVL* as the risk-neutral present value of after-tax cash flows discounted using after-tax interest rates. We also show how to compute *TPL* from first principles, and draw some useful risk management conclusions.
- **After-tax financial engineering.** We define the concept of an after-tax forward interest rate and show how it can be useful. Pre-tax and after-tax forward rates are not the same in the presence of interest rate volatility. After-tax forward rates are usually higher. This is a risk management insight with implications for pricing and ALM.
- **After-tax ALM.** We show how to value assets in a way that is consistent with market consistent liabilities**.** The key issue is that we need to put a value on future asset tax-timing differences since observed market values ignore tax-timing issues. The authors are aware that many people won't like this result.
- **A short conclusion that basically states after-tax risk management is possible and practical once the right tools have been put in place.**

We close this introduction by letting readers know what is out of scope for this paper. There are many tax practitioners who have to interpret vague tax laws and regulations that apply at either the company or policyholder level. There is always a risk that a working interpretation will be challenged by a relevant authority with adverse consequences. That issue is important but outside the scope of this paper.

High-Level Model of an Income Tax Structure

Imagine a world with no income tax at all. We have an insurance entity XYZ Corp. that has determined it needs $10 of economic capital. XYZ's economic balance sheet is seen in Table 1. The left-hand side shows the market value of assets (*MVA*) at 100. On the liability side, we have *FVL* of 90 and economic capital of 10.

Table 1. Simple Economic Balance Sheet

Assets		Liabilities	
MVA	100	*FVL*	90
		EC	10
Total	**100**	**Total**	**100**

XYZ's actuaries have engineered the insurance products so that $1 of profit margin is released each year to pay for the cost of capital, which we assume is $\pi = .10$. If the interest rate earned on surplus assets is *r*, the expected return to shareholders on economic capital is

$$\frac{10r + 1}{10} = r + 10\%.$$

Step 1. A Very Simple Tax Structure

To start, assume the tax man takes 35% of all economic income (plus or minus). At this stage in our model, we allow negative income taxes so there is complete risk sharing with the tax man. What are the consequences? The first consequence is that we no longer need to hold $10 of economic capital. Due to the risk sharing, $6.50 is now sufficient so $3.50 can be paid out immediately to the shareholder. Assuming this has been done, and the insurance product has not been repriced, the expected return to shareholders is now

$$\frac{(6.5r + 1)\ \times\ .65}{6.5}\ =\ .65r + 10\%.$$

The shareholder is, almost, neutral. The impact of the assumed tax structure is to reduce the shareholder's return by 35% of the interest earned on the after-tax capital. In the market consistent embedded value (MCEV) literature, this is referred to as frictional cost.

To fully compensate the shareholder for this frictional cost, the actuaries would have to increase the profit margin by the interest forgone on the capital the tax man has implicitly contributed, that is, 3.5*r*. Assuming *r* = 5%, the new risk margin is 1.18 = 1 + .05 × 3.5. Note this is not the same as grossing up the pre-tax profit margin to 1/(1 – .35) = 1.54 as might seem intuitive.

Two high-level conclusions at this stage of the argument are

- Income taxes are somewhat like shareholder dividends in that they compensate the tax man for implicitly contributing 35% of the economic capital. For the remainder of this article, it will be useful to think of the tax man as a special class of investor.
- The frictional cost issue is an example of a bias that favors the tax man at the expense of the common shareholder, unless the company passes the cost through to the policyholder.

Step 2. The Tax Man Introduces his own Accounting System (but we still allow negative income tax)

In most tax jurisdictions, companies must put together tax balance sheets and tax income statements that can be very different from their economic or accounting financial statements. However, in most jurisdictions, it is still possible to understand the difference between taxable income and economic income as a combination of temporary differences and permanent differences. A little bit of algebra may help here.

Let's assume we can calculate income tax as follows (we'll pick up any shortcomings of this assumption in Step 3 of our tax model).

$$\textit{Income Tax} = \textit{Tax Rate}\ [(ACF + \Delta A^{Tax} - PD^{A}) - (LCF + \Delta V^{Tax} + PD^{L})]$$

Here ACF is the asset cash flow received from invested assets and ΔA^{Tax} is the change in tax base of the company's assets. These two terms add up to the taxable investment income generated by the assets. The term $-PD^{A}$ represents a permanent difference[5] to taxable investment income arising from the assets. The taxable investment income is offset by an analogous term coming from the liability side of the balance sheet that one could think of as the tax-deductible interest along with any relevant liability-related permanent differences. LCF represents the liability cash flow while ΔV^{Tax} is the change in tax base and PD^{L} represents permanent differences arising from the liabilities.

The details of how tax values are determined, and what qualifies as a permanent difference, vary greatly by tax jurisdiction and the legal status of the taxpayer. Fortunately, we won't need to know most of these details but some life insurance examples may help to clarify the discussion. The last example in this list will be important later.

- For many jurisdictions, a bond asset is valued at amortized cost for tax purposes. In the United States, this rule is used unless the bond was bought at a discount. The U.S. tax regime does not recognize any amortization of purchase discount as taxable income until the bond is sold or matures.
- In most jurisdictions, an asset's tax base resets to market value when the asset is sold.

[5] Our sign convention for permanent differences is that a positive amount is favorable to the company.

- In the U.S., an example of a favorable permanent difference is the dividend received deduction (DRD), which allows a portion of the dividends received from assets to be deducted from taxable income.
- In Canada, life insurers must pay a federal investment income tax on behalf of their policyholders. This tax is not deductible when computing the company's corporate income tax in Quebec. This is an example of an unfavorable permanent difference.
- In the U.S., equity investments are generally valued at cost for tax purposes. In Canada, they are valued at market on the tax balance sheet.
- In most European jurisdictions, the tax base of an insurance liability resets to market if sold from one insurer to another. This is not true in the United States where the tax base of an insurance liability is effectively fixed by formulas defined in the tax code and related regulations.

How does this impact the company's relationship with the tax man? One way to analyze the situation is to break the income tax payments into three pieces that we will call asset taxes, mismatch taxes and liability taxes. This is done by adding and subtracting the economic investment income (*Econ II*) and economic required interest (*Econ Req'd I*)[6] from the basic tax equation. We then write

$$\begin{aligned} \textit{Income Tax} = \textit{Tax Rate}\ \{&[(ACF + \Delta A^{Tax} - PD^{A}) - \textit{Econ II}] && \text{Asset Tax} \\ &+ [\textit{Econ II} - \textit{Econ Req'd I}] && \text{Mismatch Tax} \\ &+ [\textit{Econ Req'd I} - (LCF + \Delta V^{Tax} + PD^{L})]\} && \text{Liability Tax} \end{aligned} \qquad (1)$$

The asset tax item captures the difference between actual taxable investment income and the economic income. The last line reflects liability issues while the middle line would, in theory, be zero if assets and liabilities were perfectly matched on an economic basis.

We take the view that an economic balance sheet should include the liability taxes in the calculation of FVL/TPL and an adjustment needs to be made to the observed market price of assets to account for the asset tax line.

Mismatch gains and losses should, on average, be zero. If this is the case, no explicit balance sheet liability is required for mismatch. This means mismatch gains and losses will fall to the bottom line as they occur and will get tax effected since they are not reserved for.

Most economic capital models have a capital requirement for mismatch losses that depends on the current state of mismatch. This capital requirement is not zero on average so it makes sense

[6] In this paper, *Econ Req'd I* includes interest on reserves and required capital.

that some provision should be held in the liabilities for this cost of capital. Some people call this the mismatch budget. Detailed discussion of this issue is beyond the scope of this paper.

The asset tax term is interesting because it can vanish if we assume all of the assets are continuously traded so that the economic and tax values are always identical. Putting a value on the asset taxes therefore requires making an assumption about how frequently assets are turned over (i.e., traded). This issue is discussed more fully in the section on after-tax ALM.

Since 2002, Canadian actuarial practice has been to use an actuarial projection platform to project future asset reinvestments/disinvestments using assumptions specified by the appointed actuary as part of the Canadian Asset/Liability Method (CALM) for valuing insurance contracts. This approach implicitly captures the value of asset tax-timing differences and permanent differences and is reported as a component of the insurance liabilities. This makes sense but will not be acceptable under IFRS.

Step 3. The Tax Man's Put Option

No doubt most readers are ready to point out that the first two steps of the tax model outlined here have missed a significant element. In terms of the tax man as shareholder concept, he not only defines his own dividend mechanism (Step 2) but he is usually able to limit his downside participation in the company's fortunes. Again, the details of how this works vary greatly from one tax jurisdiction to another. We will refer to this limit on the ability of the company to pass risk through to the tax man as the "tax man's put" option.

Some specific examples of the tax man's put at work are

- Most tax codes do not allow negative taxes per se. Tax losses can often be carried back to prior years or carried forward to future years. There are usually well-defined limits on how much of this can be done.
- In Canada (as of 2009), non-capital tax losses can be carried back three years and forward indefinitely. Capital losses can be carried back three years and forward indefinitely but can only be applied against capital gains.
- In the United States, capital losses on some asset sales can only be used to offset capital gains on similar assets.

This kind of rule puts some constraints on a company's ability to manage the asset taxes described in Step 2.

This is not entirely a one-way street. It is the authors' experience that tax specialists in many tax jurisdictions are fully aware of tools and transactions that can manage the potential impact of the tax man's put. This is often a significant activity within a company's tax department.

The tax man's put is very much an entity-specific issue so it should not be included in policy liabilities. If we think the asset or liability values described so far take too much credit for tax issues, then a special tax man's put liability on the economic balance sheet would be appropriate.

Calculating FVL/TPL

In the introduction, we defined FVL as the risk-neutral present value of

- Best estimate liability cash flows such as claims and expenses less gross premiums
- Risk loads for the cost of economic capital. The introduction showed that if these loads have the form $(\pi + r\tau)EC$, the expected after-tax return to the shareholder will be $r + \pi$. In practice, risk loads are often calculated using just πEC so the actual after-tax return to shareholders on risk capital is $r(1-\tau) + \pi$. This section will compute risk loads using the formula $(\pi_0 + \varepsilon r\tau)EC$ where $\varepsilon = 0$ or $\varepsilon = 1$.
- "Appropriate" taxes, which we now take to be the liability taxes seen in Equation (1).

We start by working through the ideas for a very simple life insurance policy with no lapses, a deterministic interest rate environment and a constant tax rate; the only risk margin is for contagion risk. Subsequent discussion will relax these simplifying assumptions.

Simple Case

Notation for this section:

- $F(t)$ is the FVL
- $V(t)$ is the TPL
- $\mu(t)$ is a traditional deterministic force of mortality
- $r,\ e,\ g$ are the interest rate, expense rate and gross premium rate, respectively
- $\pi = \pi_0 + \varepsilon r\tau$ is the cost of capital rate

The first step is to write down the basic differential equation, which states that F increases with interest and persistency and then decreases as cash flows are paid out, that is,

$$\frac{dF}{ds} = (r+\mu)F - [\mu D + e - g + \pi(1-\tau)\Delta Q(D-V)] \\ - \tau\left[rF + g - e - \mu D - PD^L - \left(\frac{dV^{Tax}}{dt} - \mu V^{Tax}\right)\right].$$

The first square bracket consists of death benefits μD, expenses e, gross premiums g and cost of capital risk margins of $\pi(1-\tau)\Delta Q(D-V)$. This last item follows from the fact that the economic capital for a mortality contagion event was shown to be $(1-\tau)\Delta Q(D-V)$ in the introduction. This cash flow is not tax deductible.

The second square bracket is our model of tax cash flow. We are assuming the interest earned on F is fully taxable, as are gross premiums. We also assume death claims and expenses can be deducted from taxable income along with any increase in the tax base. The term PD^L refers to any liability permanent differences between cash flow and taxable income as discussed earlier. Our sign convention is that PD is positive if it reduces taxable income and negative if not.

The equation is fine as a statement of first principles but is not immediately useful if you want to actually compute a number. We will now work through a sequence of algebraic reshuffles that result in something you can could actually implement in a computer program.

The first step is to collect some like terms in the formula above and rewrite it as

$$\begin{aligned}\frac{dF}{ds} &= [r(1-\tau)+\mu]F-(1-\tau)(\mu D+e-g)+\tau PD^L \\ &\quad +\tau\left(\frac{dV^{Tax}}{ds}-\mu V^{Tax}\right)-\pi(1-\tau)\Delta Q(D-V).\end{aligned} \tag{2}$$

This form of the equation is often interpreted by saying that F is the present value of after-tax cash flows using after-tax interest rates. This is useful for theoretical understanding but not for practical calculation since the risk margin cash flow depends on V.

One approach is to use the relation $V=\frac{F-\tau V^{Tax}}{1-\tau}$ in the equation above and then rearrange to get

$$\begin{aligned}\frac{dF}{ds} &= [r(1-\tau)+(\mu+\pi\Delta Q)]F-(1-\tau)[(\mu+\pi\Delta Q)D+e-g] \\ &\quad +\tau PD^L+\tau\left[\frac{dV^{Tax}}{ds}-(\mu+\pi\Delta Q)V^{Tax}\right].\end{aligned} \tag{3}$$

The effect has been to change $\mu\rightarrow(\mu+\pi\Delta Q)$ while eliminating the risk-loading term. This is an equation we can actually use to calculate F if we want to. If $F(T)$ is the fulfillment value at some future date (e.g., contract maturity), then a practical calculating formula is

$$F(t) = e^{-\int_t^T [r(1-\tau)+(\mu+\pi\Delta Q)]dv} F(T)$$
$$+ \int_t^T e^{-\int_t^s [r(1-\tau)+(\mu+\pi\Delta Q)]dv} \Big\{ (1-\tau)[(\mu+\pi\Delta Q)D + e - g]$$
$$- \tau PD^L - \tau \left[\frac{dV^{Tax}}{ds} - (\mu+\pi\Delta Q)V^{Tax} \right] \Big\} ds.$$

If we actually did this calculation, the transfer price could be calculated using $V = \frac{F-\tau V^{Tax}}{1-\tau}$.

In practice, it is common to compute V first. We can derive an equation for V by rearranging Equation (3) to get the expression

$$\frac{dF}{ds} - \tau \frac{dV^{Tax}}{ds} = r(1-\tau)F + \mu(F - +\tau V^{Tax}) + \pi(1-\tau)\Delta QV \tag{4}$$
$$-(1-\tau)[(\mu+\pi\Delta Q)D + e - g] + \tau PD^L.$$

Now divide both sides of Equation (4) by $(1-\tau)$ and note that the left-hand side will become $\frac{dV}{ds} = \frac{\frac{dF}{ds} - \tau\frac{dV^{Tax}}{ds}}{1-\tau}$. The result is

$$\frac{dV}{ds} = rF + (\mu+\pi\Delta Q)V - [(\mu+\pi\Delta Q)D + e - g] + \frac{\tau PD^L}{1-\tau}.$$

The final algebraic step is to use $F = V + \tau(V^{Tax} - V)$ in the result above to get an equation that can be solved for the transfer price V, that is,

$$\frac{dV}{ds} = [r(1-\tau) + (\mu+\pi\Delta Q)]V \tag{5}$$
$$-[(\mu+\pi\Delta Q)D + e - g] + \frac{\tau PD^L}{1-\tau} + r\tau V^{Tax}.$$

If we know the transfer price at a future date T, for example, contract maturity, the solution to the equation above can be written as

$$V(t) = e^{-\int_t^T [r(1-\tau)+(\mu+\pi\Delta Q)]dv} V(T) + \int_t^T e^{-\int_t^s [r(1-\tau)+(\mu+\pi\Delta Q)]dv} \left[(\mu+\pi\Delta Q)D + e - g - \frac{\tau PD^L}{1-\tau} - r\tau V^{Tax}\right] ds. \quad (6)$$

We will call this the "calculation" formula because there are no circular elements. This result could be used to write a computer program, once an appropriate numerical integration scheme has been chosen. Given that the transfer price has been calculated, we can calculate the fulfillment value using $F = V + \tau(V^{Tax} - V)$.

The calculation formula uses after-tax interest rates $r(1-\tau)$ and risk-loaded mortality $\mu + \pi\Delta Q$.

Readers can be forgiven if they are scratching their heads wondering what the calculation formula really means. One last algebraic reshuffle allows us to write the result in a way that is much easier to interpret.

Rewrite the differential Equation (5) defining V in the following mathematically equivalent but circular form:

$$\frac{dV}{ds} = (r+\mu)V - (\mu D + e - g) - \pi\Delta Q(D - V) + \frac{\tau PD^L}{1-\tau} + r\tau(V^{Tax} - V). \quad (7)$$

The solution to this equation can be written in a way that is easy to interpret. We call this the "presentation" formula.

$$V(t) = e^{-\int_t^T (r+\mu)dv} V(T) + \int_t^T e^{-\int_t^s (r+\mu)dv} \left[(\mu D + e - g) + \pi\Delta Q(D - V) - \frac{\tau PD^L}{1-\tau} - r\tau(V^{Tax} - V)\right] ds. \quad (8)$$

The calculated transfer price can now be interpreted/presented as the sum of

1. Best estimate liability cash flows (claims + expenses – premiums) using pre-tax interest rates and best estimate persistency; most people would call this the best estimate value
2. The pre-tax risk margin, which is the present value of grossed-up planned payments to shareholders for the cost of capital (more on this follows)
3. The impact of permanent tax differences, again grossed up to be pre-tax
4. The present value of interest on the undiscounted deferred tax liability $\tau(V^{Tax} - V)$

Practical implementations of the theory outlined above are usually engineered to produce the kind of decomposition outlined here. Variations are possible.

Some additional comments are warranted.

The risk margin term in Equation (8) should be thought of as

$$\pi\Delta Q(D-V) = \frac{(1-\tau)(\pi_0 + \varepsilon r\tau)\Delta Q(D-V)}{1-\tau}.$$

The fact that two $(1-\tau)$ factors cancel shows the risk loadings don't change much when going from a no-tax model to one with income taxes. The risk loadings changed only because we added the term $r\tau$ to π_0. We saw this happen in the example of the high-level tax model where the impact of taxing the risk margin release was largely offset by the reduction in economic capital.

Many people interpret the term $r\tau(V^{Tax} - V)$ as the interest on an interest-free loan from or to the tax man depending on the sign of the deferred tax liability. If $V^{Tax} > V$, this is a loan from the tax authority to the insurer. Pricing actuaries have been aware of this issue for decades since it often works in a company's favor.

Clearly, if $V^{Tax} < V$, the company is making an interest-free loan to the tax man and the cost of that loan is reflected in the transfer price. This can happen with older blocks of single premium immediate annuity (SPIA) business in the U.S. that have statutory and tax valuation rates high by today's (2017) interest rate standards. Standard statutory formula reserves for such blocks often turn out to be inadequate when subjected to cash flow testing analysis.

A More Complex Example: Assumption Changes

In this section, we discuss the issue of assumption changes for two reasons:

- Holding capital and risk margins for plausible assumption changes is common in most internal economic capital models and are required by some regulatory models such as Solvency II in Europe and the pending Life Insurance Capital Adequacy Test (LICAT) model in Canada.
- It forces us to consider time varying tax-rate scenarios $\tau = \tau(t)$.

The simple model discussed in the prior section showed that if holding economic capital $(1-\tau)\Delta Q(D-V)$ for contagion risk was our only risk issue, we could build in the cost of holding that capital by using a risk-loaded mortality assumption of the form $\mu + \pi\Delta Q$.

Suppose we now hold capital for plausible assumption changes $\mu \to \mu + \Delta\mu$, $\tau \to \tau + \Delta\tau$. We will briefly describe two approaches to handling this kind of issue. The first approach is motivated by Solvency II while the second approach was developed by one of the authors.[7]

If we take a Solvency II type approach to risk margins, we would calculate a best estimate value $F_0(t)$ using best estimate assumptions and then compute two shocked values $F_{1,\mu}(t)$ and $F_{1,\tau}(t)$ using shocked mortality or tax rate as appropriate. Economic capital for a mortality/tax assumption change is then calculated as $F_{1,\mu} - F_0$ or $F_{1,\tau} - F_0$. Risk margins are calculated by projecting the capital requirements into the future, under best estimate assumptions, and then computing the present value of the cost of capital.

$$
\begin{aligned}
M_\mu(t) &= \int_t^\infty e^{-\int_t^s (r+\mu)dv} \pi(s)\left[F_{1,\mu}(s) - F_0(s)\right] ds, \\
M_\tau(t) &= \int_t^\infty e^{-\int_t^s (r+\mu)dv} \pi(s)\left[F_{1,\tau}(s) - F_0(s)\right] ds.
\end{aligned}
\tag{9}
$$

Here $\pi(s)$ is the cost of capital rate. This is what most people consider the cost of capital method to mean. The process is conceptually straightforward but can be computationally expensive because of the need to project capital requirements at all future time points.

An alternative, and less computationally expensive, method is to work with risk-loaded mortality and tax rates of the form $\mu(t,s) = \mu(s) + \beta_\mu(t,s)\Delta\mu(s)$, $\tau(t,s) = \tau(s) + \beta_\tau(t,s)\Delta\tau(s)$. Here, the best estimate assumptions $\mu(s), \tau(s)$ have been augmented by dynamic risk loadings $\beta_\mu(t,s)\Delta\mu(s)$, $\beta_\tau(t,s)\Delta\tau(s)$. The quantities $\beta_\mu(t,s), \beta_\tau(t,s)$ are known as margin variables that are zero in the real world (P measure) but evolve according to certain rules when we enter the valuation world (Q measure). If we choose the risk-neutral evolution rules properly, we can get results for capital and risk margins very similar to the Solvency II type calculation seen in Equation (8).

One example of an evolution rule set is to assume $\beta_\mu(t,t) = 0, \beta_\tau(t,t) = 0$ and for $s > t$ roll the margin variables forward in time using the dynamics

$$
\begin{aligned}
\frac{d\beta_\mu(t,s)}{ds} &= \pi_0 + \varepsilon r\tau(s) - \beta_\mu(t,s)\Delta\mu(s), \\
\frac{d\beta_\tau(t,s)}{ds} &= \pi_0 + \varepsilon r\tau(s) + \beta_\tau(t,s)\Delta\tau(s).
\end{aligned}
$$

[7] B.J. Manistre, "Down but Not Out a Cost of Capital Approach to Fair Value Risk Margins," Enterprise Risk Management monograph, 2014 ERM Symposium, *https://www.soa.org/Library/Monographs/Other-Monographs/2014/september/2014-erm-symposium.aspx*.

Having developed these two sets of risk-loaded decrement and tax-rate assumptions, we do the following calculations:

1. Calculate a base fulfillment value $F(t)$ using the previously defined risk-loaded assumptions
2. Calculate a mortality shocked value $\hat{F}_\mu(t)$ using the base tax rate $\tau(s) + \beta_\tau(t,s)\Delta\tau(s)$ and shocked mortality $\mu(s) + \Delta\mu + \beta_\mu(t,s)\Delta\mu(s)$
3. Calculate a tax shocked value $\hat{F}_\tau(t)$ using the shocked base tax rate $\tau(s) + \Delta\tau + \beta_\tau(t,s)\Delta\tau(s)$ and base mortality $\mu(s)+\beta_\mu(t,s)\Delta\mu(s)$

Capital for tax-rate risk is then calculated as $\hat{F}_\tau(t) - F(t)$; capital for a mortality assumption change is given by $\hat{F}_\mu(t) - F(t)$. The base value $F(t)$ can be thought of as the best estimate value F_0 plus sufficient margin to pay for holding both capital amounts: $F(t) = F_0(t) + M$.

Unfortunately, there is not enough space in this document to derive these conclusions in detail. Most of the necessary details can be found in Manistre (2014).

Once the base and shocked fulfillment values $F, \hat{F}_\mu, \hat{F}_\tau$ have been calculated as above, it is straightforward to calculate the related transfer price quantities $V, \hat{V}_\mu, \hat{V}_\tau$ using the relation $V = \frac{F - \tau V^{Tax}}{1-\tau}$ derived earlier. This is usually the path of least resistance to get at $V, \hat{V}_\mu, \hat{V}_\tau$.

It is possible to compute a transfer price from first principles but a time-dependent tax-rate assumption creates a new technical wrinkle. If the tax-rate assumption $\tau(s)$ is smooth enough that the time derivative $\dot{\tau}(s) = \frac{d\tau}{ds}$ is continuous, the first principles presentation formula for V is

$$V(t) = e^{-\int_t^T (r+\mu)dv} V(T) + \int_t^T e^{-\int_t^s (r+\mu)dv} \left[(\mu D + e - g) + \pi\Delta Q(D - V) - \frac{\tau P D^L}{1-\tau} - (r\tau + \frac{\dot{\tau}}{1-\tau})(V^{Tax} - V) \right] ds. \tag{10}$$

The key point is that this is not the same as the presentation formula derived in Equation (8) with a time dependent tax rate. The authors invite the reader to derive the corresponding calculation formula for V and then decide whether implementing that approach is worth the effort.

After-Tax Financial Engineering: Stochastic Interest Rates

This section will briefly indicate what changes if we decide to go with a model where interest rates are stochastic. The main issue is that taking tax-timing differences into account can turn an otherwise simple, and deterministic, valuation problem into one requiring a fully stochastic approach. The good news is some problems are still simple enough they can be valued using deterministic methods, once we have the concept of an after-tax forward rate in hand.

We start by returning to the simple life insurance example we have been using but now we compute the fulfillment value using a risk-neutral expectation, that is,

$$F(t) = E^Q\left(e^{-\int_t^T [r(1-\tau)+(\mu+\pi\Delta Q)]dv} F(T) + \int_t^T e^{-\int_t^s [r(1-\tau)+(\mu+\pi\Delta Q)]dv}\left\{(1-\tau)[(\mu+\pi\Delta Q)D + e - g] - \tau P D^L - \tau\left[\frac{dV^{Tax}}{ds} - (\mu+\pi\Delta Q)V^{Tax}\right]\right\}ds\right). \tag{11}$$

If the tax rate τ and cost of capital rate π are constant (which they often are in practice), the only stochastic element in the calculation above is in the after-tax discount factor $e^{-\int_t^s [r(1-\tau)]dv}$.

We now define the after-tax forward rate $f_\tau(s)$ by

$$e^{-\int_t^s f_\tau(s)(1-\tau)dv} = E^Q\{e^{-\int_t^s [r(1-\tau)]dv}\}.$$

An equivalent definition is

$$f_\tau(s) = \frac{e^{\int_t^s f_\tau(s)(1-\tau)dv}}{1-\tau}\frac{d}{ds}E^Q\left\{e^{-\int_t^s [r(1-\tau)]dv}\right\}.$$

If $\tau = 0$, this reduces to the standard definition of forward rate.

Under our stated simplification (τ, π are constant), we can take the risk-neutral expectation operator through the large parentheses in Equation (11) to write

$$F(t) = e^{-\int_t^T [f_\tau(1-\tau)+(\mu+\pi\Delta Q)]dv} F(T) + \int_t^T e^{-\int_t^s [f_\tau(1-\tau)+(\mu+\pi\Delta Q)]dv} \left\{(1-\tau)[(\mu+\pi\Delta Q)D + e - g] - \tau P D^L - \tau\left[\frac{dV^{Tax}}{ds} - (\mu+\pi\Delta Q)V^{Tax}\right]\right\} ds. \tag{12}$$

This is a deterministic calculation that is clearly more practical to implement than a stochastic Monte Carlo approach. A useful implication of this result is that the presentation formula introduced in Equation (8) continues to apply as long as we use the after-tax forward rates instead of the pre-tax forward rates.

In general, the after-tax forward rates will have to be estimated by Monte Carlo simulation. The computational advantage is that this may only need to be done once and then applied many times over to different contracts.

If the stochastic interest rate model being used is simple enough,[8] it is possible to write down a closed form expression for the after-tax forward rates. As a simple example, assume we are using the well-known Vasicek single factor model $dr = \alpha[\theta(s) - r]ds + \sigma dz(s)$. Here α, σ are constants and $\theta(s)$ is a deterministic function used to calibrate the model to a given set of pre-tax forward rates $f_0(s)$. It can be shown that the relationship between after-tax forward rates and pre-tax forward rates, for this model, is given by

$$f_\tau(s) = f_0(s) + \frac{\tau\sigma^2}{2\alpha^2}[1 - e^{-\alpha(s-t)}]^2. \tag{13}$$

In this formula, the valuation is being performed at time t and $s \geq t$.

For more complex models, it is still true that $f_\tau(s) \geq f_0(s)$ and the difference gets bigger as the interest rate model's volatility assumption is increased. This has a potentially significant risk management implication: If we shock up the interest rate volatility assumption, the liabilities will increase if there are any interest rate options or guarantees present. That increase will be offset by the change in value of tax timing differences on all liabilities. A natural hedge is at work here.

Even if a computational shortcut like Equation (13) is not available, a practical implication of the analysis above is that combining an after-tax valuation model with stochastic interest rates can have a material impact. For most insurance liabilities, the effect is to reduce the transfer price, increase the duration and lower the interest rate convexity.

[8] This can usually be done for an affine interest rate model.

After-Tax ALM

In practice, most North American insurers use a variety of metrics to manage day-to-day issues. Some examples include:

- Insurance product development/pricing is often done using an after-tax model of the form described in this paper.
- Earnings management is often driven by a local accounting standard such as US GAAP or IFRS.
- Asset/Liability Management is often driven by a cash flow testing model specified by local regulators.

Part of the authors' rant at the beginning of this paper was driven by the obvious inconsistencies outlined above. There are, of course, practical reasons for these inconsistencies. The purpose of this section is to outline what an ALM process might look like if it were engineered to be consistent with the model developed here.

Assets are different from insurance liabilities and tax issues are one of the main reasons. Suppose we have a vanilla bond whose observed transfer price in the market is TPA. How did the market determine the TPA? The answer is that bond traders do not take future tax-timing differences into account when calculating transfer prices because the tax base of an asset usually resets to market when traded. They do take permanent differences into account, if they apply.

If a bond was purchased at some time in the past, it will usually have a tax base A^{Tax}, which is different from its current transfer price. Most accounting models would assign a total market value of $MVA = TPA + \tau(A^{Tax} - TPA)$ to the asset since this is the amount of cash on hand if the asset were sold immediately. A starting point for the ALM process is then to set the MVA of the assets equal to the fulfillment value of the liabilities, that is,

$$MVA = TPA + \tau(A^{Tax} - TPA) = FVL = TPL + \tau(V^{Tax} - TPL).$$

So far, so good. If by ALM we mean something like duration matching under a yield curve or equity shock, then after-tax ALM would mean $\Delta TPA\ (1 - \tau) = \Delta TPL\ (1 - \tau)$ if A^{Tax} and V^{Tax} were unaffected by a market shock. The obvious conclusion is that the dollar duration of the TPA should be managed relative to the dollar duration of the liabilities as measured by TPL not FVL.

The only thing wrong with this ALM model is that it assumes the assets will be sold immediately. The problems created by this oversimplification would become clear to anyone actually managing off this model once they started doing a rigorous earnings-by-source analysis.

To the extent assets were not actually being traded on a regular basis, the asset tax-timing differences would start hitting the economic bottom line as they occurred (plus or minus).

Whatever asset turnover assumption we make will probably be wrong but using an assumption with a known bias does not make sense from a risk management perspective. There are a number of situations where continuous asset trading is not realistic. Here are two examples.

- The US GAAP accounting model can result in classifying assets as being either hold to maturity (HTM) or available for sale (AFS). There can be material accounting penalties for trading an asset in the HTM bucket. The result is that it is reasonable to assume assets assigned to the HTM bucket will not be traded. A version of the FVL/TPL valuation model should be used for assets in the HTM bucket.
- Imagine an insurer that has engineered the asset cash flows backing a block of insurance liabilities to be an exact after-tax match. This means the asset cash flows received in any time interval are equal to the pre-tax liability cash flows plus all related taxes on the combined asset/liability block. Using what is hopefully a transparent notation,[9] this is

$$ACF_t = LCF_t + \tau[ACF_t + A^{Tax}{}_t - A^{Tax}{}_{t-1} - PD^A{}_t - (LCF_t + V^{Tax}{}_t - V^{Tax}{}_{t-1} + PD^L{}_t)]. \quad (14)$$

  On rearrangement, this becomes the statement that after-tax asset cash flows are equal to after-tax liability cash flows, that is,

$$ACF_t(1-\tau) - \tau(A^{Tax}{}_t - A^{Tax}{}_{t-1} - PD^A{}_t) = LCF_t(1-\tau) - \tau(V^{Tax}{}_t - V^{Tax}{}_{t-1} + PD^L{}_t). \quad (15)$$

  As we saw in Equation (3), the present value of after-tax liability cash flows discounted at after-tax interest rates is the fulfillment value of the liabilities FVL. It therefore makes sense to define the fulfilment value of an asset FVA as the present value of after-tax asset cash flows using after-tax interest rates. There are practical problems with this idea that would need to be wrestled to the ground before it can be put into practice.[10] Once we get past those issues, we can define the concept of a going-concern value or GCV for an asset by $FVA = GCV(1-\tau) + \tau A^{Tax}$. The GCV for assets would then be calculated using the same principles as the TPL for liabilities. The fundamental ALM equation then becomes

[9] An equation to be taken seriously but not literally.

[10] One issue, among others, is how to handle credit and liquidity spreads in the process. That issue is outside the scope of this document.

$$FVA = GCV(1-\tau) + \tau A^{Tax} = FVL = TPL(1-\tau) + \tau V^{Tax}.$$

Duration matching now means matching the dollar duration of GCV to that of TPL.

Once we start calculating the present value of tax-timing differences on assets, we run into a number of the same issues that arose when valuing liabilities, in particular, income tax increases the discount rate when combined with stochastic interest rates. While this was good news for liabilities, it could be bad news for assets and needs to be addressed on a case-by-case basis.

In an after-tax world, the asset turnover assumption is an actuarial assumption just like mortality or lapse rates. If we know the asset will be sold immediately, the right way to value the asset is to report the MVA described above. At the other extreme, if we know the asset will be held to its maturity date (like an insurance liability), the FVL/TPL model developed earlier in this paper should apply because it takes into account future tax-timing and permanent differences.

We develop this idea in a little more detail. Let $A(t)$ be an asset's observed transfer price or fair value. We assume there is a corresponding known tax base $A^{Tax}(t)$, which depends on the tax jurisdiction we are working in. If we actually sell the asset, the after-tax cash flow received is $A(t)(1-\tau) + \tau A^{Tax}(t)$. While the insurer owns the asset, the after-tax cash flow is the sum of contractual cash flows $CF(t)$ adjusted for income tax in the amount $\tau[CF + \frac{dA^{Tax}}{dt} - PDA(t)]$. Here PDA is a permanent difference originating from the terms of the asset. It could be zero.

We will define the fulfillment value F of the asset to be the risk-neutral present value of after-tax cash flows using after-tax interest rates. This requires us to model the asset turnover rate using a traditional actuarial decrement rate μ. The fundamental equation defining F is then

$$\begin{aligned}\frac{dF}{ds} &= [r(1-\tau) + \mu]F \\ &\quad - \left\{CF - \tau\left[CF + \frac{dA^{Tax}}{ds} - PDA(s)\right] + \mu[A(s)(1-\tau) + \tau A^{Tax}(s)]\right\}.\end{aligned} \tag{16}$$

Assuming we know how to do this calculation, we can then define the analog of the transfer price of a liability by $G = \frac{F - \tau A^{Tax}}{1-\tau}$. We will call this the going-concern value of the asset.

Substituting $F = G(1-\tau) + \tau A^{Tax}$ into the equation gives us, after some simplification,

$$\frac{dG}{ds} = (r + \mu)G - \left[CF + \frac{\tau PDA}{1 - \tau} + \mu A + r\tau(G - A^{Tax})\right].$$

The fair value itself will satisfy an evolution equation of the form

$$\frac{dA}{ds} = rA - \left(CF + \frac{\tau PDA}{1 - \tau}\right).$$

This assumes the market value A as the present value of contractual cash flows adjusted for any grossed-up permanent tax differences. We can take the difference of the two equations to get

$$\frac{d(G - A)}{ds} = (r + \mu)(G - A) - r\tau(G - A^{Tax}).$$

This allows us to write

$$G(t) = A(t) + \int_t^T e^{-\int_t^s (r+\mu)dv}\, r\tau[G(s) - A^{Tax}(s)]ds.$$

While circular, this equation does allow us to understand the difference between the going-concern value G and the market or exit value A as the present value of tax on the interest that could be earned on the undiscounted deferred tax asset $\tau(G - A^{Tax})$.

If we sell the assets immediately (i.e., $\mu = \infty$), there is no interest to discount so $G = A$.

At the other extreme, if we know we will hold the asset to maturity, $\mu = 0$, we get an adjustment that depends on the relative differences $G - A^{Tax}$. Since $G \approx A$, this suggests a strategy of deferring the recognition of taxable gains on sale while accelerating the recognition of taxable capital losses. That conclusion won't be news to asset managers or tax authorities.

Conclusion

In this paper, we have presented an approach to after-tax risk management consistent with current actuarial thinking in a risk-neutral setting.

In the preparation for this paper, the authors considered why to discount distributable earnings with $r + \pi$ instead of $r(1 - \tau) + \pi$. The former approach seems to be favored by MCEV practitioners and by the American Academy of Actuaries' Public Policy Practice Note on the subject[11] and the latter was not discussed here. A proper discussion of this issue could easily be a paper on its own.

We presented a discussion of three risk management questions:

a. Balance sheet value
b. Sale/purchase value
c. Which to use

This is followed by a description of a high-level model of an income tax structure. The fulfillment value of a liability (FVL) was proposed as the solution to (a); the transfer price of the liability (TPL) was proposed as the solution to (b); and it was suggested for (c) that the transfer value is usually the one to use.

We then gave a formal derivation of fulfillment value of the liability and transfer price of the liability. For convenience, continuous assumptions were used (including the assumption that tax reserve at issue is 0). The discussion was generalized to include varying assumptions (like income tax) and stochastic interest rates.

Finally, there was a brief section on after-tax asset liability management.

None of the topics discussed was developed in enough detail for the ideas or formulas to be taken literally, but they should be taken seriously.

[11] American Academy of Actuaries, "Market Consistent Embedded Values," *Public Policy Practice Note*, March 2011, *https://www.actuary.org/files/MCEV%20Practice%20Note%20Final%20WEB%20031611.4.pdf/MCEV%20Practice%20Note%20Final%20WEB%20031611.4.pdf.*

Appendix: Some Numerical Examples

Product: 10-year endowment, continuous premiums and decrement, issue age 65, lapses ignored

Continuous fair value and transfer price assumptions

Pre-tax interest:	7.00%
Tax rate:	35.00%
Best estimate mortality:	CIA[12] 9704 Male Aggregate Age Nearest Birthday
Best estimate expense per premium (e):	20% in year 1, 2% thereafter
ΔQ:	1.5 per 1,000 flat
Cost of capital rate:	6.00%

Continuous tax reserve assumptions (ignores cash value floor)

Interest:	6.50%
Mortality:	CIA 8692 Male Aggregate Age Nearest Birthday
Method:	Full preliminary term

Table 2. Calculated Values

Policy Year	FVL	TPL	Tax Base	Death Benefit	Premium
0	–$75.63	–$116.35	$0.00	$1,000.00	$95.00
1	–$31.37	–$48.26	$0.00	$1,000.00	$95.00
2	$56.48	$40.32	$86.49	$1,000.00	$95.00
3	$149.26	$134.17	$177.27	$1,000.00	$95.00
4	$247.52	$233.83	$272.94	$1,000.00	$95.00
5	$351.89	$339.89	$374.18	$1,000.00	$95.00
6	$463.14	$453.12	$481.77	$1,000.00	$95.00
7	$582.22	$574.43	$596.68	$1,000.00	$95.00
8	$710.31	$705.00	$720.19	$1,000.00	$95.00
9	$848.94	$846.25	$853.93	$1,000.00	$95.00
10	$1,000.00	$1,000.00	$1,000.00	$1,000.00	$95.00

These may be reconciled using an approach discussed by Gould:[13]

[12] Canadian Institute of Actuaries

[13] G. E. Gould, "Control of Explicit Valuations of Individual Life Insurance: An Integrated Approach to Valuation, Forecasting and Earnings Analysis," *Proceedings: Canadian Institute of Actuaries* XXV, no. 2 (March 1993–94).

Table 3. Reconciliation of Transfer Price of Liabilities (to within about a cent)

Policy Year	Year Start TPL	Premiums	Expenses	Claims	Interest on DToL	Pre-tax Interest	Release on Death	Year End TPL
1	–$116.35	$94.81	–$18.96	–$3.94	$1.95	–$5.58	–$0.19	–$48.26
2	–$48.26	$94.75	–$1.89	–$5.30	$1.12	–$0.31	$0.21	$40.32
3	$40.32	$94.68	–$1.89	–$6.73	$1.05	$5.84	$0.90	$134.17
4	$134.17	$94.60	–$1.89	–$8.33	$0.97	$12.35	$1.95	$233.83
5	$233.83	$94.52	–$1.89	–$10.13	$0.87	$19.26	$3.44	$339.89
6	$339.89	$94.42	–$1.89	–$12.17	$0.74	$26.61	$5.51	$453.12
7	$453.12	$94.31	–$1.89	–$14.46	$0.60	$34.44	$8.31	$574.43
8	$574.43	$94.19	–$1.88	–$17.02	$0.44	$42.84	$12.00	$705.00
9	$705.00	$94.05	–$1.88	–$19.88	$0.27	$51.87	$16.82	$846.25
10	$846.25	$93.90	–$1.88	–$23.05	$0.09	$61.64	$23.05	$1,000.00

Table 4. Reconciliation of Fulfillment Value of Liabilities (to within about a cent)

Policy Year	Year Start FVL	Premiums: AT Cash Flows	Expenses: AT Cash Flows	Claims: AT Cash Flows	Tax Rate x ΔV^{Tax}	After-tax Interest	Release on Death	Year End FVL
1	–$75.62	$61.63	–$12.33	–$2.56	$0.00	–$2.36	–$0.12	–$31.36
2	–$31.36	$61.59	–$1.23	–$3.44	$30.11	$0.53	$0.30	$56.49
3	$56.49	$61.54	–$1.23	–$4.37	$31.35	$4.48	$1.00	$149.27
4	$149.27	$61.49	–$1.23	–$5.41	$32.69	$8.66	$2.06	$247.53
5	$247.53	$61.44	–$1.23	–$6.58	$34.10	$13.08	$3.56	$351.90
6	$351.90	$61.37	–$1.23	–$7.91	$35.60	$17.78	$5.64	$463.15
7	$463.15	$61.30	–$1.23	–$9.40	$37.20	$22.78	$8.42	$582.22
8	$582.22	$61.22	–$1.22	–$11.06	$38.94	$28.13	$12.09	$710.32
9	$710.32	$61.13	–$1.22	–$12.92	$40.87	$33.89	$16.88	$848.94
10	$848.94	$61.04	–$1.22	–$14.98	$43.06	$40.12	$23.05	$1,000.00