Exam FM   Financial Mathematics
Spring 2009

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Exam FM is a three-hour multiple choice examination and is identical to CAS Exam 2. The examination is jointly sponsored and administered by the SOA, CAS, and the Canadian Institute of Actuaries (CIA). The examination is also jointly sponsored by the American Academy of Actuaries (AAA) and the Conference of Consulting Actuaries (CCA).

Exam FM is administered as a computer-based test. For additional details check http://www.beanactuary.org/exams/cbt.cfm.

The goal of the syllabus for this examination is to provide an understanding of the fundamental concepts of financial mathematics, and how those concepts are applied in calculating present and accumulated values for various streams of cash flows as a basis for future use in: reserving, valuation, pricing, asset/liability management, investment income, capital budgeting and valuing contingent cash flows. The candidate will also be given an introduction to financial instruments, including derivatives, and the concept of no-arbitrage as it relates to financial mathematics.

Exam FM assumes a basic knowledge of calculus and an introductory knowledge of probability.

The following learning objectives below are presented with the understanding that candidates are allowed to use specified calculators on the exam. The education and examination of candidates reflects that fact. In particular, such calculators eliminate the need for candidates to learn and be examined on certain mathematical methods of approximation.

Check the Updates section of the SOA Web site for any changes to the exam or syllabus.

LEARNING OBJECTIVES

1. Candidates will know definitions of key terms of financial mathematics: inflation; rates of interest [simple, compound (interest and discount), real, nominal, effective, dollar-weighted, time-weighted, spot, forward], term structure of interest rates; force of interest (constant and varying); equivalent measures of interest; yield rate; principal; equation of value; present value; future value; current value; net present value; accumulation function; discount function; annuity certain (immediate and due); perpetuity (immediate and due); stocks (common and preferred); bonds (including zero-coupon bonds); other financial instruments such as mutual funds, and guaranteed investment contracts.

Specifically, candidates are expected to demonstrate the ability to:
   a. Choose the term, given a definition.
   b. Define a given term.
   c. Determine an equation of value, given a valuation problem involving one or more sets of cash flows at specified times.

2. Candidates will understand key procedures of financial mathematics: determining equivalent measures of interest; discounting; accumulating; determining yield rates; estimating the rate of return on a fund; and amortization.

Specifically, candidates are expected to demonstrate the ability to:
   a. Calculate the equivalent annual effective rate of interest or discount, given a nominal annual rate and a frequency of interest conversion, discrete or continuous, other than annual.
   b. Calculate the equivalent effective rate of interest or discount per payment period given a payment period different from the interest conversion period.
   c. Estimate the interest return on a fund.

d. Calculate the appropriate equivalent <u>single</u> value [present value, net present value, future (accumulated) value or combination], given a set of cash flows (level or varying), where the cash flows may occur as frequently or more frequently than interest or discount is accrued, an appropriate term structure of interest rates, the method of crediting interest (e.g., portfolio or investment year) as necessary, an appropriate set of inflation rates as necessary, and accounting for reinvestment interest rates as necessary.
For example:
   i. Calculate the loan amount or outstanding loan balance, given a set of loan payments (level or varying) and the desired yield rate (level or varying).
   ii. Calculate the price of a bond (callable or non-callable), given the bond coupons, the redemption value, the term of the bond (constant or varying), the coupon interest rate, and the desired yield rate (level or varying).
   iii. Calculate the value of a stock, given the pattern of dividends and the desired yield rate (level or varying).
   iv. Calculate the net present value, given a set of investment contributions and investment returns.
e. Calculate a unique yield rate, when it exists, given a set of investment cash flows.
f. Calculate the amount(s) of investment contributions, given there is more than one contribution, and given a set of yield rates, the amount(s) and timing of investment return(s), and the desired timing of the investment contributions.
g. Calculate the amount(s) of investment returns, given there is more than one return, and given a set of yield rates, the amount(s) and timing of investment contribution(s) and the desired timing of the investment returns; for example:
   i. Calculate loan payments, given the loan amount(s), the term of the loan, and the desired yield rate (level or varying).
   ii. Calculate the principal and interest portions of a loan payment, given the loan amount, the set of loan payments (level or varying), and a set of interest rates (level or varying).
   iii. Calculate bond coupons or redemption values, given the bond price, the term of the bond, and the desired yield rate (level or varying).
h. Calculate the term of an investment, given a set of cash flows (level or varying), and a set of interest rates (level or varying); for example:
   i. Calculate the length of time required to accumulate a given amount, given the yield rate and an initial amount.
   ii. Calculate the length of time to repay a given loan amount, given the loan payments and the loan interest rate(s).
   iii. Calculate the time to maturity of a bond, given the price of the bond, the coupon payments, redemption value, and yield rate.

3. Candidates will know <u>definitions</u> of key terms of modern financial analysis at an introductory and intuitive level, and be able to complete basic calculations involving such terms: yield curves, spot rates, forward rates, duration, convexity, and immunization.

Specifically, candidates are expected to demonstrate the ability to:
a. Choose the term, given a definition.
b. Write the definition, given a term.
c. Perform calculations such as:
   i.   measuring interest rate risk using duration and convexity.
   ii. basic immunization calculations.
   iii. cash flow matching calculations (the terms dedication and asset-liability matching are used in the readings as equivalent to cash flow matching).

4. Candidates will know <u>definitions</u> of key terms of financial economics at an introductory level: derivatives, forwards, futures, short and long positions, call and put options, spreads, collars, hedging, arbitrage, and swaps.

Specifically, candidates are expected to demonstrate the ability to:
   a. Explain the need for financial risk management. Explain how derivative securities can be used as tools to manage financial risk. Explain the reasons to hedge and not to hedge.

   b. Define, evaluate payoff, and evaluate profit of basic derivates contracts [forward contracts, futures contracts, American and European put and call options, simple commodity swaps, and interest rate swaps].

   c. Define, evaluate payoff, and evaluate profit of basic trading strategies [floors, caps, covered puts and calls, synthetic forwards, spreads (including bull, bear, box, and ratio spreads), collars (including zero-cost collars), and straddles (including strangles, written straddles, and butterfly spreads)].

   d. Explain no-arbitrage pricing including put-call parity and pricing of prepaid forward contracts on stocks (with and without dividends). Explain arbitrage with respect to synthetic forward contracts and the effect of transaction costs.

   e. Determine forward price from prepaid forward price. Explain the relationship between forward price and futures price. Explain the relationship between forward price and future stock price.

Textbooks

Knowledge and understanding of financial mathematics concepts are significantly enhanced through working out problems based on those concepts. Thus in preparing for the Financial Mathematics examination, whichever of the source of textbooks candidates choose to use, candidates are encouraged to work out the textbook exercises related to the listed readings.

Suggested Textbooks for topics in Learning Outcomes 1 through 3
There is not a single textbook required for the topics covered by Learning Objectives 1 through 3. The texts listed below are representative of the textbooks available to cover the material on which the candidate may be tested. Not all topics may be covered at the same level in each text. The candidate may wish to use one or more texts in their preparation for the examination. The # indicates new or updated material or changes in the sections selected.

<u>**REFERENCES**</u>

- # Broverman, S.A., *Mathematics of Investment and Credit* (Fourth Edition), 2008, ACTEX Publications:

  Chapter 1 (1.1-1.7)
  Chapter 2 (2.1 -2.4 excluding 2.4.2 and 2.4.3)
  Chapter 3 (3.1-3.3, excluding 3.2.1 and 3.2.2)
  Chapter 4 (4.1-4.3.1)
  Chapter 5 (5.1-5.3 excluding 5.1.4 and 5.3.2)
  Chapter 6 (6.1-6.3 excluding 6.2)
  Chapter 7 (7.1-7.2)
  Chapter 8 (8.1, 8.3.1 and 8.4.1–8.4.2)

NOTE: Candidates may also use the Third Edition of *Mathematics of Investment and Credit.* The following chapter references apply:

Chapter 1 (1.1-1.6)
Chapter 2 (2.1 -2.4 excluding 2.4.2 and 2.4.3)
Chapter 3 (3.1-3.3 excluding pp. 188–189)
Chapter 4 (4.1-4.3.1)
Chapter 5 (5.1-5.3 excluding 5.1.4 and 5.3.2)
Chapter 6 (6.1-6.3 excluding 6.2)
Chapter 7 (7.1-7.2)
Chapter 8 (8.2.1, 8.2.4, 8.3.1–8.3.2).

- # Daniel, J.W. and Vaaler, L.J.F., *Mathematical Interest Theory,* (Second Edition) 2009,   The Mathematical Association of America,

  Chapter 1 (1.3-1.12, 1.14)
  Chapter 2 (2.2-2.7)
  Chapter 3 (3.2-3.9, 3.11, 3.13)
  Chapter 4 (4.2-4.6)
  Chapter 5 (5.2-5.4)
  Chapter 6 (6.2-6.6, 6.9)
  Chapter 7 (7.1)
  Chapter 8 (8.3)
  Chapter 9 (9.1-9.5)

  NOTE: Candidates may also use the First Edition of *Mathematical Interest Theory* (Publisher: Prentice Hall). The same chapter references apply.

- Kellison, S.G., *The Theory of Interest* (Third Edition), 2008, Irwin/McGraw-Hill,

  Chapter 1 (1.2-1.10)
  Chapter 2 (2.3-2.6)
  Chapter 3 (3.2-3.8)
  Chapter 4 (4.2-4.9)
  Chapter 5 (5.2-5.6)
  Chapter 6 (6.2-6.7, 6.10)
  Chapter 7 (7.2-7.7)
  Chapter 9 (9.4)
  Chapter 10 (10.2-10.5)
  Chapter 11 (11.2-11.8)

- Ruckman, C, and Francis, J., *Financial Mathematics: A practical Guide for Actuaries and other Business Professionals* (Second Edition), 2005, BPP Professional Education,

  Chapter 1
  Chapter 2
  Chapter 3 (3.1-3.9)
  Chapter 4 (4.1-4.5)
  Chapter 5
  Chapter 6 (6.1-6.3 excluding 6.1.6-6.1.7)

Chapter 7 (7.1-7.9)
Chapter 8 (8.1-8.3)

Textbook for the topics in Learning Outcome 4
- McDonald, R.L., *Derivatives Markets* (Second Edition), 2006, Addison- Wesley,

Chapter 1 (1.1-1.4)
Chapter 2 (2.1-2.6 and Appendix 2.A)
Chapter 3 (3.1-3.5)
Chapter 4 (4.1-4.4)
Chapter 5 (5.1-5.4 and Appendix 5.B)
Chapter 8 (8.1-8.2)

Study Notes: The # indicates new or updated material.

Code	**Title**
	Derivatives Markets, Errata, 2006 Second Edition, by R. McDonald *http://www.kellogg.northwestern.edu/faculty/mcdonald/htm/typos2e.html*
	Mathematics of Investment and Credit, Errata, 2004 Third Edition, by S. Broverman http://www.actexmadriver.com/client/client_pages/actex_errata2.cfm
	Notation and terminology used for Exam FM/ Exam 2
Past Exams	All released exam papers, since 2000, can be found at: http://www.soa.org/education/exam-req/syllabus-study-materials/ edu-multiple-choice-exam.aspx
FM-09-05	Exam FM Sample Questions and Solutions
FM-09-07	Sample Questions and Solutions for *Derivatives Markets*
FM-22-05	Review of Calculator Functions for the Texas Instruments BA-35
FM-23-05	Review of Calculator Functions for the Texas Instruments BA II Plus