Carbon Credit Program Report

SOA 2020 Student Research Case Study Challenge

Renmin University of China
No.59 Zhongguancun Avenue,
Haidian District, Beijing, China

RUC WZLLY Actuarial Consulting

Ruotong Wang	2017201627@ruc.edu.cn
Yuang Li	2017201595@ruc.edu.cn
Jinzhao Yu	2017201639@ruc.edu.cn
Yajing Liu	2017201125lyj@ruc.edu.cn
Yuanyuan Zhang	zhangyy_candy@163.com

Faculty Advisor: Yan Wang

Contents

Executive Summary

The government of Pullanta recently set a goal of reducing carbon emissions to 25% below the 2018 level by the end of the year 2030. Our team design a carbon credit program for Pullanta's Department of Environment Concerns. In the report, we first formulate the number of available carbon credits of each year, then design the allocation of carbon credits and establish a phased, sector-specific program to reduce emissions. Besides, we establish the operating mechanism of carbon credits auction market. After that, we model the carbon spot price by using the jump fractional Brownian process. Based on this, we design three carbon credit financial instrument and give pricing models. In addition, we show some important results from our data analysis, and conduct a sensitivity analysis. We also analyze the specific risks that the various stakeholders will face. Finally, we state some model limitations and suggestions for the carbon credit program.

1.Purpose and Background

1.1 The urgency of Pullanta to implement the emission reduction project

Analyzing the biocapacity(EC) and ecological footprint(EF) of Pullanta, we know the biocapacity per person fluctuates slightly and the overall trend is declining as shown in Table1.1 and Figure1.1 The ecological footprint per person fluctuates more obviously and generally shows an upward trend. What's more important, an ecological deficit occur, that is, ecological footprint of each year is greater than biocapacity. And the deficit is increasing, which means Pullanta is consuming natural resources faster than the ecosystem can recover, putting a lot of pressure on the ecosystem and going against the concept of sustainable development. This means it's urgent for

Pullanta to take measures to reduce emission.

Table1.1 Biocapacity, ecological footprint & ecological deficit per person of Pullanta.

Year	Biocapacity per person (hm^2 /cap)	Ecological Footprint per person(hm^2 /cap)	Ecological deficit per person (hm^2 /cap)
1995	5.5121	9.0492	-3.5371
1996	5.533	9.5367	-4.0036
1997	5.4075	9.4643	-4.0568
1998	5.7738	9.6754	-3.9016
1999	5.4086	8.9827	-3.5741
2000	5.3076	9.4485	-4.1409
2001	5.1828	8.1414	-2.9586
2002	5.2676	9.0019	-3.7343
2003	5.0959	9.1272	-4.0312
2004	5.2217	9.4254	-4.2037
2005	5.207	9.8137	-4.6067
2006	5.0652	10.2619	-5.1967
2007	4.8598	10.7688	-5.909
2008	4.748	10.4412	-5.6931
2009	4.9461	10.095	-5.1489
2010	4.9658	10.6634	-5.6976
2011	4.9109	11.1364	-6.2255
2012	4.8381	10.9305	-6.0924
2013	4.8363	10.5552	-5.7189
2014	4.695	10.541	-5.846
2015	4.797	10.6985	-5.9015
2016	4.7101	10.8628	-6.1528
2017	4.656	10.7699	-6.1139
2018	4.7033	10.8783	-6.175
2019*	4.6966	10.8538	-6.1571

Note: Biocapacity per person=Biocapacity/population,

Ecological Footprint per person=Ecological/population,

Ecological deficit per person=Biocapacity per person- Ecological Footprint per person

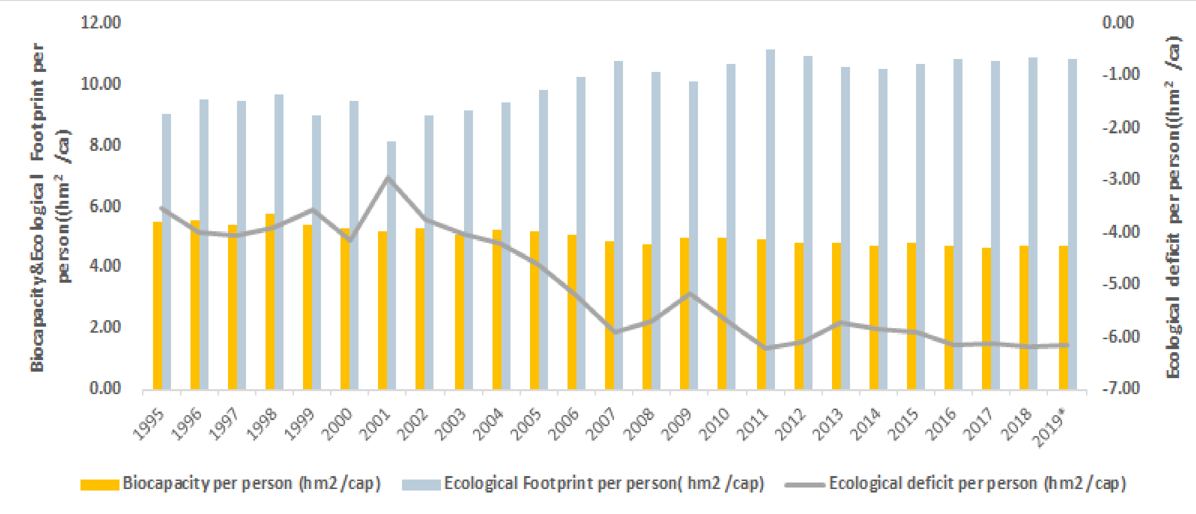

Figure1.1 Biocapacity, ecological footprint & ecological deficit per person of Pullanta.

According to Ecological Footprint Index(EFI) proposed by WWF *Living Planet Report*, that is, the ratio of ecological deficit / surplus to biocapacity. The calculation formula is: $EFI = \frac{EC-EF}{EC}$.This index can reflect the ecosystem's ability to withstand pressure as shown in Table1.2 We calculated Pullanta's EFI from 1995 to 2019 as shown in figure1.2, and according to the grade standards given, it can be seen that Pullanta's ecological sustainability is declining, which also illustrates the urgency of implementing the emission reduction project in Pullanta.

Table1.2 Rank standard of ecological footprint index

	Rank				
Item	I	II	III	IV	V
EFI	0.5~1	0~0.5	0	-1~0	<-1
State	Strong sustainability	Weak sustainability	critical point	unsustainable	Seriously unsustainable

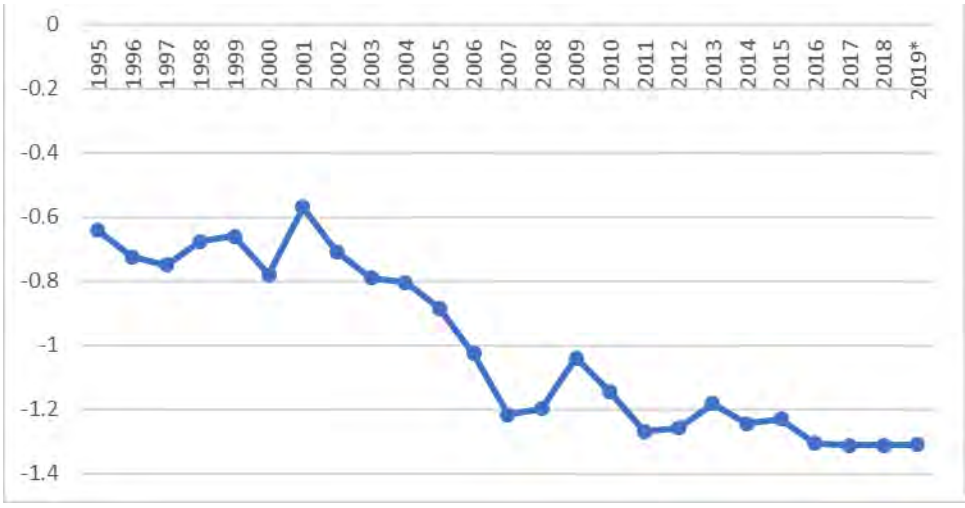

Figure1.2 Ecological Footprint Index (EFI) in Pullanta

1.2 Introduction to Carbon Market

The market for trading carbon credits is called Carbon Market. The supply and demand of carbon credits in the Carbon Market are greatly influenced by policy makers. Many countries in the world have established a series of emission reduction mechanisms. In particular, the trading of carbon credits has developed rapidly since the implementation of *Kyoto Protocol*. As a consequence, Carbon Market is growing in size. According to the trading principle, the international carbon credits trading market system is arranged as shown in Figure 1.3 below.

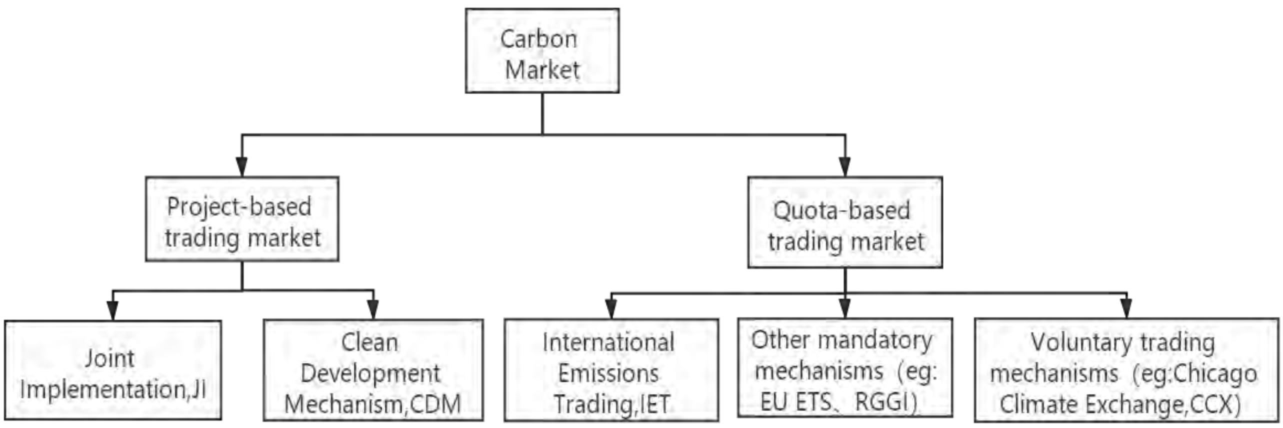

Figure1.3 International carbon credits trading market system

1.3 Introduction to carbon financial market

With the development of carbon trading, a large number of carbon financial products and derivatives such as forwards, futures, options and swaps have emerged as the time requires. A series of financial activities supporting carbon trading are collectively referred to carbon finance. Specifically, it refers to various financial transaction activities and related financial system aiming to achieve emission reduction targets.

The broad carbon financial market includes not only the purchase and sale of carbon credits, but also the investment and financing market for carbon financial products, derivatives and emission reduction projects.

According to different classification standards, carbon financial markets can be further subdivided into the categories in Figure 1.4.

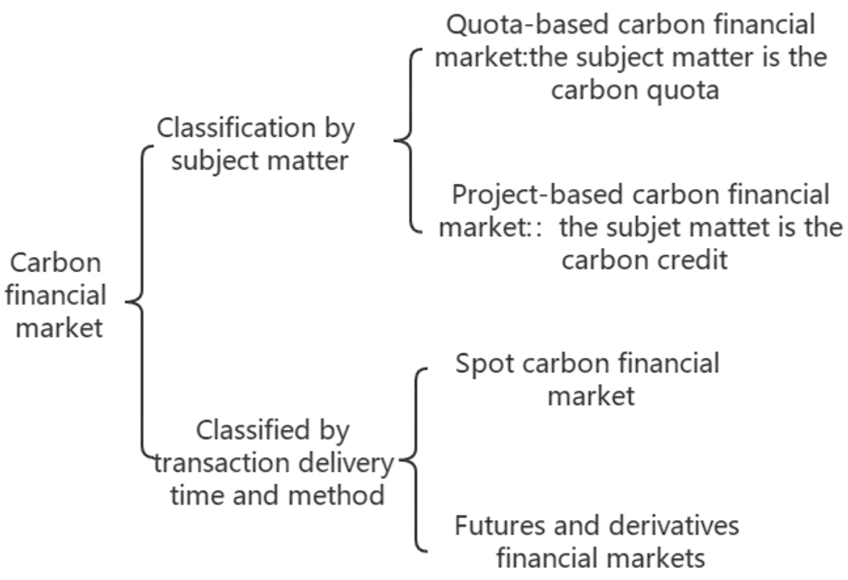

Figure1.4.Classification of carbon financial market

1.4 Research Purpose

Our team will design a carbon credit plan for Pullanta to help it achieve emission reduction targets and improve its ecological environment. At the same time, it will allow the government to obtain revenue, and then invest in energy-saving and emission-reduction projects such as clean energy and equipment renovation. With I view to alleviating the pressure on ecosystems and climate conditions.

We will also study the mechanism of carbon credit price formation, the price operation and pricing mechanism of carbon spot, futures, and options. With a view to providing decision reference for the construction and development of Pullanta's carbon financial market.

In addition, we also hope that Pullanta’s plan can provide some experience for other countries in the world.

1.5 Research Route

The research roadmap of this paper is shown in Figure 1.5.

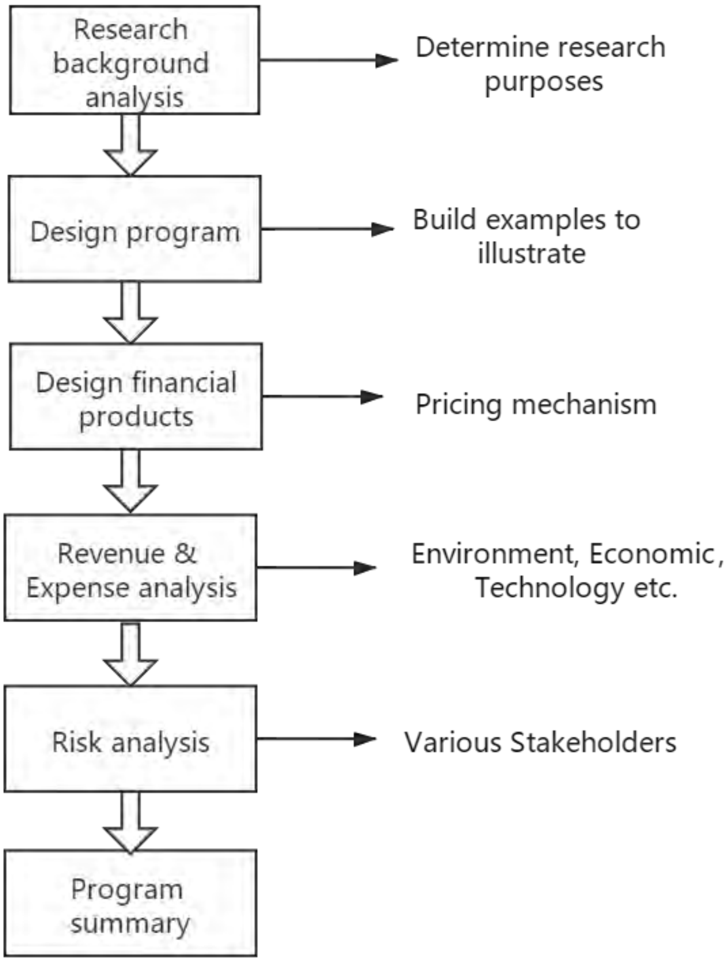

Figure1.5 Research roadmap

2. Model Construction

2.1 Program Framework

To encourage reducing carbon emissions and generate revenue to fund climate change mitigation, our team has designed a carbon credit program with a comprehensive implementation plan. The framework of our plan is bellowing:

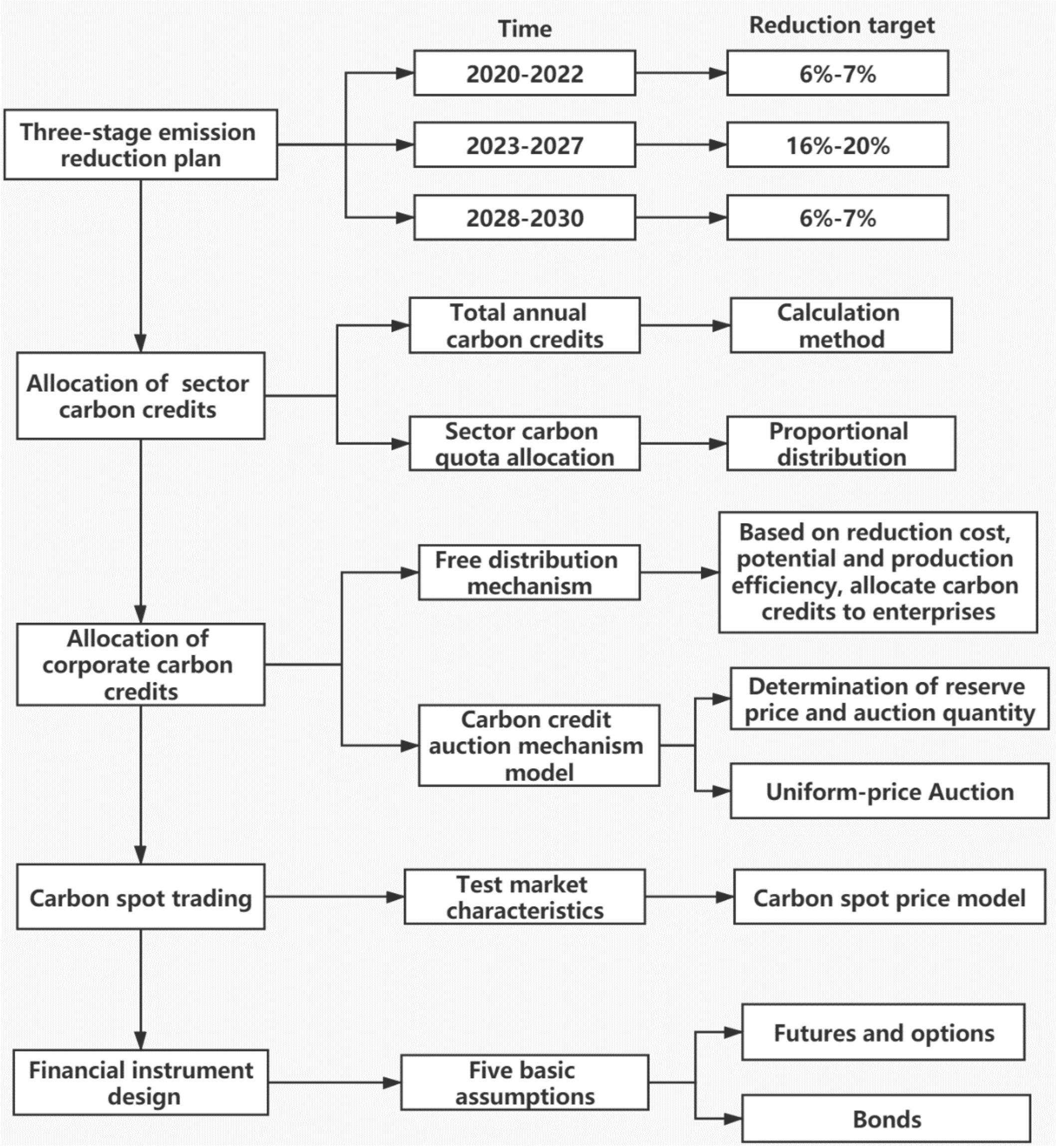

Figure2.1 Program framework

2.2 Allocation of Carbon Credits

2.2.1 Three-stage emission reduction plan

Based on the total emission reduction target and the actual situation of Pullanta, our team has formulated the emission reduction plan divided into three stages:

Table2.1 Three-stage emission reduction plan

Stage	Length	Stage target	Minimum annual target	Explanation
2020-2022	3 years	(l_1, r_1)	a_1 a_2 a_3	Lack of new technology and equipment
2023-2027	5 years	(l_2, r_2)	a_4 a_5 a_6 a_7 a_8	Large space for emission reduction & New equipment and technologies are gradually put into large-scale use
2028-2030	3 years	(l_3, r_3)	a_9 a_{10} a_{11}	Shrink of the space for emission reduction & New equipment and technologies are basically replaced

The stage targets should meet: $75\% * 90\% < \prod_{i=1}^{3}(1-r_i) < \prod_{i=1}^{3}(1-l_i) < 75\%$

Assuming that the minimum annual target is achieved exactly every year, the total future emissions can be predicted as:

$$C_i = C_0 * \prod_{i=1}^{11}(1-a_i)$$

where C_i is the total carbon emission in year i of the emission reduction plan, and C_0 is the total carbon emission in 2018

2.2.2 Allocation of Carbon Credits

The average growth rates of the ratio of carbon emission in each sector from 1995 to 2018 are calculated as follows:

Table2.2 Average growth rate in each sector

Sector	Average growth rate
Buildings & Land Use	-0.31%
Energy, Manufacturing & Construction	-0.12%
Industrial Processes & Product Use	0.10%
Other	0.00%
Transport	0.32%
Waste	0.00%

Assume steady development, the carbon emission ratio of each industry in 2020-2030 is predicted as follows:

Table2.3 The proportion of carbon emission by sector (including B)

Year	Buildings & Land Use	Energy, Manufacturing& Construction	Industrial Processes& Product Use	Other	Transport	Waste
2020	14.27%	52.82%	10.01%	0.33%	18.29%	4.29%
2021	13.97%	52.70%	10.10%	0.33%	18.62%	4.29%
2022	13.67%	52.58%	10.19%	0.33%	18.95%	4.29%
2023	13.37%	52.46%	10.28%	0.33%	19.28%	4.29%
2024	13.07%	52.34%	10.37%	0.33%	19.61%	4.29%
2025	12.77%	52.22%	10.46%	0.33%	19.94%	4.29%
2026	12.47%	52.10%	10.55%	0.33%	20.27%	4.29%
2027	12.17%	51.98%	10.64%	0.33%	20.60%	4.29%
2028	11.87%	51.86%	10.73%	0.33%	20.93%	4.29%
2029	11.57%	51.74%	10.82%	0.33%	21.26%	4.29%
2030	11.27%	51.62%	10.91%	0.33%	21.59%	4.29%

However, our team found that no company exists in sector B, so the emission reduction task of B needs to be shared by the other five sectors according to the proportion of each sector. The results are as follows:

Table2.4 The proportion of carbon emission by sector (excluding B)

year	Energy, Manufacturing & Construction	Industrial Processes & Product Use	Other	Transport	Waste
2020	61.61%	11.67%	0.38%	21.34%	5.00%
2021	61.25%	11.74%	0.38%	21.65%	4.99%
2022	60.90%	11.80%	0.38%	21.95%	4.97%
2023	60.55%	11.86%	0.38%	22.26%	4.95%
2024	60.20%	11.92%	0.37%	22.56%	4.94%
2025	59.86%	11.99%	0.37%	22.86%	4.92%
2026	59.52%	12.05%	0.37%	23.16%	4.90%
2027	59.18%	12.11%	0.37%	23.46%	4.89%
2028	58.84%	12.17%	0.37%	23.75%	4.87%
2029	58.50%	12.23%	0.37%	24.04%	4.85%
2030	58.17%	12.29%	0.37%	24.33%	4.84%

The carbon quota of each sector can be obtained by multiplying the proportion of each sector by the total carbon emission each year. After obtaining the sector quota forecast, carbon quota is allocated to enterprises in each sector according to their carbon emission ratio. Enterprises obtain carbon credits through auction and free allocation from the government of Pullanta.

As for the proportion of free allocation and auction in each sector, different proportions can be set at each stage or each year according to the actual situation of the emission reduction cost and emission reduction potential of each sector.

2.2.3 Understanding of the emission reduction goal

Pullanta has a goal of reducing carbon emissions to 25% below the 2018 level by the end of the year 2030 and needs a carbon credit program that is expected to result in total carbon emissions staying within 90% of Pullanta's goals with 90% certainty. From the perspective of probability statistics, the probability of the total annual carbon emissions of 2030 is less than 75% of the total annual carbon emissions of 2018 is 1, and the probability of the total annual carbon emissions of 2030 is less than 67.5% (=90%*75%) of the total annual carbon emissions of 2018 is 0.9.

In our program, the actual annual carbon emission reduction (the amount of actual annual carbon emission reduction compared with 2018) is taken as the random variable X. X_t represents the actual emission reduction in year t. According to the overview, there is a lower limit for the emission reduction, that is, the emission reduction must be greater than a certain value, in order to be meaningful. The lower limit varies from year to year and is determined by the reduction target for that year. Considering the characteristics of carbon emissions reduction, X, this article assumes that X obeys the Single-parameter Pareto distribution.

The probability density function of the Single-parameter Pareto distribution is:

$$f(x) = \frac{\alpha\theta^{\alpha}}{x^{\alpha+1}}, \qquad x > \theta$$

And the cumulative distribution function is,

$$F(x) = 1 - (\theta/x)^{\alpha}, \qquad x > \theta$$

In our program, we are interested in its tail distribution function, which is,

$$\bar{F}(x) = \Pr(X > x) = (\theta/x)^{\alpha}, \qquad x > \theta$$

where, θ is the lower limit of carbon emissions reduction. Taking year of 2030 as an example, the situation is as follows.

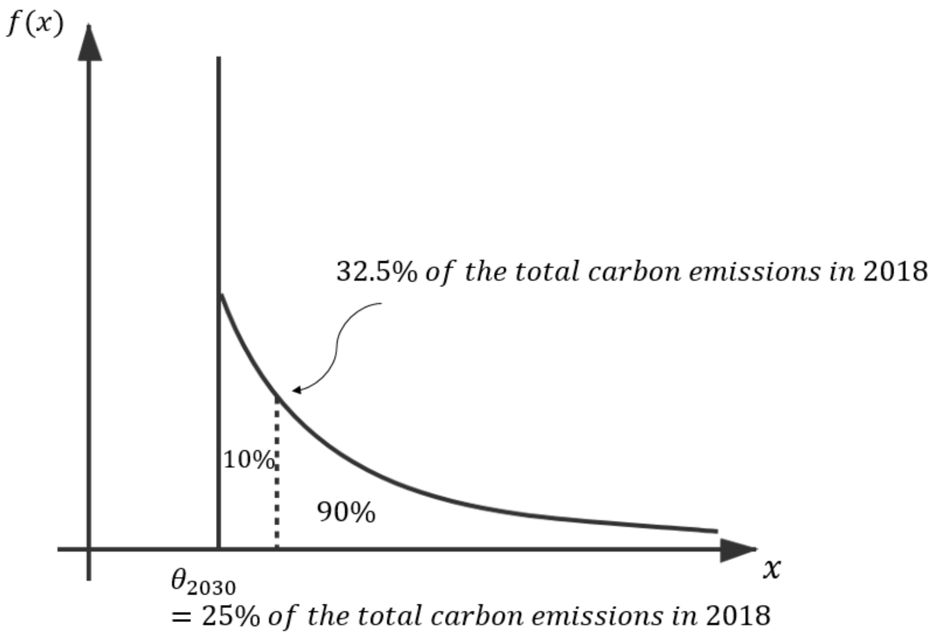

Figure2.2 Distribution of the actual carbon emissions in 2030

We can solve for $\theta = 230610266.1, \alpha = 0.4016$.

Since the actual annual carbon emissions are also random variables, there must be a superior limit of the total amount of carbon credits from government each year to ensure that the target can be realized. In this model, the annual superior limit is equal to the difference between carbon emissions in 2018 and the annual lower limit, θ_t. The superior limit of the total amount of carbon credits issued in the year t is Y_t, because emission reduction will cause the increase of production costs of enterprises, which will lead to the increase of social costs, these increases will be passed on to consumers. Considering the cost minimization, the total amount of carbon emission rights issued by the government every year is Y_t.

2.3Auction model for carbon credit

Due to differences in emission reduction targets and freely allocated carbon credits in different sectors, auctions will be conducted in different sectors. Pullanta can establish an online auction system and require companies to submit bid prices and bid quantities within the prescribed time once a quarter.

2.3.1 Liability of auction market agent

The relationship between auction market agents as shown below:

(1) Because the government is the seller and supervisor, it needs to:

①Determine the total carbon credits Q_t for each auction according to the overall emission reduction targets of Pullanta;

②Determine the reserve price RP_t and maximum floating ratio α_t for the t_{th} auction. We can set the price based on revelent factors and pricing model which are mentioned in 2.4 as original price ORP_t . Then the government determines downscale β_t. Finally,

The reserve price RP_t= original price $ORP_t \times (1-\beta_t)$.

The setting of these two ratios is mainly based on the actual situation in Pullanta. The government should take the implementation effect of program, the feedback of enterprises, the operational condition of carbon market, the emission pressure of enterprises and so on into consideration. Besides, the government should refer to the setting of other international emission reduction programs.

(2) Companies are bidders.The model assumes that all companies make bidding strategies based on profit-seeking purposes. And they make decisions independently, that is, they don't depend on other companies' bidding information. In order to obtain the maximum economic benefits, the bidder I needs to:

①adjust bidding strategies based on its own situation and previous external market information;

②determines the bid price $Pb_{i,t}$ and quantity $q_{i,t}$ for the t_{th} auction.

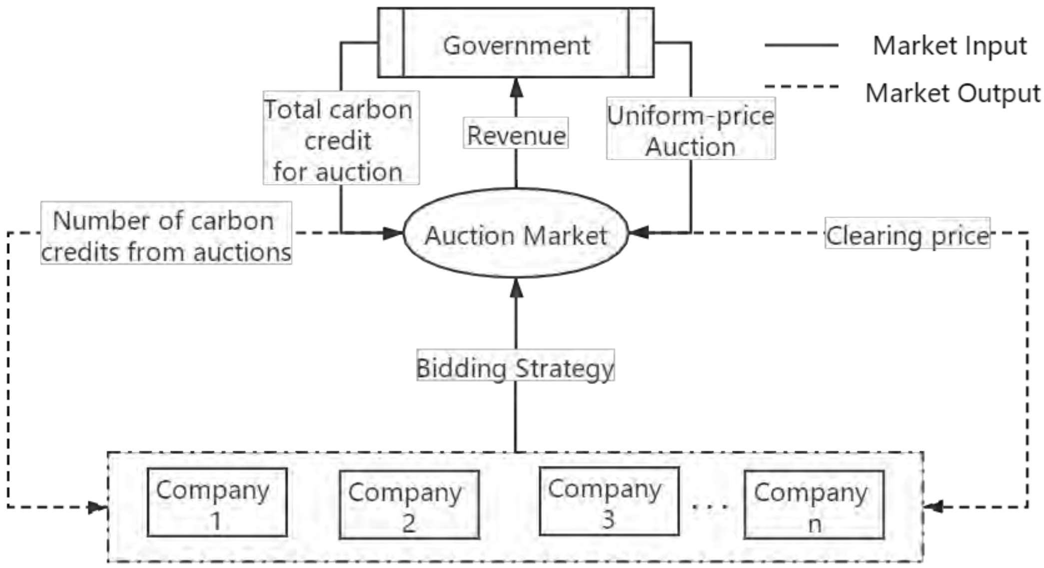

Figure2.3 Relationship between carbon credit right auction market agents for any sector

2.3.2 Auction market rules

Rules of the auction market as shown below:

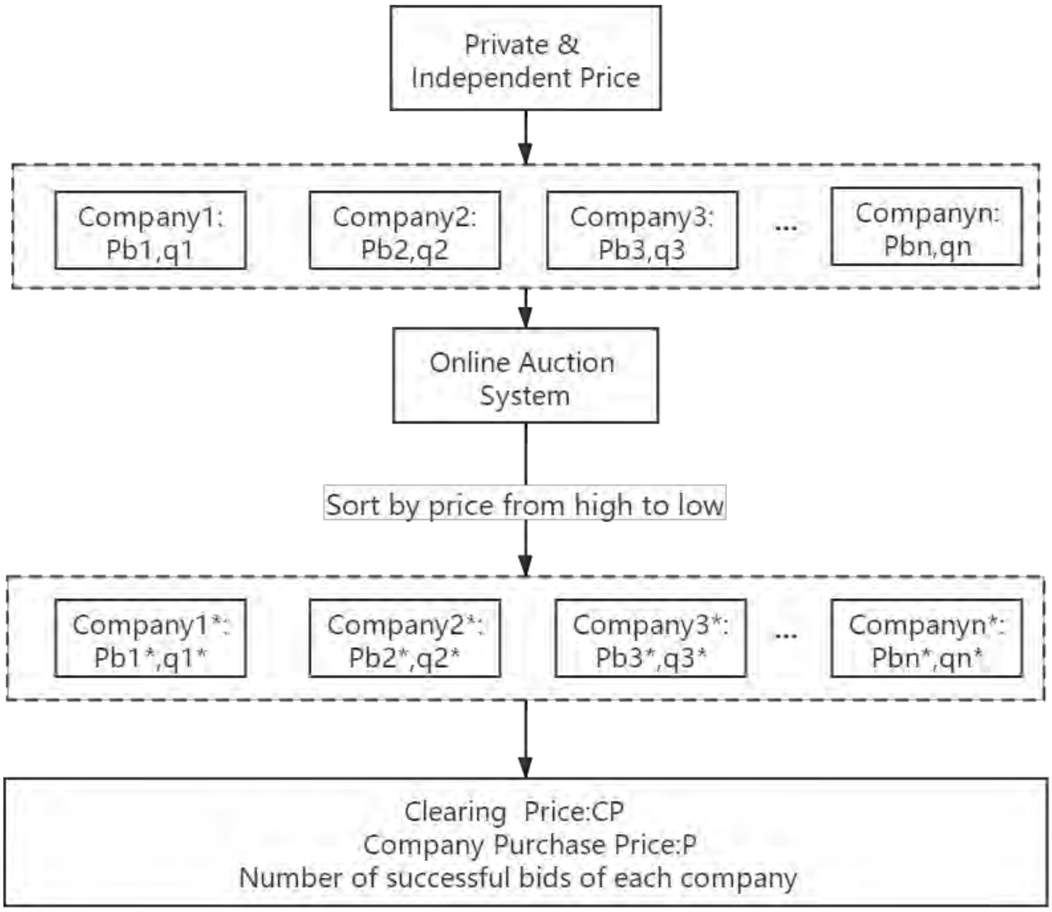

Figure2.4 Process of auction of any time

There is a more detailed explanation about the purchase price $P_{i,t}$ and market clearing price CP_t as follows. Note that purchase price $P_{i,t}$ is the same as market clearing price CP_t because the government adopts uniform-price auctions.

$$P_{i,t} = CP_t = \begin{cases} Pb^*_{u+1,t}, \ if \sum_{i=1}^{u} q^*_{i,t} < Q_t, \ \sum_{i=1}^{u+1} q^*_{i,t} \geq Q_t \ , i \leq u+1 & (a) \\ +\infty, \quad if \sum_{i=1}^{u} q^*_{i,t} < Q_t, \ \sum_{i=1}^{u+1} q^*_{i,t} \geq Q_t \ , i \geq u+1 & (b) \\ Pb^*_{n,t}, \quad if \sum_{i=1}^{n} q^*_{i,t} < Q_t & (c) \end{cases}$$

Figure 2.5 Clearing & purchase prices under different circumstances

Table2.5 Explanation of signals

Signal	Explanation
$P_{i,t}$	Purchase price of company i^* in the t_{th} auction
$q_{i,t}^*$	Number of successful bids of company i^*
$Pb_{u+1,t}^*$	Bid price of company $u+1^*$ in the t_{th} auction
Pb_n^*	Bid price of company n^* in the t_{th} auction

Table2.6 Explanation of different circumstances

(a)	When the supply in the auction market is less than demand, the top u + 1 companies win the t_{th} auction. The bid price of the u + 1_{th} company is the uniform transaction price of all the winning companies.
(b)	When the supply in the auction market is less than demand, there is no transaction price for unsuccessful bidders. So the transaction price is expressed as infinity ∞.
(c)	When the supply in the auction market is more than demand, the uniform transaction price is the lowest price Pb_n^* of all n bidders.
Note: purchase price $P_{i,t}$ is the same as market clearing price CP_t because the government adopts uniform-price auctions .	

2.3.3 Bidding learning process for bidders

By the way, there is a bidding learning process for bidders as shown in Figure2.6 First, the bidder bids according to its own bidding function. After the auction, the company finds the personal best bid prices and market best bid prices based on its personal historical bidding results and market transaction information, and then adjusts the bidding function to wait for the next auction.

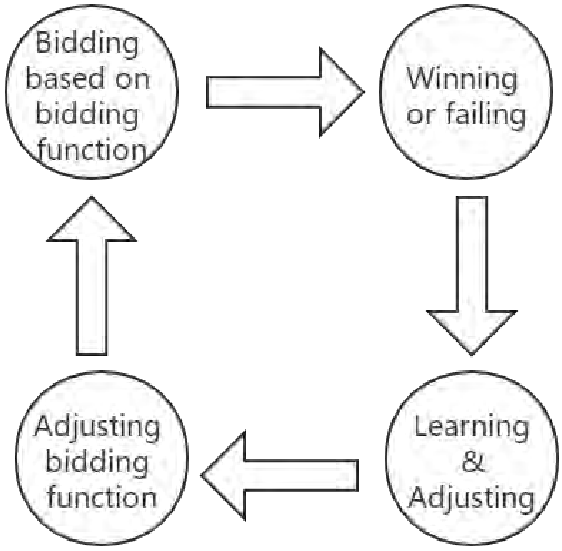

Figure2.6 Bidding learning process for bidders

2.4 Auction Base Price Model

In the auction market where the government is the seller, the government needs to set a base price at every auction, and the bidding price of the enterprise shall not be lower than this base price. The set of the base price is influenced by economic conditions, environmental pressure, emission reduction technology and other aspects. The base price directly affects the pricing of carbon spot and derivatives in the secondary market. The rationality of pricing is related to the determination of enterprises and the development of industry, and directly affects the whole economic entity.

2.4.1 Introduction and comparison of credit trading pricing model

At present, researches on carbon credit trading pricing model mainly include '*Shadow Price Model*', '*Marginal Abatement Cost Pricing Model*', and '*Break-Even Pricing Model*' based on financial perspective.

It is worth noting that these models can only be established under strict assumptions, and the calculated prices are not the prices that are actually applied in the real world. They are only used as a reference for pricing, and the actual pricing needs to consider about various uncertain factors.

The following table compares these three kinds of models.

Table2.7 Three kinds of carbon credit pricing models

Pricing Methods	Shadow Price Model	Marginal Abatement Cost Pricing Model	Break-Even Pricing Model
Details	Shadow price, also known as optimal plan price or optimal calculated price. The United Nations believes that shadow prices is a kind of opportunity cost for enterprises from putting financial resources, material resources and human resources into their production. Shadow prices actually measure the marginal benefit, or marginal contribution, of an extra unit of resources, reflecting the scarcity and true value of resources.	Marginal Abatement Cost (MAC) refers to the economic cost of reducing one unit of CO_2 emission at a certain emission level. With the increase of emission reduction, the difficulty of emission reduction gradually increases, and the marginal abatement cost increases (decreases first and then increases later). Marginal abatement cost is a pricing method based on marginal cost theory. Enterprises whose marginal abatement cost is lower than the market price of carbon credit will choose to reduce emissions as much as possible, while other enterprises will choose to buy carbon credit to meet the emission standards. All enterprises will optimize their behaviors for the maximum profit and ultimately achieve 'Pareto Optimization'.	Break-Even pricing methods, refers to when the sales is certain, the price of products must reach a certain level to achieve break-even. The key of break-even analysis is to determine the break-even point, which is the state when profit is zero.
Assumption	Perfectly competitive market		Sales volume is the only cost driver of market clearing
Calculate Methods	Dual solutions for linear programming; Lagrange multiplier method	By deducing the marginal abatement cost curve of a certain place or a certain industry through the model, and then the marginal abatement cost of CO_2, namely carbon credit price, can be further eliminated by maximizing corporate profits.	Cost-Volume-Profit Analysis, CVP, the price of the equilibrium point is the price when the company's sales revenue is equal to total cost (fixed cost + variable cost)

Ad-vantages	The advantage of shadow price is that it can provide the correct and forward-looking price standard for the reasonable allocation of resources and the scientific and reasona-ble use of enterprises, so that enterprises can avoid using their own subjective to determine the price.	The emission of CO_2 comes from production, so it is reasonable to trace the marginal abatement cost back to the source based on production. This method does not have a large number of complicated calculations, and the data is available, and can ensure the uniqueness of emis-sion reduction costs.	Calculate the break-even production capacity (refers to the emission reduction capacity, i.e. the produc-tion capacity of carbon credit) and the price from the perspective of enter-prise break-even, so as to facilitate enterprises to make short-term decisions and long-term plans. Fi-nancial data are readily available to businesses.
Disad-vantages	When calculating the shadow price, because the constraint condi-tions generally include the matrix with input coefficient, the com-plexity of the matrix leads to complicated calculation process, and it can't reflect the demand preference of the buyer well.	The selection of the model is directly related to the fitting ef-fect of MAC curve and the ac-curacy of carbon credit price. The cost of reducing emissions does not consider fixed costs, so the calculated marginal cost is lower than the average cost.	For enterprise pricing, it does not apply to industry and local. And the market clearing assumption that production equals sales is not easily satisfied.

To sum up, by comparing the advantages and disadvantages of the above three pricing models, we believes that the ‘Marginal Abatement Cost Pricing Model’ more fits the design of policies and products, and its feasibility and applicability are higher.

2.4.2 Model Choose

There are three main approaches to determine the marginal abatement cost curve:

Table2.8 The determination method of marginal abatement cost curve

Methods	Details
Professional MAC Curve	Based on the engineering method, the emission reduction potential and emission reduction cost of different countries and industries are estimated based on a certain advanced technology, and the MAC curve is formed by ranking the cost. The most typical example is the global marginal CO_2 reduction cost curve published by McKinsey. Professional MAC curve is easy to understand, but the interaction of technology, inter-temporal factors and non-technical factors on the cost of emission reduction are not considered, and the analysis results are subjective.
MAC curve based on economic-energy model	Set a series of assumptions, construct an equilibrium model that satisfies the assumptions, and then deduce the emission reduction cost under different emission levels by changing the conditions in the assumptions, forming the marginal emission reduction cost curve. It takes into account the cross-influences of inter-temporal dynamic factors and various factors, but still has the limitations of over-dependence on model selection and sensitivity to assumptions and parameters.
MAC curve based on production theory	The principle of this method is to describe the relationship between marginal emission reduction cost and emission reduction under a certain technical and economic environment, assuming a certain production feasible set, and calculate the marginal emission reduction cost by the method of extreme value.

We adopt the third method, in which there are two commonly used function models of production function: one is the cost function, the other is the distance function. The distance function is more widely used because it does not need to consider the price of each input and output factor. In this paper, Directional Distance Function (DDF) is selected to fit the marginal abatement cost curve.

There are two methods to determine the directional distance function, one is the parametric method, the other is the non-parametric method. Compared with non-parametric method, the advantage of parametric method is that it can guarantee the derivability and uniqueness of emission reduction cost. For the directional distance function of the parametric method, the model setting is usually quadratic or superlogarithmic form, and a series of studies show that the quadratic form is better than the superlogarithmic form. Therefore, we choose the quadratic directional distance function to estimate the marginal abatement cost of CO_2, so as to provide the reference price of carbon credit in the auction market.

2.4.2 Derivation of carbon credit price

Assuming that the directional vector is $\boldsymbol{g} = (g_y, -g_b)$then the directional distance function can be defined as,

$$D\left(x, y, b; g_y, -g_b\right) = max\left\{\ \beta: \left(y + \beta g_y, b - \beta g_b\right) \in P(x)\ \right\}$$

Where y is the consensual output and b is the non-consensual output. $P(x) = \{(y, b)\}$ represents the production feasible set capable of producing (y, b).

The directional distance function can be explained by the following process: in the plane coordinates, the horizontal axis is defined as b, the vertical axis is defined as y, and the original production point is $A(y, b)$. In the production possibility set $A(y, b)$, the producer can map the original production point to the production front along the direction vector $\boldsymbol{g} = (g_y, -g_b)$, and the process can be realized by improving the technical efficiency.

The maximum value of β is the value of the directional distance function, which is the maximum value that can be achieved by the reverse scaling of the consensual output and the non-consensual output with the same proportion, that is, the optimal output at a certain time of input. In this case, the consensual output is as large as possible in the production feasible set, while the non-consensual output is as small as possible in the production feasible set. When the original production point was moved to the production front, the desired output increased by βy; At the same time, non-desirable outputs were reduced by βb. On the leading edge, the directional distance function is 0. At this point, the technical efficiency reaches the maximum, and the slope of the tangent line on the direction front is $-\frac{q}{p}$, where q is the price of the non-desirable output, and p is the price of the desirable output.

Let R represent the maximum return that the producer can get, that is, the return

$$R(x,p,q) = max\{\ py - qb{:}\, D\left(x,y,b;g_y,-g_b\right) \geq 0\ \}$$

When $\boldsymbol{g} = \left(g_y,-g_b\right) = (1,-1)$, the expression for the return can be expressed as,

$$R(x,p,q) = \max_{y,b}\{\ py - qb{:}\, D(x,y,b;1,-1) \geq 0\ \}$$

Construct the Lagrangian function and take the derivative,

$$\begin{cases} (p\times 1 + q\times 1)\times \dfrac{\partial D(x,y,b;1,-1)}{\partial y} = -p \\ (p\times 1 + q\times 1)\times \dfrac{\partial D(x,y,b;1,-1)}{\partial b} = q \end{cases}$$

If we take the ratio of the two equations, we can get the price of undesirable output as follows,

$$q = -p\times \frac{\partial D/\partial b}{\partial D/\partial y}$$

In our program, desirable output y is the GDP, output b represents CO_2 emissions, GDP is priced by Pulo (Ᵽ), the price of p can be normalized to 1, then the price of CO2 marginal cost is,

$$q = -\frac{\partial D/\partial b}{\partial D/\partial y}$$

The specific form of quadratic directional distance function is as follows,

$$D(x,y,b;1,-1) = \alpha_0 + \sum_{n=1}^{3}\alpha_n x_n + \beta_1 y + \gamma_1 b + \frac{1}{2}\sum_{n=1}^{3}\sum_{m=1}^{3}\alpha_{nm}x_n x_m + \frac{1}{2}\beta_2 y^2 + \frac{1}{2}\gamma_2 b^2 + \sum_{n=1}^{3}\delta_n x_n y + \sum_{n=1}^{3}\sigma_n x_n b + \mu yb$$

Thus, the expression of marginal emission reduction cost is derived as,

$$q = -\frac{\frac{\partial D}{\partial b}}{\frac{\partial D}{\partial y}} = -\frac{\gamma_1 + \gamma_2 b + \sum_{n=1}^{3}\sigma_n x_n + \mu y}{\beta_1 + \beta_2 y + \sum_{n=1}^{3}\delta_n x_n + \mu b}$$

3. Case interpretation

3.1 Assign the emission reduction goals of three stages

Firstly, assign the stage target and the minimum annual target and the specific three-stage emission reduction plan is obtained as follows:

Table3.1 Three-stage emission reduction plan

Stage	Lengrh	Stage target	Minimum annual target
2020- 2022	3 years	6%-7%	1.5%
			2.0%
			2.5%
2023-2027	5 years	16%-20%	3.0%
			3.5%
			4.0%
			3.5%
			3.0%
2028-2030	3 years	6%-7%	2.5%
			2.0%
			1.5%

If the emission reduction plan is implemented, assuming that minimum annual target is exactly achieved, the total emission is predicted to be:

Table3.2 Future total emission forecast

Year	Total
2018	922441064.3
2020	908604448.4
2021	890432359.4
2022	868171550.4
2023	842126403.9
2024	812651979.8
2025	780145900.6
2026	752840794
2027	730255570.2
2028	711999181
2029	697759197.3
2030	687292809.4

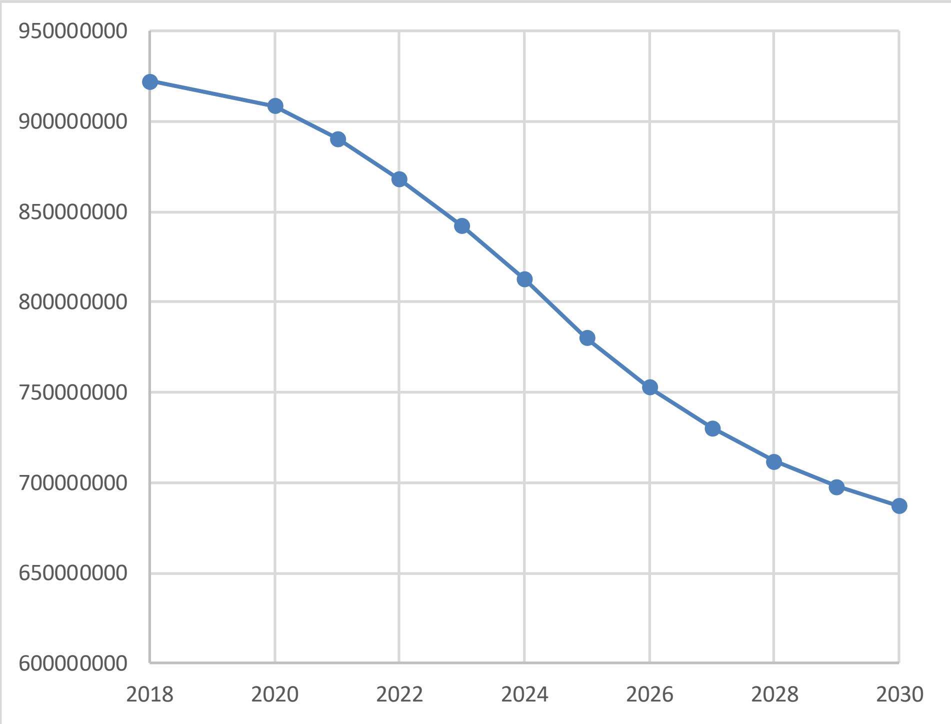

Figure 3.1 Future total emission forecast

The carbon quota of each sector in each year can be obtained by multiplying the proportion of each sector as mentioned above by the total annual carbon emission, as shown in the table below:

Table3.3 Carbon emission quota for each sector (excluding B)

Year	Energy, Manu-facturing & Construction	Industrial Processes & Product Use	Other	Transport	Waste
2020	559765070	106050099	3449287	193867097	45472896
2021	545414796	104498202	3368514	192742809	44408038
2022	528724708	102436767	3272888	190589811	43147376
2023	509920412	99894453	3163707	187439814	41708018
2024	489253231	96906822	3042441	183340146	40109339
2025	466994555	93515519	2910699	178352588	38372540
2026	448073082	90707263	2799197	174358665	36902587
2027	432148600	88433809	2705947	171293971	35673243
2028	418941130	86656561	2629318	169109158	34663014
2029	408222611	85345473	2567990	167768613	33854511
2030	399810255	84478191	2520918	167249499	33233947

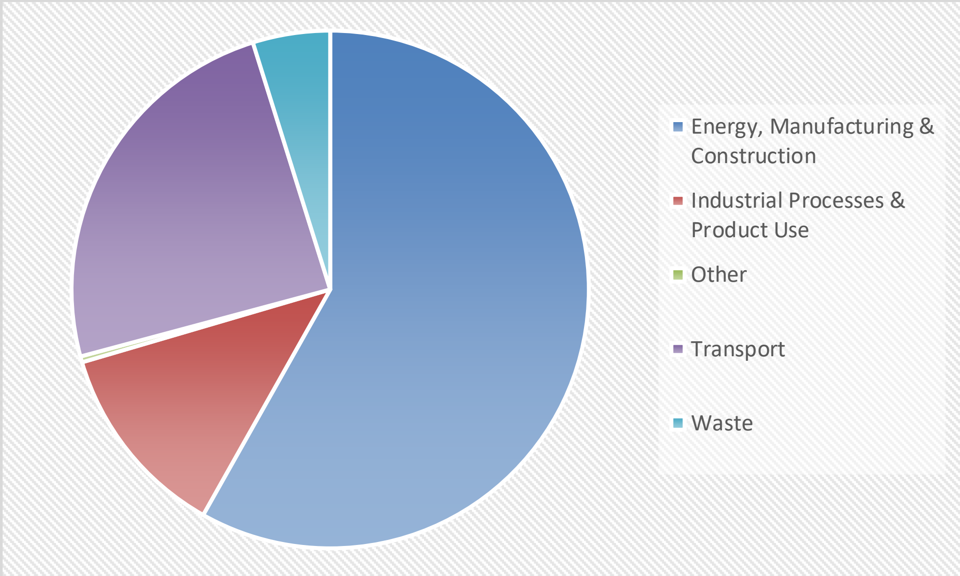

Figure 3.2 Pie chart of carbon emission quota for each sector in 2030

3.2 Design the proportion of free allocation and auction in each sector

As for the proportion of free allocation and auction in each sector, we have said we can set different proportions at each stage or each year according to the actual situation of each sector.

Here our team take the setting of different proportions at the three stages in each sector as an example. Because we learn that the sector E (Energy, Manufacturing & Construction) has the highest carbon emission ratio in each year by analyzing the proportion of carbon emissions by sector as shown in table3.3. This may mean that sector E has a greater rigid demand for carbon emissions, so the proportion of free allocation in sector E should be larger. The proportion of free allocation in other sectors can also be considered in this way. Besides, consider the different characteristics of the three stages—the first stage is the adaptation period and the program is still in the trial operation stage, the program and carbon credits market will become more and more matured in the latter two stages. As a consequence, the proportion of free allocation at the first stage should be larger for each sector. Finally, combining with the experience of other emission reduction programs in the world, we list the table of auction proportions of each sector at three stages as an example.

Table3.4 Auction proportion of each sector at three stage

Sector	Stage 1	Stage 2	Stage 3
E	0	10%	20%
T	0	20%	30%
I	0	20%	30%
W	0	20%	50%
O	0	20%	50%

Finally, we talk about downscale β_t and floating ratio α_t mentioned in 2.3.1. The setting of these two ratios is mainly based on the actual situation in Pullanta. The government should take the implementation effect of program, the feedback of enterprises, the operational condition of carbon market, the emission pressure of enterprises and so on into consideration. Besides, the government should refer to the setting of other international emission reduction programs. Here we set $\beta_t: 5\% - 20\%$, $\alpha_t: 20\% - 40\%$ at the second and third stage.

4. Carbon Credit Financial Instruments

4.1 Basic assumptions

In the design of the following financial instruments, we set the following five basic assumptions, and on this basis, we design these financial instruments. The basic assumptions are as follows:

a. There is no market friction.

b. The market participants do not bear the counterpart risk.

c. The market is perfectly competitive.

d. Market participants are risk averse and want as much wealth as possible.

e. There is no arbitrage opportunity in the market.

4.2 Carbon spot price model

Analyzing existing empirical results, we know that the carbon trading market has fractal and jump characteristics. Therefore, the jump fractional Brownian process may be a more suitable stochastic process for modeling carbon trading prices. The jump fractal process is further applied to the valuation of carbon financial products.

4.2.1 Testing of market characteristics

Due to the limited availability of data and considering that EU ETS is more mature, we use the BlueNext Exchange CER cash and futures trading data from 2011 to 2013 as research objects. And we mainly carry out empirical test of the fractal and jump characteristics of carbon trading market.

4.2.1.1Testing of fractal characteristic

To test the fractal characteristic, we use R/S method proposed by Hurst (1951). Hurst discovered the relation in 1951: $\log\left(\frac{R}{S}\right) = Hlogn + logc$, where R / S is the recalibration domain, c is a constant, n is the number of observations, H is the Hurst index. Different values of Hurst index can reveal different problems, as shown in Table4.1

Table4.1. Different values of Hurst index H

H=0.5	The variables are independent of each other, and the time series follows a random process, and the market is efficient at this time.
0<H<0.5	Negative correlation between variables. And the closer H is to 0, the stronger the degree of negative correlation is. The time series is anti-persistent and mean reverting. If the sequence of the previous period goes up, the next period is likely to go down and vice versa. At this time, the time series is unstable and easily fluctuates.
0.5<H<1	Positive correlation between variables. The closer H is to 1, the stronger the degree of positive correlation is. Time series is characterized by persistence or trend enhancement. This positive correlation, the long-term memory structure, makes the time series self-similar, that is, they have similar statistical characteristics on different time scales.

Because the R / S calculation method of the Hurst index is complicated and the length of the paper is limited, we won't show it. Our team uses SAS to calculate that H of the CER spot and futures markets. We found the Hurst indices are greater than 0.5, which indicates that the market has long memory and obvious fractal characteristic.

4.2.1.2 Testing of jump characteristic

To test the jump characteristic, we use the jump test method in statistic. Commonly used non-parametric methods for jump testing include BN-S testing (Barndorff-Nielsen, Shepshard, 2006), Z testing (Andeisen, 2007), J-O testing (Jiang-Oomen, 2008), A-J testing (Ait- Sahalia, Jacob, 2009) and LM testing (Lee, Mykland, 2008). We adopt the A-J method to perform a non-parametric test of the jumping behavior of the carbon market. The A-J method determines whether jumps occur by calculating the ratio of p-time variation at different sampling frequencies. The test statistic is: $\hat{S}(p,k,\triangle_n)_t = \frac{\hat{B}(p,k\triangle_n)_t}{\hat{B}(p,\triangle_n)_t}$.

In the test statistic, p is the power of calculating variation, $\triangle_n$ is the minimum sampling frequency, k is the maximum sampling frequency and it is the time length of the time series sample.

And $\hat{B}(p,\triangle_n)_t$ is the p-time variation of the sample sequence $\{X_0, X_{\triangle_n}, \dots X_{n\triangle_n}\}$, which can be defined as:

$$\hat{B}(p,\triangle_n)_t = \sum_{i=1}^{\left[\frac{t}{\triangle_n}\right]} |\triangle_i^n X|^p$$

$$\triangle_i^n X = X_{i\triangle_n} - X_{(i-1)\triangle_n}$$

Ait-Sahalia and Jacob (2009) proved that if the null hypothesis H_0 is there is no jump in the price process, let:

$$\hat{V}_{n,t}^c = \frac{\triangle_n M(p,k)\hat{A}(2p,\triangle_n)}{\hat{A}(p,\triangle_n)^2}$$

then,

$$\frac{\hat{S}(p,k,\triangle_n) - k^{\frac{p}{2}-1}}{\sqrt{\hat{V}_{n,t}^c}} \sim N(0,1), \qquad when\ \triangle_n \to 0$$

As a result, if we set the significance level is α, then when

$$N\left(\frac{\hat{S}(p,k,\triangle_n) - k^{\frac{p}{2}-1}}{\sqrt{\hat{V}_{n,t}^c}}\right) < \alpha,$$

we reject the null hypothesis that the sample data has no jump.

Based on the above principles, we test the daily yield data of the CER spot and futures of the BlueNext Exchange to observe if there is jump. And according to the recommendations of Ait-Sahalia and Jacob (2009), we set p = 4, k = 2, ω = 0.48, and the time series frequency is 1 day. The trading day of one year is set to 252 days, so $\triangle_n = 1/252$. The test results show that, at a significant level of 1%, there has been a significant jump in the CER spot during 2011-2013, and there has also been a significant jump in the various types of CER futures.

So far, we have verified that it is appropriate to model carbon prices by using the jump fractional Brownian process.

4.2.2 Modeling carbon spot price

In order to represent the model conveniently, we firstly give the symbols used in this section in the table below.

Table 4.2 Explanation of signals

Signal	Meaning	Signal	Meaning
S_t	Carbon spot price at time t	$\{B_t^H\}$	Fractional Brownian motion with Hurst index H
$\{N_t\}$	Poisson process with intensity λ	$\{Y_t\}$	The jump amplitude following the lognormal distribution, ln(Yt)~N(μy,σy)
X	Option strike price	r_f	Pulo's risk-free interest rate, set to constant
σ_B	Yield volatility of underlying asset	r_q	Euro's risk-free interest rate, set to constant
Δt	Time interval of time series data: Year	T	Option expiration date

Suppose that the carbon spot price obeys the geometric fractal Brownian motion. The price process under risk neutrality is:

$$dS_t = r_q S_t dt + \sigma S_t dB_t^H, t \geq 0$$

From the Ito formula under Wick-Ito-Skorohod integral, we can get:

$$dlnS_t = r_q dt + \sigma dB_t^H - \sigma^2 H t^{2H-1} dt$$

so

$$var\left(\triangle \, lnS(t)\right) = \sigma^2 H (\triangle \, t)^2$$

As for the price volatility of carbon spot, it can be estimated by using historical carbon price data. The estimation formula is:

$$\sigma^2 = \frac{1}{(\triangle t)^{2H}} * \frac{1}{n-1}\sum_{i=1}^{n}(r_{ti} - \bar{r})^2$$

where, $r_{ti} = lnS_{ti} - lns_{ti-1}, \bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_{ti}$.

4.3 Design of Futures and Options

The carbon credit futures contract is essentially the same as other futures contracts in Pullanta's financial futures market, and the subject matter is carbon credit. The contract is also the same as other ordinary future contracts, and is highly standardized. At the same time, we consider the subject matter, carbon credit, to be a non-yielding asset. According to the "Spot-Futures Parity Theorem", the theoretical delivery price in the futures contract of carbon credit can be obtained as:

$$F = Se^{r\ (T-t)}$$

where: F is the theoretical delivery price, S is the spot price, r is the risk-free interest rate, T is the delivery date, t is the current date; At the same time, in the design of futures, we need to assume that the risk-free interest rate remains constant.

The carbon credit option contract is similar to the futures contract, which is the same as other options. In this design of option contract, the carbon credit option belongs to the European option, that is, the option contract cannot be exercised before expiration. There are many kinds of option pricing models. We use Black-Scholes pricing model to price the carbon credit option contract. The pricing model is detailed as follows:

$$C = S_0 N(d_1) - Xe^{-it}N(d_2)$$

$$d_1 = \frac{\left[\ln\left(\frac{S_0}{X}\right) + \left(i + \frac{\sigma^2}{2}\right)t\right]}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

where: C is the price of the call option, S_0 is the initial price of the subject matter, X is the strike price, i is the risk-free interest rate, t is the length of the contract time, and S is the standard deviation of the change of the corresponding asset price;

4.4 Design of Bonds

4.4.1 Profit and loss structure of products

According to the calculation of the CER daily cash from 2011 to 2013, we know the average price volatility reaches about 32%. It shows carbon spot prices are highly volatile. Therefore, the bonds are designed to give investors the lowest return, that is, our products are guaranteed financial products. At the same time, in order to control financing costs and avoid excessive carbon credits issuance, we set a ceiling for returns.

We design two bonds and both bonds pay interest at the end of each year. They are both 5-year spread-guaranteed bonds. Because 5 years is a key term in the term structure of interest rates. They are:

①Bond A: a floating-rate carbon bond embedded in European bull spread options, which is equivalent to a combination of 1 fixed-rate bond and 5 European bull spread options;

②Bond B: a floating-rate carbon bond embedded in Asian bull spread options, which is equivalent to a combination of 1 fixed-rate bond and 5 Asian bull spread options. So bond B has a strong path-dependent nature of the carbon spot price, which can reduce the impact of price fluctuations to a certain extent.

The interest payment profit and loss charts of the two products in each period are similar, as shown in Figures 3.3below.

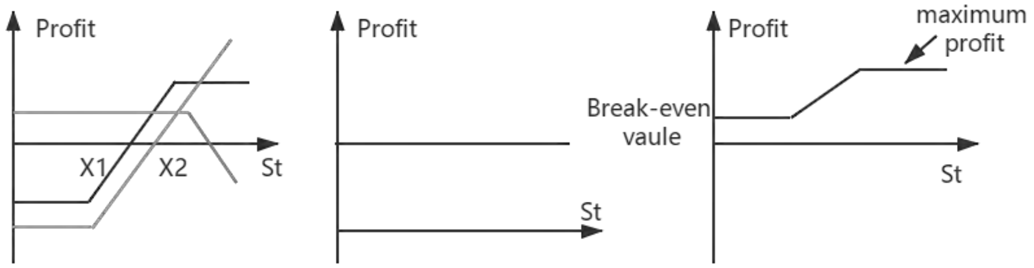

Figure 3.3. Bull spread & Fixed-rate bond & Spread-guaranteed bond

According the structure of bond A and B, we can write the formula of profit and loss at maturity, consisting of two parts:

①the part of fixed income:

$$P_B = \sum_{t=1}^{T} \frac{\eta B}{(1+i)^t} + \frac{B}{(1+i)^T}$$

②the part of floating income:

a. European bull spread option:

$$P_{C\_euro} = \begin{cases} 0, & if \ \ S_t < X_1 \\ B \times \theta \left(\dfrac{S_t - X_1}{S_0} \right), & if \ \ X_1 \le S_t < X_2 \ , t = 1, \dots 5 \\ B \times \theta \left(\dfrac{X_2 - X_1}{S_0} \right), & if \ \ S_t \ge X_2 \end{cases}$$
$$= \frac{B \times \theta}{S_0} (\max(S_t - X_1, 0) - \max(S_t - X_2, 0))$$

We set $C_1^*(T) = \max(S_T - X_1, 0)$, $C_2^*(T) = \max(S_T - X_2, 0)$, to represent the value of two embedded options at expiration.

b. Asian bull spread option:

$$P_{C\_asian} = \begin{cases} 0, & if \ \ Average_{st} < X_1 \\ B \times \theta \left(\dfrac{Average_{st} - X_1}{S_0} \right), & if X_1 \le Average_{st} < X_2 \ , t = 1, \dots, 5 \\ B \times \theta \left(\dfrac{X_2 - X_1}{S_0} \right), & if Average_{st} \ge X_2 \end{cases}$$

The average value $Average_{st}$ here is the average of the carbon spot prices at the end of each month in each interest-paying year.

In the formula above, B represents face value, η is the principal guaranteed rate and θ represents the participation rate. X_1，X_2 are the lower and upper limits of the price of the underlying asset. The purpose of setting η and θ here is to show that the product can be designed as a financial product.

4.4.2 Basic terms design of carbon bonds

The fixed-income characteristics of two bonds are achieved through the determination of coupon rates. As for the parts linked to carbon spot price, we assume a group of European or Asian bull spread options with carbon credit prices as the target are purchased.

4.4.2.1. Terms design of coupon bonds

Face value: B =100

Expiry date: T=5 year，and interest is paid once a year.

Coupon rate/ principal guaranteed rate：η, that is, the $face\ \ vaule\ \ *\eta$ is paid as interest at the end of each year.

Enforcement terms：it is not callable, not puttable and it can't be converted into equity upon maturity.

4.4.2.2 Terms design of options

Strike price：it is a call option. The parity method is adopted, which means the price of the underlying asset on the product issue date is set as the strike price, that is,$X_1 = S_0$ is set. This setting can reflect the volatility value of options better. At the same time, suppose the maximum return limit of Pullanta's financial products is δ, we can set $X_2 = (1+\delta) * S_0$.

Underlying asset：the CER spot price of BlueNext Exchange

Because BlueNext Exchange is the largest CER trading market in the world and may have a great impact on the running of Pullanta's carbon credits trading market, we set CER spot of BlueNext Exchange as underlying asset. However, it should be noted that the price of CER is denominated in Euro, and the bonds are issued in Pullanta and denominated in Pulo.

The basic terms of the carbon bonds are shown in table 4.3

Table4.3 Basic terms of carbon bonds

Coupon bonds	Options
Face value： B=100	Strike price： $X_1=S_0$， $X_2=(1+\delta)*S_0$
Expiry date： T=5year	Underlying asset： carbon credits in exchange
Principal guaranteed rate: η	Participation rate： θ
Non-callable, non-puttable, non-convertible, due to be executed at the expiry date; The denomination currency is Pulo	

4.4.3 Option valuation

In the options part, for Bond A- -the valuation of European floating-rate bond, the options part is the sum of five bull spread options for one year, two years, three years, four years, and five years. For Bond B--the Asian floating-rate bond, the options part involves the characteristics of forward and path dependence, so it is necessary to use numerical simulation methods to value forward options.

Combined with the foregoing, we use the European option pricing formula of the jump fractal Brownian process to calculate the value of the options part. The calculation of Bond B is similar, but some improvements need to be added.

4.4.3.1 Valuation process of European bull spread options

For European bull spread options, we only need to compare the relationship between the carbon spot price at the expiration date and the strike price to make a valuation.

In the case that the carbon spot price obeys the geometric fractal Brownian motion, the fractal BS formula can be used to calculate the value of the European bull spread option. Refer to the fractal BS formula of the European call option at any time $t \in [0,T]$ given by Necula (2002):

$$C_1^*(t) = S_t N(d_1) - X_1 e^{-r_q(T-t)} N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_t}{X_1}\right) + r_q(T-t) + \frac{1}{2}\sigma^2(T^{2H} - t^{2H})}{\sqrt[\sigma]{T^{2H} - t^{2H}}}$$

$$d_2 = d_1 - \sqrt[\sigma]{T^{2H} - t^{2H}}$$

where $C_1^*(t) = E_t^*\left(e^{-r_q(T-t)} \max(S_T - X_1, 0)\right)$, is the value of a standard Euro-denominated European call option at time t.

Assuming that the Euro-Pulo exchange rate is independent of the carbon spot price, the value of the two embedded options in carbon bonds can be calculated by the following formulas. Since the pricing process of the two options $C_1(t)$ and $C_2(t)$ is similar, we just give the pricing formula of option 1--$C_1(t)$ as an example. According to Hu and ksendal (2011), under the risk-neutral measure, the current value of option 1--$C_1(t)$ is:

$$C_1(t) = E_t^*\left(e^{-r_f(T-t)} C_1(T)\right) = \frac{B\theta}{S_0} e^{(r_q - r_f)(T-t)} C_1^*(t)$$

4.4.3.2 Valuation process of Asian bull spread options

For Asian floating-rate bonds, the payment of the floating-rate part of each period is not related to the carbon price on the date of interest payment, but to the average carbon price over the entire floating-rate interval. Because such options have forward options and are path-dependent, pricing should be approximated by numerical methods. With reference to Asmussen (1999), our team uses Monte Carlo method to simulate the five-year carbon spot price under the fractal Brownian motion, and then evaluates the Asian bull spread option. Specific steps are as follows:

a. assume that carbon spot price obeys the geometric fractal Brownian motion just like

$$dlnS_t = r_q dt + \sigma dB_t^H - \sigma^2 H t^{2H-1} dt,$$

then discretize Eulerian as:

$$\Delta lnS_t = r_q \Delta t + \sigma \Delta B_t^H - \sigma^2 H t^{2H-1} \Delta t \quad (*)$$

Due to the incremental correlation of fractal Brownian motion, when the validity period of claims is divided into n parts, using the autocorrelation feature of fractal Brownian motion, the variance-covariance matrix between the increments of n fractal Brownian motion is:

$$\Sigma = \frac{1}{2}(\Delta t)^{2H} \begin{Bmatrix} C_{11}, C_{12}, \dots, C_{1n} \\ \dots \\ C_{n1}, C_{n2}, \dots, C_{nn} \end{Bmatrix}$$

$where\ \ C_{ij} = |i-j+1|^{2H} + |i-j-1|^{2H} - 2|i-j|^{2H}, i, j = 1, \dots, n.$

Therefore, an incremental random number of fractal Brownian motion on a path can be obtained by extracting an n-dimensional joint normal distribution random number whose mean vector is 0 vector and variance-covariance matrix is Σ, and then obtain a path of carbon credits price by recursion (*);

b. use Monte Carlo simulation method to generate enough (assuming 100,000) five-year carbon credits price paths. For each path, take 60 observation points, that is, N = 60, then there are 12 observation points each year, $\triangle t = T/N$;

c. for each path, on the floating interest payment date at the end of each year, the arithmetic average of the carbon price of 12 observation points of the year is calculated. Then compare it with the strike price, and then discount through the term structure of the Pulo's risk-free interest rate to obtain the Asian bull spread. So, a sample of the value of the option is obtained;

d. the arithmetic mean of the valuation of 100,000 paths is the valuation of the Asian bull spread option.

5.Social cost and government revenue

5.1 Social cost

One of the social costs of not implementing the program is the increase in carbon emission. We use the single variable linear regression analysis method, combining with the total emissions data from 1995 to 2018, fitting for the total emission curves with time. The result shows that the total emission increases $8.63 * 10^6(\mathrm{mt})$ annually on average. Compared with the maximum emission with program, the difference is the social cost of not implementing the program. The total social cost of carbon emission is about $2.034 * 10^9(\mathrm{mt})$ accumulated to the end of 2030.

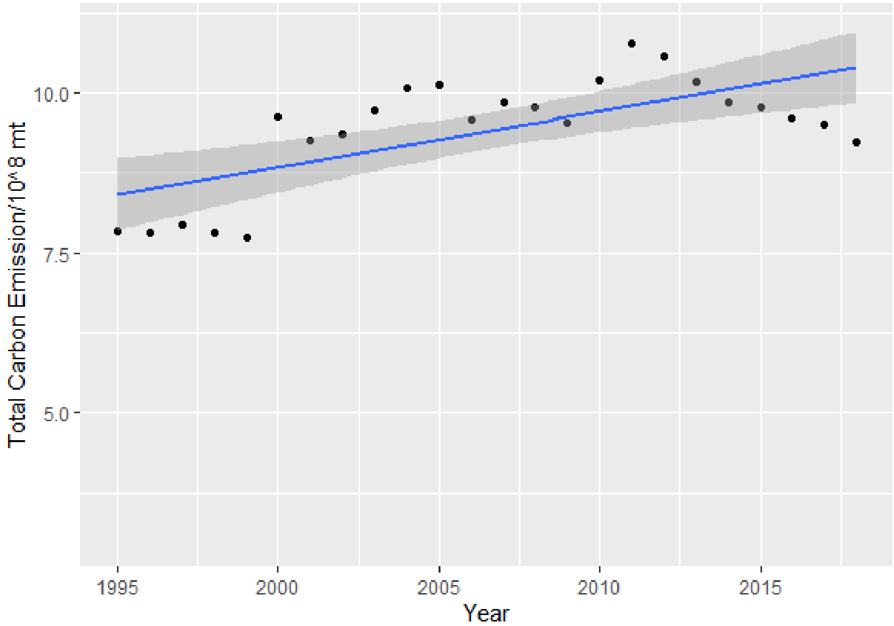

Figure 5.1.Regression fitting line of total carbon emission

Table 5.1 Social cost: Carbon emissions

Year	Average emission without program	Maximum emission with program	difference
2018	922441064.3	922441064.3	0
2020	931070064.3	908604448.4	22465615.9
2021	939699064.3	890432359.4	49266704.9
2022	948328064.3	868171550.4	80156513.9
2023	956957064.3	842126403.9	114830660.4
2024	965586064.3	812651979.8	152934084.5
2025	974215064.3	780145900.6	194069163.7
2026	982844064.3	752840794	230003270.3
2027	991473064.3	730255570.2	261217494.1
2028	1000102064	711999181	288102883.3
2029	1008731064	697759197.3	310971867
2030	1017360064	687292809.4	330067254.9
		Sum	2034085513

Enterprises may adopt two types of emission reduction measures: one is to reduce production, and the other is to develop emission reduction technologies. The former has led to a reduction in GDP, while the latter has led to an increase in the production costs of enterprises. Both are social costs. It is difficult to predict the increased costs of enterprises due to the development of emission reduction technologies, and it depends on the actual situation each year. The following is a forecast of the increased social costs caused by the reduction of GDP.

The figure below shows the historical data of the country's GDP growth rate, and found that it has a large fluctuation. The average GDP growth rate is 4.96%, and the average growth rate in the past 10 years is 2.58%. Using the annual GDP growth rate of 2.58% to predict the GDP of the next 11 years, we get Table 5.2

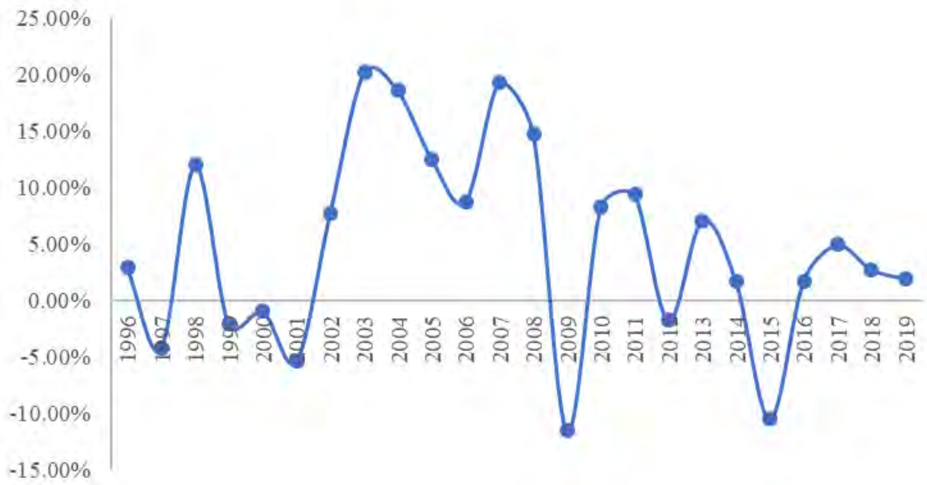

Figure 5.2. GDP Growth Rate from 1996 to 2019

Table 5.2 Social cost: the reduction of GDP

Year	GDP without program	GDP with program	Difference
2020	744471634445	757341830908	12870196463
2021	763679002613	766651759174	2972756561
2022	783381920881	777412343526	-5969577355
2023	803593174440	789505584888	-14087589552
2024	824325878340	802800088211	-21525790129
2025	845593486001	817153459639	-28440026362
2026	867409797940	828054467486	-39355330454
2027	889788970727	836866115506	-52922855221
2028	912745526172	843739200990	-69006325182
2029	936294360747	848790918772	-87503441975
2030	960450755254	852396991551	-1.08054E+11
Total			-4.11022E+11

By the end of 2030, the country's total GDP reduction due to the implementation of emission reduction program is expected to be -4.11022E + 11.

5.1.2 Government revenue

The government's revenue from the program includes revenue from auctioning carbon emission rights, fines from companies with excessive emissions, and platform transaction fees.

Table 5.3 Government revenue

Revenue Type	Content	Formula	Symbol Description
Revenue from auctioning carbon emission rights	Since this project uses the auction mechanism with the market clearing price as the uniform price, the income from each auction is the uniform price multiplied by the auction volume.	$\sum_{t=1}^{T} CP_t Q_t$	t: the t_{th} auction CP_t: market clearing price for the t_{th} auction Q_t: total carbon credits sold for the t_{th} auction
Fines from companies with excessive emissions	On the liquidation date, if the actual emissions of the enterprise are greater than the carbon emission holdings, the enterprise will pay a penalty higher than the market price for the excess emissions, and the price is accumulated in a stepwise manner.	$\sum_{i=1}^{I}\sum_{j=1}^{J} F_j \min\{(M_j - H_{j-1}), H_j\}$	i: the i_{th} company j: the j_{th} level M_j: Excess emissions the i_{th} company producted H_j: maximum emissions for the j_{th} level F_j: fine per ton for the j_{th} level
Platform transaction fees	When enterprises conduct carbon trading on a carbon trading platform, they must pay the platform transaction fees to the platform side. Fees are multiplied by the rate and the actual transaction amount, and are charged in both directions.	$\sum_{n=1}^{N} rA_n$	n: the n_{th} auction deal r: Transaction fee rate A_n: actual transaction amount the n_{th} auction deal

6. Sensitivity Analysis

The emission reduction plan used in the case in the third part of this article is a three-stage emission reduction. The intensity of emission reduction increases first and then decreases. It is formulated by predicting the development of emission reduction technology in Pullanta in the future. If the development status does not meet our assumptions, the emission reduction plan will change accordingly, and the following four plans (including one used in this article) will be analyzed for sensitivity.

The first is *"linear emission reduction"*, which means that the same amount of carbon emissions are reduced each year compared to the previous year; the second is *"equal reductions"*, which means that the annual reduction ratio compared to the previous year is equivalent; the third is *"growth-type emission reduction"*, and the annual reduction ratio is increasing compared with the previous year, indicating that the country's emission reduction technology has always flourished at this stage; the fourth is adopted in this article *"Three-stage emission reduction"*.

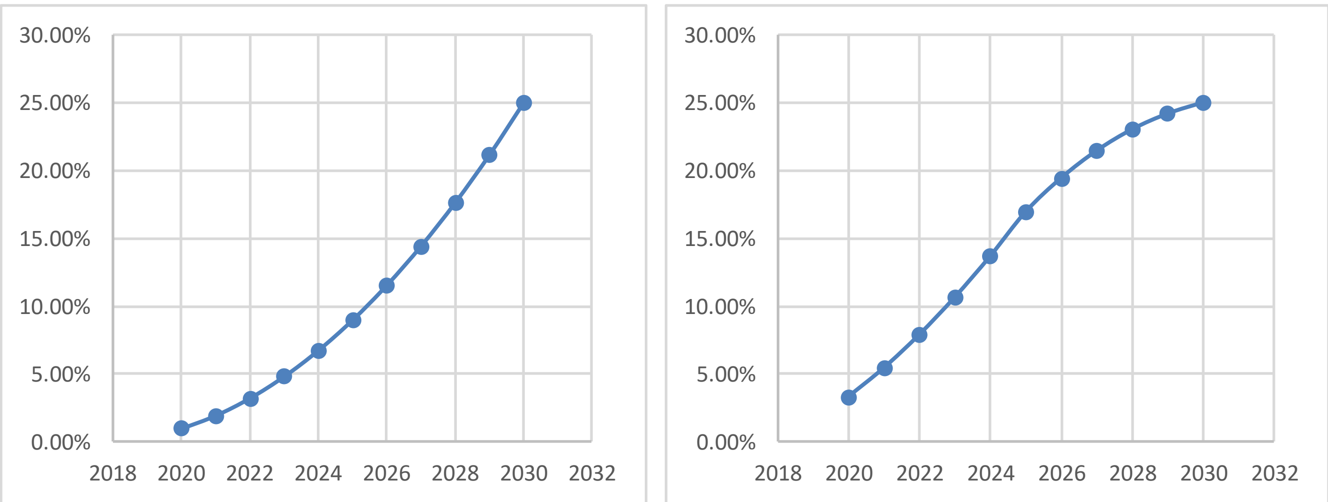

Figure 6.1 Annual targets for Plan Three & Our Plan (compared to 2018)

6.1 Carbon emission quota

The annual carbon emission quotas corresponding to the four plans are shown in the Figure 6.2.

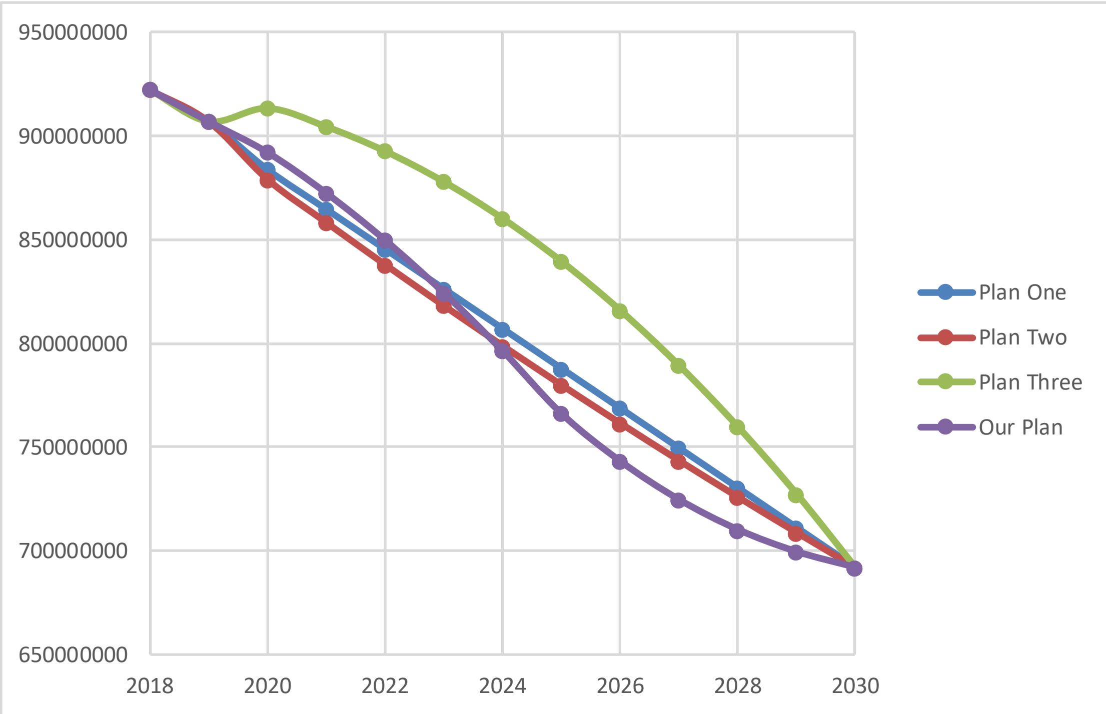

Figure 6.2 Annual carbon emission quotas under different plans

6.2 Emission reduction

The estimated annual emission reductions corresponding to the four plans are shown in Figure 6.2 It can be clearly seen that the "three-stage emission reduction" adopted in this paper brings the largest total emission reduction and the best emission reduction effect.

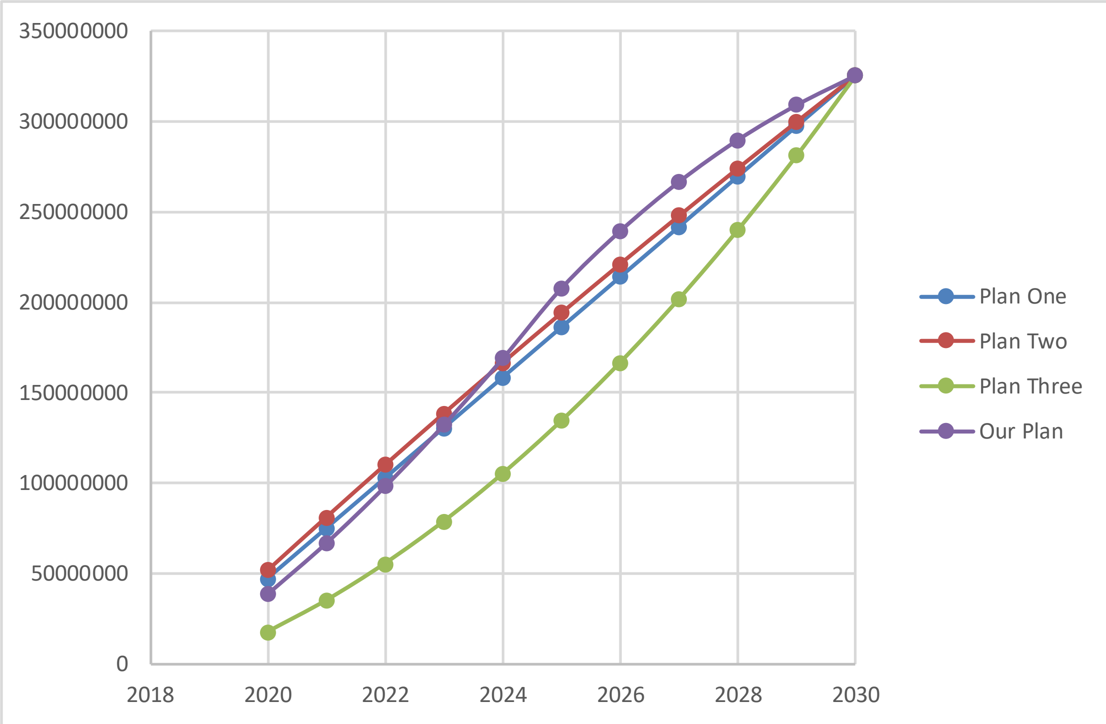

Figure 6.3 Estimated annual emission reductions under different plans

6.3 GDP reduction

The GDP reductions corresponding to the four plans are shown in Table 6.1. It can be found that the "three-stage emission reduction" caused the smallest reduction in GDP and the smallest social cost.

Table 6.1 GDP reductions under different emission reduction plan

Year	Plan One	Plan Two	Plan Three	Our Plan
2020	16779969023	19032663190	2761808342	12870196463
2021	6687139673	9703749074	-12239017845	2972756561
2022	-3901239777	-354720119.8	-26333213535	-5969577355
2023	-14997954566	-11149986452	-39533563824	-14087589552
2024	-26616119648	-22689752979	-51853183857	-21525790129
2025	-38769188492	-34982189469	-63305526959	-28440026362
2026	-51470961613	-48035937904	-73904393695	-39355330454
2027	-64735595629	-61860118539	-83663940634	-52922855221
2028	-78577612257	-76464336106	-92598689587	-69006325182
2029	-93011908014	-91858686483	-1.00724E+11	-87503441975
2030	-1.08054E+11	-1.08054E+11	-1.08054E+11	-1.08054E+11
Total	-4.56667E+11	-4.26713E+11	-6.49447E+11	-4.11022E+11

It is not difficult to see from the sensitivity analysis that the "three-stage emission reduction" plan is effective.

7. Risks and Alternatives

7.1 Risks and its Impact of Stakeholders

There are five types of stakeholders involved in the carbon credit market, as shown in the figure below.

Figure 7.1 Five types of stakeholders involved in the carbon credit market

In Carbon Credit Program, there are risks brought by policy, market, liquidity and other factors, leading to sharp price fluctuations. And many carbon credit financial instruments easily lead to sharp fluctuations in market prices. Especially, it is difficult to ensure the complete transparency of relevant information disclosure and the unimpeded communication channels, resulting in information asymmetry, adverse selection and even moral hazard, such as financial fraud. The following table details the risks associated with the Carbon Credit Program and its impact on stakeholders.

Table 7.1: Risks and its Impact of Stakeholders

Risk	Description	Impact
Policy and Political Risk	Policy Risk: the risk of losses due to the uncertainty and irrationality of policies Political Risk: the risk of loss due to changes in international political relations.	(1) Emission reduction requirements are unreasonable, and the speed of domestic technological innovation does not match, leading to economic recession; (2) The social cost of carbon emissions is miscalculated, resulting in the lack of motivation. (4) The different carbon emission and climate policies at home and abroad increase the trading risk of multinational enterprises.
Market Risk	Due to changes in the country's macroeconomic environment and the global macroeconomic environment, the risk of loss is caused by changes in market factors such as interest rate and exchange rate.	(1) Carbon credit trading has a certain inter-temporal nature, so the interest rate and exchange rate fluctuations will cause the interests of both sides of carbon credit trading to change greatly; (2) the macroeconomic environment effect the interest of enterprise，leading to affect the stability of carbon price。
Operational Risk	The risk caused by system failure, human operation error, management error and the possibility of loss caused by external emergencies.	(1) the supervision of carbon credit trading process is not in place, and market loopholes exist; (2) Whether the carbon credit trading system run stably and safely will affect the fairness of the transaction; (3) Many participating enterprises lack understanding of carbon trading, leading poor management.
Moral Hazards	Information asymmetry and the electronic nature of carbon credits lead to adverse selection, market manipulation and insider trading, which disrupt the trading order of carbon credit trading market	(1) The carbon credit management platform is attacked by hackers, resulting in data chaos. (2) Firms lie to get more quotas (especially free ones) (3) The lack of market transparency allows companies to advertise their carbon credits at will, leading to the risk of ponzi schemes.
Liquidity risk	Without increasing costs or asset value loss, the liquidity needs of stakeholders cannot be met in time, thus causing the possibility of losses.	The lack of liquidity in carbon credit trading would hamper the freedom of both sides of the trade, increasing the additional transaction cost - the liquidity cost.

Without a carbon credit program, society will face the risk of global warming, increased global desertification and declining biodiversity caused by carbon emissions. The cost of these risks will be non-financial.

7.2 Alternatives Comparison

According to the traditional welfare economics, there are two alternatives to the negative externalities caused by carbon emissions -- imposing pigouvian taxes and imposing administrative controls. Their pros and cons are shown in the table below:

Table 7.2： Alternative Approach and its pros and cons

Alternative Approach	Description	Pros	Cons
Imposing pigouvian taxes	Taxes and subsidies imposed by the government to offset the externalities of carbon emissions	(1) to some extent curb corporate carbon emissions; (2) tax revenue being transferred to the undertaker of externalities.	(1) The motivation of some enterprises emission reduction is insufficient; (2) Monopolistic industries will raise product prices so that externalities are transferred to consumers; (3) low efficiency, high implementation costs, and poor results (4) government failure and the overall social welfare loss
Imposing administrative controls.	The government sets clear environmental standards for carbon emissions and imposes economic fines and administrative penalties on companies that violate them	to some extent curb corporate carbon emissions;	low efficiency, high implementation costs, and poor results

To prevent and avoid these risks, ensure that the total carbon emissions will not be broken through, and regulate the monopoly of the carbon financial market is inseparable from effective information disclosure system, emission verification system, joint registration system and other means, so as to make the carbon credit trading market more fair and efficient.

8. Data Limitations and assumptions

The data provided by Pullanta's Department of Environmental Concerns can provide a general picture of the country's economy and carbon emissions, but the implementation of the carbon credit program needs to be adjusted to the unknowns and uncertainties of some decisive data. The following table details the data limitations, their impact, and the adjustments made.

Table 8.1 data limitations, impact, and justification

Data Limitation	Impact	Assumption and Justification
Data on Pullanta's energy usage structure and clean energy usage was not provided.	Inability to accurately measure the economic aspect of social costs of implementing the program (GDP trends)	We assume that energy usage structure and clean energy usage will not change. We only used the univariate linear regression method to analyze the GDP of Pullanta.
	The future trend of carbon spot price cannot be accurately predicted	We assume that the structure of energy usage and the situation of clean energy are in line with the situation in the EU We refer to the trend of carbon spot price in the EU.
Data of GDP by sector was not provided.	It is hard to accurately discuss the action ratio setting for each industry	We assume that the share of GDP by sector is the same as in EU. We refer to the proportion of auctions for sectors in the EU.
Historical data of specific emission reduction technology of Pullanta was not provided	We are unable to fit the emission reduction cost curve and calculate auction reserve price	We use Directional Distance Function to fit the theoretical formula of marginal emission reduction cost curve and derive the theoretical formula of auction reserve price of carbon emission rights
Historical data on international carbon prices was not provided	The accurate measurement of the geometric Brownian fractal motion of the carbon spot price is hindered	We assume that future Inflation of carbon prices is similar with that in EU. The estimation of carbon spot price volatility σ^2 was based on the historical carbon spot prices in the European Union
We are not told about the specifics of Pullanta's legal environment and the carbon emission regulation mechanism.	We are unable to precisely discuss the risks and cost of implementing carbon credit programs.	We assume that laws and regulations are enforced very strictly in Pullanta
Historical data of Pullanta's exchange rate was not provided	It is hard to discuss the impact of exchange rates on the price of carbon derivatives.	We assume that the exchange rate and the price of carbon derivatives are independent of each other
There is no data about how major economic event would impact this market	We cannot accurately discuss the implementation of the carbon credit program and the achievement of emission reduction targets	We assume that the macroeconomic environment is stable during the implementation of the carbon credit program.

9. Conclusion

Based on the analysis and model construction above, we can believe that the carbon credit program above can complete the emission reduction plan of the Department of Environmental Concerns of Pullanta, and can provide revenue for the government during the emission reduction process, promote the diversity and liquidity of the financial market of Pullanta. We model carbon credit price by using the jump fractional Brownian process, and use ‘Marginal Abatement Cost Pricing Model’ to construct the auction market of carbon credit. Meanwhile, secondary market construction are adopted in this plan to ensure the normal operation of Pullanta's carbon credit market. However, there are still some limitations in the formulation of this plan, which still needs to be adjusted according to the actual situation of Pullanta and the implementation of the plan.

10. References

[1] Pauline Barrieu & Max Fehr. （2011）. Integrated EUA and CER price modeling and application for spread option pricing. Retrieved from https: //ssrn.lom/abstract=1737637.

[2] Hurst H. (1951). Long term storage capacity of reservoirs .*Transactions of the American Society of Civil Engineers*， 116，770-799.

[3] George Daskalakis. & Raphael N. Markellos. (2008). are the European carbon market efficient［J］. *Review of Futures Markets*，17(2)，103-128.

[4] Ciprian Necula. (2002) Option Pricing in a Fractional Brownian Motion Environment. *Academy of Economic Studies Bucharest, Romania*.

[5] Pauline Barrieu & Max Fehr (2011)， Integrated EUA and CER Price Modeling and Application for Spread Option Pricing. *working paper*.

[6] S. Asmussen. (1999)，Stochastic Simulation with a View Towards Stochastic Processes. *Notes of a Concentrated Advance Course at MaPhySto*， Aarhus Denmark, 2

[6] Dutschkea. Metal. (2004). Value and Risks of Expiring Carbon Credits from CDM Afforestation and Reforestation *HWWA Discussion Paper*. 290.

[7] Seifert, J. Uhrig-Homburg, M. & Wagner, M. (2008). Dynamic behavior of co2 spot prices. *Journal of Environmental Economics and Management*, 56.

[8] Labatt, S., & White, R. R. (2015). Carbon Finance: The Financial Implications of Climate Change.

[9] The Wrong Sort of Recycling， THE ECONOMIST. (2010, Mar. 25). Retrieved Feb 21, 2020, from http: / /www. economist. com /node /15774368.

[10] Michael Szabo. (2008, Dec 2). DNV Suspension Another Jab at Battered CO2 Scheme，REUTERS， Retrieved Feb 21, 2020, from http: / /www. reuters. com /article /2008 /12 /02 /us － carbon － dnv － idUSTRE4B04K120081202.

[11] Mark Schapiro. (2010). Conning the Climate: Inside the Carbon Trading Shell Game. *Harper's Magazine*，21, 31-39

[12]Zheng, Y.H.(2016). Tan jinrong shichang de dingjia yu jiage yunxing jizhi yanjiu [Research on the Pricing Mechanism and Price Operation Mechanism of Carbon Financial Market]. Zhongguo kuangye daxue(Beijing)

[13]Feng, J.F. & Xia, C.X. & Wang, C.X.(2018). Tan paifang quan jiage jianmo yu tan zhaiquan guzhi. [Carbon Emission Right Price Modeling and Carbon Bond Valuation]. Hebei jingmao daxue xuebao, 39(01), 66-72.

[14]Gao, S. & Li, M.Y. & Duan, M.S. et al.(2019). Bali Xieding xia de guoji tan shichang jizhi: jiben xingshi he qianjing zhanwang. [International market mechanisms under Paris Agreement: basic form and future prospects]. Qihou bianhua yanjiu jinzhan, 15 (3), 222-231

[15] Chen, J.M. & Li H.D. & Wang, X.S.(2013). Aodaliya tan paifang jiaoyi tixi de tese fenxi ji qishi. [Characteristics Analysis and Enlightenment of Carbon Emission Trading System in Australia].Shengtai jingji, 04, 70-74+87.

[16]Feng, J.F. & Chen, J. & Du, D.Y. & Liu, Q.M.(2015). Tan zhaiquan de guzhi ji minganxing fenxi. [Carbon Bond Pricing and Sensitivity Analysis]. Kexue juece, 10, 17-32.

[17]Zhang, Y. & Li, Y. (2018). Guoji tan paifang quan jiaoyi tixi yanjiu ji dui zhongguo de qishi. [Research on the International Carbon Emission Trading Scheme and Its Implications to China]. Shengtai jingji, 34(02):66-70.

[18]Chen, N. & Lian, N.N. (2019). Qianxi woguo tan paifangquan jiaoyi shichang youhua lujing——jiyu tan paifangquan jiage shijiao. [Analysis on Optimized Path to Trading Market of Carbon Emission in China——from the price perspective of carbon emission rights]. Hunan shewai jingji xueyuan xuebao, 19(03):35-44.

[19] Onno Kuik, Luke Brander & Richard S.J. Tol. (2009). Marginal abatement costs of green-house gas emissions: A meta-analysis. *Energy policy*, 37(4): 1395-1403.

[20] Liao, C., H. & Onal, M. H. Chen. (2009). Average shadow price and equilibrium price: case study of tradable pollution permit markets. *European Journal of Operational Research,* 196(3):1207-1213.

A. Appendix: R Codes & Summaries

```
> lm <- lm(GDP ~ Total + Percent_of_Population_in_Urban_Areas_ + Energy_Use, data = dat)
> summary(lm)

Call:
lm(formula = GDP ~ Total + Percent_of_Population_in_Urban_Areas_ +
    Energy_Use, data = dat)

Residuals:
       Min         1Q     Median         3Q        Max
-7.382e+10 -3.192e+10 -9.565e+09  3.076e+10  7.934e+10

Coefficients:
                                       Estimate Std. Error t value Pr(>|t|)
(Intercept)                          -1.876e+12  2.546e+11  -7.369 4.05e-07 ***
Total                                -4.743e+02  2.259e+02  -2.100   0.0486 *
Percent_of_Population_in_Urban_Areas_  8.646e+12  7.406e+11  11.675 2.21e-10 ***
Energy_Use                             2.744e+07  1.062e+07   2.583   0.0178 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.402e+10 on 20 degrees of freedom
Multiple R-squared:  0.9537,    Adjusted R-squared:  0.9467
F-statistic: 137.2 on 3 and 20 DF,  p-value: 1.649e-13

> Predict1<- predict(lm,data.frame(Total = predict_data$Plan_One,
+                                  Percent_of_Population_in_Urban_Areas_ = predict_data$Perce
nt_of_Population_in_Urban_Areas,
+                                 Energy_Use = predict_data$Energy_Use))
> Predict2<- predict(lm,data.frame(Total = predict_data$Plan_Two,
+                                  Percent_of_Population_in_Urban_Areas_ = predict_data$Perce
nt_of_Population_in_Urban_Areas,
+                                  Energy_Use = predict_data$Energy_Use))
> Predict3<- predict(lm,data.frame(Total = predict_data$Plan_Three,
+                                  Percent_of_Population_in_Urban_Areas_ = predict_data$Perce
nt_of_Population_in_Urban_Areas,
+                                  Energy_Use = predict_data$Energy_Use))
> Predict4<- predict(lm,data.frame(Total = predict_data$Our_Plan,
+                                  Percent_of_Population_in_Urban_Areas_ = predict_data$Perce
nt_of_Population_in_Urban_Areas,
+                                  Energy_Use = predict_data$Energy_Use))
> Predict <- cbind(Predict1, Predict2, Predict3, Predict4)
> Predict
       Predict1     Predict2     Predict3     Predict4
1  761251603468 763504297635 747233442787 757341830908
2  770366142286 773382751687 751439984768 766651759174
3  779480681104 783027200761 757048707346 777412343526
4  788595219874 792443187988 764059610616 789505584888
5  797709758692 801636125361 772472694483 802800088211
6  806824297509 810611296532 782287959042 817153459639
7  815938836327 819373860036 793505404245 828054467486
8  825053375098 827928852188 806125030093 836866115506
9  834167913915 836281190066 820146836585 843739200990
10 843282452733 844435674264 835570823722 848790918772
11 852396991551 852396991551 852396991551 852396991551
```

```
> dat$Total <- dat$Total/100000000
> dat$GDP <- dat$GDP/10000000000
> reg_GDP <- lm(GDP ~ Year, data = dat)
> summary(reg_GDP)

Call:
lm(formula = GDP ~ Year, data = dat)

Residuals:
       Min         1Q     Median         3Q        Max
-9.692e-10 -5.292e-10  3.108e-11  4.759e-10  1.249e-09

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.086e-07  3.650e-08  -13.93 2.14e-12 ***
Year         2.559e-10  1.819e-11   14.07 1.78e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.169e-10 on 22 degrees of freedom
Multiple R-squared:  0.8999,	Adjusted R-squared:  0.8954
F-statistic: 197.9 on 1 and 22 DF,  p-value: 1.778e-12
```

```
> reg_Total <- lm(Total ~ Year, data = dat)
> summary(reg_Total)

Call:
lm(formula = Total ~ Year, data = dat)

Residuals:
       Min         1Q     Median         3Q        Max
-1.173e-08 -6.219e-09  2.107e-09  5.288e-09  9.735e-09

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.637e-06  4.041e-07  -4.051 0.000532 ***
Year         8.629e-10  2.014e-10   4.284 0.000301 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.83e-09 on 22 degrees of freedom
Multiple R-squared:  0.4548,	Adjusted R-squared:   0.43
F-statistic: 18.35 on 1 and 22 DF,  p-value: 0.0003014
```

```
> library(ggplot2)
> p <- ggplot(dat, aes(x=Year,y=Total)) + geom_point(shape=19) +
+   xlab("Year") + ylab("Total Carbon Emission/10^8 mt")
> p + geom_smooth(method = lm) + ylim(3, 11)
```

```
> p <- ggplot(dat, aes(x=Year,y=GDP)) + geom_point(shape=19) +
+   xlab("Year") + ylab("GDP/10^10")
> p + geom_smooth(method = lm)
```