QFI-Investment Risk Management Formula Package

Spring and Fall 2023

Exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula package was developed by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not in the formula package.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes. In sources where some equations are numbered and others are not, the page number is provided instead.

Quantitative Enterprise Risk Management, Hardy & Saunders, 2022

Chapter 1 (Black-Scholes formula to be used for questions of other chapters)

P. 7

(1.3), (1.4)

$$p_0 = Ke^{-rT}\Phi(-d_2(0,T)) - S_0\Phi(-d_1(0,T))$$

$$d_1(t_1,t_2) = \frac{Ln(S_{t_1}/K) + (r + \sigma^2/2)(t_2 - t_1)}{\sigma\sqrt{t_2 - t_1}}$$

$$d_2(t_1,t_2) = d_1(t_1,t_2) - \sigma\sqrt{t_2 - t_1}$$

Chapter 3 Risk Measures

P. 79

(3.7)

$$ES_\alpha = \frac{(\beta' - \alpha)Q_\alpha + (1-\beta')E\left[L \middle| L > Q_\alpha\right]}{1-\alpha}$$

P.83

(3.11) (3.12)

$$ES_\alpha = \mu + \frac{\sigma}{1-\sigma}\phi(\frac{Q_\alpha - \mu}{\sigma}) = \mu + \frac{\sigma}{1-\sigma}\phi(z_\alpha)$$

Chapter 6 Copulas

P. 169 - 170

$$F_j(x_j) = \Pr\left[X_j \le x_j\right]$$

$$C(F_1(x_1),....,F_m(x_m)) = \Pr\left[X_1 \le x_1,....,X_m \le x_m\right]$$

for j=1 ,….,m

$$C(u_1,u_2,...,u_m) = u_1 * u_2 * ... * u_m$$

$$C(u_1,u_2,...,u_m) = \min(u_1,u_2,...,u_m)$$

$$C(u,v) = \max(u+v-1,0)$$

P. 181 - 182

$$C(u_1,u_2,...,u_m) = (u_1^{-\theta} + u_2^{-\theta} + ... + u_m^{-\theta} - (m-1))^{-\frac{1}{\theta}}, \theta > 0$$

$$C(u_1,u_2,...,u_m) = Exp\left\{-((-Ln(u_1))^\theta + (-Ln(u_2))^\theta + ... + (-Ln(u_m))^\theta)^{\frac{1}{\theta}}\right\}, \theta > 1$$

$$C(u_1,u_2,...,u_m) = \Phi_\rho(z_{u_1}, z_{u_2}, ..., z_{u_m}), -1 \le \rho \le 1, z_\alpha = \Phi^{-1}(\alpha)$$

$$\overline{C}(u,v) = \Pr\left[F_x(X) > u, F_y(Y) > u\right], \widehat{C}(u,v) = \overline{C}(1-u,1-v)$$

$$\rho(X,Y)=\frac{Cov\left[X,Y\right]}{\sqrt{Var\left[X\right]Var\left[Y\right]}}$$

$$\rho_s(X,Y)=\rho(U,V)$$

$$r=\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{r_{x_i}-\overline{r_x}}{s_{r_x}}\right)\left(\frac{r_{y_i}-\overline{r_y}}{s_{r_y}}\right)$$

P. 185 - 189

$$\tau_k=\Pr\left[(X-X^*)(Y-Y^*)>0\right]-\Pr\left[(X-X^*)(Y-Y^*)<0\right]$$

$$=E\left[sign((X-X^*)(Y-Y^*))\right]$$

$$sign(z)=\{-1,z<0,0,z=0,1,z>0\}$$

$$t_k=\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}sign((x_i-x_j)(y_i-y_j))$$

$$C(u,v)=\phi^{-1}(\phi(u)+\phi(v))$$

$$\tau_k=1+4\int_0^1\frac{\phi(t)}{\frac{d}{dt}\phi(t)}dt$$

$$\lambda_U=2-2\lim_{t\to 0}\frac{\frac{d}{dt}\phi^{-1}(2t)}{\frac{d}{dt}\phi^{-1}(t)}$$

P. 193 - 194

$$\lambda_L=2\lim_{t\to\infty}\frac{\frac{d}{dt}\phi^{-1}(2t)}{\frac{d}{dt}\phi^{-1}(t)}$$

$$\tau_k=\frac{\theta}{\theta+2},\lambda_U=0,\lambda_L=2^{\frac{-1}{\theta}}$$

$$\tau_K=\frac{\theta-1}{\theta},\lambda_U=2-2^{1/\theta},\lambda_L=0$$

$$\phi(u)=\frac{(u^{-\theta}-1)^{\delta}}{\theta^{\delta}}$$

Chapter 8 Market Risk Models

P. 233

$$dS_t=\mu^*S_tdt+\sigma S_tdW_t,\sigma>0$$

$$\frac{S_{t+k}}{S_t} \sim LogN(k(\mu^* - \frac{\sigma^2}{2}), \sqrt{k}\sigma), k > 0$$

$$\Rightarrow Log(\frac{S_{t+k}}{S_t}) \sim N(k(\mu^* - \frac{\sigma^2}{2}), k\sigma^2)$$

P. 235

$$Y_{t,h} \sim N(h\mu, h\sigma^2)$$

$$e^{Y_{t,h}} = \frac{S_t}{S_{t-h}} \sim LnN(h\mu, \sqrt{h}\sigma)$$

P. 236

(8.1) (8.2)

$$Y_t = \mu + \sigma_t \varepsilon_t$$

$$\sigma_t^2 = a_0 + a_1(Y_{t-1} - \mu)^2 + b\sigma_{t-1}^2, a_0, a_1, b > 0$$

P. 238

$$E_0\left[\sigma_t^2\right] = a_0(\frac{1-(a_1+b)^{t-1}}{1-(a_1+b)}) + (a_1+b)^{t-1}\sigma_1^2$$

For

$$a_1 + b \neq 1$$

(8.6)

$$\lim_{t\to\infty} a_0(\frac{1-(a_1+b)^{t-1}}{1-(a_1+b)}) + (a_1+b)^{t-1}\sigma_1^2 = \frac{a_0}{1-(a_1+b)}$$

P. 244, 246

(8.9) (8.10) (8.11) (8.12)

$$Y_t \left| \{\rho_t = 1\} \right. \sim N(\mu_1, \sigma_1^2)$$

$$Y \left| \{\rho_t = 2\} \right. \sim N(\mu_2, \sigma_2^2)$$

$$\Pr\left[\rho_t = 1 \middle| \rho_{t-1} = 1\right] = p_{11}$$

$$\Pr\left[\rho_t = 1 \middle| \rho_{t-1} = 2\right] = p_{21}$$

$$\Pr\left[\rho_t = 2 \middle| \rho_{t-1} = 1\right] = p_{12} = 1 - p_{11}$$

$$\Pr\left[\rho_t = 2 \middle| \rho_{t-1} = 2\right] = p_{22} = 1 - p_{21}$$

$$\pi_1 = \frac{p_{21}}{p_{12} + p_{21}}$$

$$\pi_2 = \frac{p_{12}}{p_{12} + p_{21}}$$

P. 250

(8.13) $$l(\theta)=\sum_{i=1}^{n} Lnf(x_i \,|\, x_1,...,x_{i-1};\theta)$$

Chapter 9 Short-Term Portfolio Risk

P. 263

$$L \sim N(\mu_L,\sigma_L^2)$$
$$VaR_\alpha(L)=\mu_L+z_\alpha\sigma_L$$
$$ES_\alpha(L)=\mu_L+\frac{\phi(z_\alpha)}{1-\alpha}\sigma_L$$

(9.1) $$VaR_\alpha(L_{t,h})=Q_\alpha(L_{t,h})=-Q_{1-\alpha}(R_{t,h})V(t)=Q_\alpha(-R_{t,h})V(t)$$

P. 266 - 267

$$R_{j,t,h}=\frac{(S_j(t+h)-S_j(t))}{S_j(t)}$$
$$L_{t,h}=-\sum_{j=1}^{n} a_j S_j(t) R_{j,t,h}$$
$$L_{t,h} \sim N(0,h\sigma_L^2)$$
$$\sigma_L^2=\sum_{k=1}^{n}\sum_{j=1}^{n} a_j a_k S_j(t) S_k(t)\rho_{jk}\sigma_j\sigma_k$$
$$VaR_\alpha(L_{t,h})=z_\alpha\sqrt{h}\sigma_L$$
$$ES_\alpha(L_{t,h})=\frac{\phi(z_\alpha)}{1-\alpha}\sqrt{h}\sigma_L$$

P. 268 - 269

$$V(t+h)-V(t)\approx\frac{\partial V}{\partial S}(S(t+h)-S(t))+\frac{1}{2}\frac{\partial^2 V}{\partial S^2}(S(t+h)-S(t))^2+....+\frac{\partial V}{\partial t}h$$
$$L_{t,h}=V(t)-V(t+h)\approx-\Delta(t)S(t)R_{t,h}$$
$$\sigma_L=|\Delta(t)|S(t)\sigma_S$$
$$VaR_\alpha(L_{t,h})\approx z_\alpha\sqrt{h}\sigma_L=z_\alpha\sqrt{h}\,|\Delta(t)|\,S(t)\sigma_S$$
$$ES_\alpha(L_{t,h})\approx\frac{\phi(z_\alpha)}{1-\alpha}\sqrt{h}\sigma_L=\frac{\phi(z_\alpha)}{1-\alpha}\sqrt{h}\,|\Delta(t)|\,S(t)\sigma_S$$

P. 273

$$\Gamma_j(t)=\frac{\phi(d_1(t))}{S_j(t)\sigma\sqrt{T-t}}$$
$$V(t)-V(t+h)\approx-\Delta_j(t)S_j(t)R_{j,t,h}-\frac{1}{2}\Gamma_j(t)S_j(t)^2R_{j,t,h}^2$$
$$L_{t,h}\approx-\Delta_j(t)S_j(t)R_{j,t,h}-\frac{1}{2}\Gamma_j(t)S_j(t)^2R_{j,t,h}^2$$

(9.14) (9.15)

$$VaR_\alpha \approx h\mu_L + z_\alpha \sqrt{Var\left[L_{t,h}\right]}$$
$$= -h\frac{1}{2}\Gamma_j(t)S_j(t)^2\sigma_j^2 + z_\alpha \sqrt{\Delta_j(t)^2 S_j(t)^2 \sigma_j^2 h + \frac{1}{2}\Gamma_j(t)^2 S_j(t)^4 \sigma_j^4 h^2}$$
$$ES_\alpha \approx \mu_L h + \frac{\phi(z_\alpha)}{1-\alpha}\sqrt{Var\left[L_{t,h}\right]}$$
$$= \frac{-h}{2}\Gamma_j(t)S_j(t)^2\sigma_j^2 + \frac{\phi(z_\alpha)}{1-\alpha}\sqrt{\Delta_j(t)^2 S_j(t)^2 \sigma_j^2 h + \frac{1}{2}\Gamma_j(t)^2 S_j(t)^4 \sigma_j^4 h^2}$$

Chapter 19 Behavioral Risk Management

P. 581

$$EU = \sum_j u(W + x_j)\, p(x_j)$$

$$EIU = \sum_j (u(W + x_j) - u(W))\, p(x_j)$$

P. 583

(19.1)

$$v(x^*) = \begin{cases} (x^*)^\alpha & x^* > 0 \\ -\lambda((-x^*)^\beta) & x^* \le 0 \end{cases}$$

P. 584

$$w(q) = \frac{q^\gamma}{(q^\gamma + (1-q)^\gamma)^\gamma}$$

P. 585

$$\pi(x_1) = w\big(F(x_1)\big),$$

$$\pi(x_j) = w(F(x_j)) - w(F(x_{j-1})) \qquad j = 2, 3, \ldots, n.$$

QFII-110-15: The Devil is in the Tails: Actuarial Mathematics and the Subprime Mortgage Crisis

P. 4

(4.1)

$$C_{\rho}^{gau}(u,v) := \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left\{-\frac{s^2 - 2\rho st + t^2}{2(1-\rho^2)}\right\} dsdt$$

where

$$u, v \in [0,1], |\rho| < 1$$

QFII-129-23: Ch 2, Risk Budgeting Approach of Introduction to Risk Parity and Risk Budgeting

P. 74

(2.1)

$$SD_c(x) = -x^{\mathrm{T}}\mu + c * \sqrt{x^{\mathrm{T}} \sum x}$$

(2.2)

$$VaR_{\alpha}(x) = -x^{\mathrm{T}}\mu + \Phi^{-1}(\alpha)\sqrt{x^{\mathrm{T}} \sum x}$$

P. 75

(2.3)

$$ES_{\alpha}(x) = -x^{\mathrm{T}}\mu + \frac{\sqrt{x^{\mathrm{T}} \sum x}}{1-\alpha} \phi(\Phi^{-1}(\alpha))$$

P. 80

(2.10)

$$RC_i = x_i * (-\mu_i + c \frac{(\sum x)_i}{\sqrt{x^{\mathrm{T}} \sum x}})$$

(2.11)

$$RC_i = x_i * (-\mu_i + \Phi^{-1}(\alpha) \frac{(\sum x)_i}{\sqrt{x^{\mathrm{T}} \sum x}})$$

(2.12)

$$RC_i = x_i * (-\mu_i + \frac{(\sum x)_i}{(1-\alpha)\sqrt{x^{\mathrm{T}} \sum x}} \phi(\Phi^{-1}(\alpha)))$$

Fixed Income Securities: Valuation, Risk, and Risk Management, Veronesi, 2010

Chapter 5 Interest Rate Derivatives: Forwards and Swaps

P.157

(5.5)

$$F(t,T_1,T_2)=\frac{1}{(1+\frac{f_n(t,T_1,T_2)}{n})^{n*(T_2-T_1)}}$$

(5.6)

$$F(t,T_1,T_2)=e^{-f(t,T_1,T_2)(T_2-T_1)}$$

P. 160

(5.14)

$$f(0,T,T+\Delta)=r(0,T)+(T+\Delta)*\frac{r(0,T+\Delta)-r(0,T)}{\Delta}$$

P. 161

(5.21)

$$r(0,T_n)=\frac{1}{T_n}\sum_{i=1}^{n}f(0,T_{i-1},T_i)*\Delta$$

P. 166

(5.25)

Value of FRA at t= $V^{FRA}(t)=V^{fixed}(t)-V^{floating}(t)=N*\left[M*Z(t,T_2)-Z(t,T_1)\right]$

P. 170

(5.34)

$$P_c^{fwd}(0,T,T^*)=\frac{c}{2}*\sum_{i=1}^{n}P_z^{fwd}(0,T,T_i)+P_z^{fwd}(0,T,T_n)$$

P. 175

(5.40)

$$V^{swap}(t_i;c,T)=100-(\frac{c}{2}*100*\sum_{j=i+1}^{M}Z(T_i,T_j)+Z(T_i,T_M)*100)$$

P. 176

(5.43)

$$c = n * \left(\frac{1 - Z(0, T_M)}{\sum_{j=1}^{M} Z(0, T_j)}\right)$$

P. 181

(5.51)

$$f_n^s(0, T, T^*) = n * \frac{1 - F(0, T, T^*)}{\sum_{j=1}^{M} F(0, T, T_j)}$$ (This is a correction to the text formula)

Chapter 6 Interest Rate Derivatives: Futures and Options

P. 203

(6.4)

P&L from futures at t=k*contract size*[$P^{fut}(t, T) - P^{fut}(t - dt, T)$]

Handbook of Fixed Income Securities, Fabozzi, Frank J., 2021

Chapter 69 Credit Derivative Valuation and Risk

P. 1693

$$V_{\text{Premium}} = \frac{C(T)}{2} \sum_{i=1}^{N} \Delta(t_{i-1}, t_i) Z(t_i)(Q(t_i) + Q(t_{i-1}))$$

P. 1964

$$V_{\text{Protection}} = (1 - R) \sum_{n=1}^{M} Z(t_n)(Q(t_{n-1}) - Q(t_n))$$

P. 1696

$$Q(T) = \exp\left(-\int_0^T h(t)dt\right)$$

P. 1700

$$\text{Spread DV01} = -A(T, S + 1bp) \cdot 1bp + (S - C) \cdot (A(T, S) - A(\boldsymbol{T}, S + 1bp))$$

P. 1704

$$A(\bar{S},T) \approx \frac{1-\exp\left(-\left(W+\frac{\bar{S}}{(1-R)}\right)T\right)}{W+\frac{\bar{S}}{(1-R)}} \times \frac{365}{360}$$

P. 1706

$$V^{Index} = \frac{1}{P}\sum_{p=1}^{P}(C_I(T)-S_p(T))A_p(T) = (C_I(T)-\bar{S}_I(\boldsymbol{T}))\bar{A}_I(T)$$

P. 1707

$$Q_I(t) = \frac{1}{P}\sum_{p=1}^{P}Q_p(t) = N_I(t)$$

Investment Risk Management, Baker & Filbeck, 2015

Chapter 8 Liquidity Risk

P. 144

(8.1)

$$LR_{it} = \frac{\sum_{t=1}^{T} P_{it}V_{it}}{\sum_{t=1}^{T}\left|P_{it}-P_{it-1}\right|}$$

(8.2)

$$ML_t = \sum_{i=1}^{N}\frac{(P_{it}-P_{it-1})^2}{V_{it}}$$

(8.3)

$$HH_i = \frac{(P_{i\max}-P_{i\min})/P_{i\min}}{V/N * P_{avg}}$$

P. 145

(8.4)

$$TR_{it} = \frac{\sum_{i=1}^{N} P_{it} * Q_{it}}{P_{avg} * V}$$

(8.5)

$$PCTR_{it} = \frac{|\%\Delta P|}{TR_{it}}$$

(8.6)

$$ILLIQ_{it} = \frac{|R_{it}|}{\sum_{i=1}^{N} P_{it} * Q_{it}}$$

P. 146

(8.9)

$$VR_i = \frac{VAR(R_{it})}{K * VAR(r_{it})}$$

P. 147

(8.10)

$$MR_i = \frac{1}{K}\sum_{k=1}^{K}\left|\frac{P_{ik} - P_{ik-1}}{P_{ik}}\right| * 100$$

Chapter 25 Futures

P. 488

$$N_f = \frac{N_A}{qS_0}, \qquad (25.4)$$

P. 490

$$N_f = \frac{hN_A}{qS_0}, \qquad (25.9)$$

P. 494

$$p = \frac{i_t^f - i_t^{\text{pre}}\left(\frac{d_1}{B} + \frac{d_2}{B}\right)}{\left(i_t^{\text{post}} - i_t^{\text{pre}}\right)\frac{d_2}{B}}. \qquad (25.15)$$

Credit-Risk Modelling, Bolder

Chapter 1

(1.3) $\mathrm{VaR}_\alpha(L) = \inf\Big(x : \mathbb{P}(L \le x) \ge 1-\alpha\Big)$

(1.6) $\mathrm{VaR}_\alpha(L) = F_L^{-1}(1-\alpha)$

Chapter 2

(2.26) $\widehat{\mathbb{E}(L_N)} = \frac{1}{M}\sum_{m=1}^{M} L_N^{(m)}$

(2.27) $\widehat{\sigma(L_N)} = \sqrt{\frac{1}{M-1}\sum_{m=1}^{M}\left(L_N^{(m)} - \widehat{\mathbb{E}(L_N)}\right)^2}$

(2.28) $\widehat{\mathrm{VaR}_\alpha(L_N)} = \tilde{L}_N(\lceil \alpha \cdot M \rceil)$

(2.29) $\widehat{\mathcal{E}_\alpha(L_N)} = \frac{1}{M-\lceil \alpha\cdot M\rceil+1}\sum_{m=\lceil \alpha\cdot M\rceil}^{M} L_N^{(m)}$ This is a correction to an error in the text.

(2.45) $\mathbb{E}\Big((\mathbb{D}_N - Np)^2\Big) = Np(1-p)$

(2.60) $f_{\mathbb{D}_N}(k) = \mathbb{P}(\mathbb{D}_N = k) = \frac{e^{-N\lambda}(N\lambda)^k}{k!}$

(2.61) $F_{\mathbb{D}_N}(m) = \mathbb{P}(\mathbb{D}_N \le m) = \sum_{k=0}^{m}\frac{e^{-N\lambda}(N\lambda)^k}{k!}$

Chapter 3

(3.10) $\mathbb{E}(\mathbb{D}_N) = N\bar{p}$

(3.11) $\mathrm{var}(\mathbb{D}_N) = N\bar{p}(1-\bar{p}) + N(N-1)\mathrm{var}(p(Z))$

(3.17) $f_Z(z) = \frac{1}{B(\alpha,\beta)} z^{\alpha-1}(1-z)^{\beta-1}$

(3.18) $B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt$

(3.19) $B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

(3.20) $\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}dx$

(3.21) $\mathbb{E}(Z) = \frac{\alpha}{\alpha+\beta}$

(3.22) $\mathrm{var}(Z) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

(3.26) $\rho_{\mathcal{D}} = \rho\left(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}\right) = \frac{1}{\alpha+\beta+1}$

(3.27) $\mathbb{E}(Z) = \bar{p} = \frac{\alpha}{\alpha+\beta}$

(3.35) $p_1^{-1}(z) \equiv y = \frac{1}{\sigma_1}\left(\ln\left(\frac{z}{1-z}\right) - \mu_1\right)$

(3.36) $p_2^{-1}(z) \equiv y = \frac{\Phi^{-1}(z)-\mu_2}{\sigma_2}$

(3.39) $f_{p_1(Z)}(z) = \frac{1}{z(1-z)\sqrt{2\pi\sigma_1^2}} exp\left[-\frac{\left(\ln\left(\frac{z}{1-z}\right)-\mu_1\right)^2}{2\sigma_1^2}\right]$

(3.52) $\mathbb{P}(\mathbb{D}_N = k) = \int_{\mathbb{R}_+}\frac{e^{-\lambda(s)}\lambda(s)^k}{k!} f_S(s)ds$

(3.53) $f_S(s) = \frac{b^a e^{-bs}s^{a-1}}{\mathit{\Gamma}(\alpha)}$

(3.54) $\mathbb{P}(\mathbb{D}_N = k) = \frac{\mathit{\Gamma}(a+k)}{\mathit{\Gamma}(k+1)\mathit{\Gamma}(a)} q_1^a(1-q_1)^k, \quad q_1 = \frac{b}{b+1}$

(3.81) $\sigma(p_n(S)) = p_n\sqrt{\frac{\omega_1^2}{a}}$

(3.83) $\mathbb{E}(\mathbb{I}_{\{X_n \ge 1\}}\mathbb{I}_{\{X_m\ge 1\}}) = p_n p_m\left(\omega_0^2 + 2\omega_0\omega_1 + \omega_1^2\mathbb{E}(S^2)\right)$

(3.84) $\mathbb{E}(S^2) = 1 + \frac{1}{a}$

(3.85) $\mathbb{E}(\mathbb{I}_{\{X_n \geq 1\}} \mathbb{I}_{\{X_m \geq 1\}}) = p_n p_m \left(1 + \frac{\omega_1^2}{a}\right)$

(3.86) $\rho(\mathcal{D}_n, \mathcal{D}_m) = \left(\frac{\omega_1^2}{a}\right) \left(\frac{p_n p_m}{\sqrt{p_n(1-p_n)}\sqrt{p_m(1-p_m)}}\right)$

(3.95) $\sigma(p_n(\mathbf{S})) = p_n \sqrt{\sum_{k=1}^{K} \frac{\omega_{n,k}^2}{a_k}}$ This is a correction to an error in the text.

(3.99) $\rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \left(\sum_{k=1}^{K} \frac{\omega_{n,k}\omega_{m,k}}{a_k}\right) \left(\frac{p_n p_m}{\sqrt{p_n(1-p_n)}\sqrt{p_m(1-p_m)}}\right)$

Chapter 4

(4.1) $y_n = aG + b\epsilon_n$

(4.5) $Y_n = a \cdot G + \underbrace{\sqrt{1-a^2}}_{b} \epsilon_n$

(4.7) $y_n = \sqrt{a} \cdot G + \sqrt{1-a}\epsilon_n$

(4.18) $p_n(G) = \Phi\left(\frac{\Phi^{-1}(p_n) - \sqrt{\rho}G}{\sqrt{1-\rho}}\right)$

(4.22) $\rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \frac{\mathbb{P}(\mathcal{D}_n \cap \mathcal{D}_m) - p_n p_m}{\sqrt{p_n p_m (1-p_n)(1-p_m)}}$

(4.23) $\mathbb{P}(\mathcal{D}_n \cap \mathcal{D}_m) = \Phi\left(\Phi^{-1}(p_n), \Phi^{-1}(p_m); \rho\right)$

(4.30) $\rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \frac{\overbrace{\Phi\left(\Phi^{-1}(p_n), \Phi^{-1}(p_m); \rho\right)}^{\mathbb{P}(\mathcal{D}_n \cap \mathcal{D}_m} - p_n p_m}{\sqrt{p_n p_m (1-p_n)(1-p_m)}}$

(4.31) $\rho(\mathbb{I}_{\mathcal{D}_n}, \mathbb{I}_{\mathcal{D}_m}) = \frac{\Phi\left(\Phi^{-1}(\bar{p}), \Phi^{-1}(\bar{p}); \rho\right) - \bar{p}^2}{\bar{p}(1-\bar{p})}$

(4.44) $\text{var}(\breve{\mathbb{D}}_N | G) = \frac{p(G)(1-p(G))}{N}$

(4.46) $\text{F(x)} = \mathbb{P}(G \leq -p^{-1}(x))$

(4.49) $F(x) = h(b(x))$

(4.50) $h(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{\mu^2}{2}} du$ This is a correction to an error in the text

(4.51) $b(x) = \frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}}$

(4.55) $F'(x) = \sqrt{\frac{1-\rho}{\rho}} e^{\frac{(\Phi^{-1}(x))^2}{2} - \frac{b(x)^2}{2}}$

(4.69) $y_n = \sqrt{\frac{\nu}{W}} \left(\sqrt{\rho}G + \sqrt{1-\rho}\epsilon_n\right)$

(4.72) $\mathbb{E}\left(\frac{1}{W}\right) = \frac{1}{\nu-2}$

(4.73) $\text{cov}(y_n, y_m) = \rho\left(\frac{\nu}{\nu-2}\right)$

(4.74) $\rho(y_n, y_m) = \rho$

(4.81) $y_n = \sqrt{V} \left(\sqrt{\rho}G + \sqrt{1-\rho}\epsilon_n\right)$

(4.87) $p_n = F_{\mathcal{NV}}(K_n)$

(4.103) $y_n = \underbrace{a_{n,K+1}\epsilon_n}_{\text{Idiosyncratic}} + \underbrace{\sum_{k=1}^{K} a_{n,k} Z_k}_{\text{Systematic}}$

(4.107) $\text{cov}(y_n, y_m) = a_n^T a_m$

(4.108) $$p_n(Z) = \Phi\left(\frac{\Phi^{-1}(p_n) - a_n^T Z}{a_{n,K+1}}\right)$$

(4.110) $$y_n = \sqrt{h(V)}\left(a_{n,K+1}\epsilon_n + \sum_{k=1}^{K} a_{n,k} Z_k\right)$$

(4.113) $$\rho(y_n, y_m) = a_n^T a_m$$

(4.115) $$p_n(Z, V) = \Phi\left(\frac{\sqrt{\frac{1}{h(V)}} F_{\mathcal{N}\mathcal{V}}^{-1}(p_n) - a_n^T Z}{a_{n,K+1}}\right)$$

(4.116) $$y_n(k) = \sqrt{h(V)}\left(\underbrace{\sqrt{a}G + \sqrt{(1-a)b_k}R_k}_{\text{Systematic element}} + \underbrace{\sqrt{(1-a)(1-b_k)}\epsilon_n}_{\text{Idiosyncratic element}}\right)$$

(4.120) $$\rho\left(y_n(k), y_m(j)\right) = a + \mathbb{I}_{k=j}(1-a)\sqrt{b_k b_j}$$

(4.123) $$p_n(G, R_k, V) = \Phi\left(\frac{\sqrt{\frac{1}{h(V)}} F_{\mathcal{N}\mathcal{V}}^{-1}(p_n) - \sqrt{a}G - \sqrt{(1-a)b_k}R_k}{\sqrt{(1-a)(1-b_k)}}\right)$$