ILA LAM Model Solutions
Spring 2022

1. Learning Objectives:

1. The candidate will understand, evaluate and use stochastic, generalized linear, multi-state, projection and transition matrix models. The candidate will demonstrate an understanding of their underlying methodologies, strengths, limitations, and applications.

Learning Outcomes:

(1a) With respect to stochastic models:

- Explain and apply the stochastic modeling methodology, including measurement metrics (e.g., CTE).
- Describe and apply the theory and uses of real world versus risk neutral assumptions.
- Describe and apply the techniques of Monte Carlo simulation (including variance reduction and importance sampling).
- Describe and evaluate Random Number Generator models, and explain their uses, advantages, and theory.
- Describe and evaluate how stochastic models may be used to understand mortality and policyholder behavior risks and inform the use of reinsurance.
- Describe the technique of nested stochastic projections and explain why they are needed, and evaluate implementation issues.

(1b) With respect to generalized linear models:

- Describe and apply the basic principles of GLMs, and evaluate where GLMs might be useful in a Life Insurance context.

(1c) With respect to multi-state and transition matrix models:

- Describe and apply the methodologies for constructing multi-state and transition models in an insurance context.

(1d) With respect to multi-state and transition matrix models:

- Describe and apply the modeling methodology in an LTC product context.

Sources:

LAM-135-19: Stochastic Modeling, Theory and Reality from and Actuarial Perspective, sections I.A, I.B-I.B.3.a, I.B.4 & I.D-I.D.3

LAM-142-19: Case Study: LTC Insurance First Principles Modeling: Mortality Assumptions

1. Continued

Stochastic Modeling is on the Rise, Product Matters, Nov 2016

Stochastic Analysis of Long-Term Multiple-Decrement Contracts, 2008 (excluding Attachments)

Common Misunderstandings of Risk-Neutral Valuation, Stroman, Financial Reporter, SoA, June 2019

Commentary on Question:
Candidates are expected to demonstrate solid understanding of various concepts related to stochastic modelling, including tail risk metrics, risk-neutral vs. real-world scenarios, and their applications in real life practice.

Many candidates were successful at reiterating theoretical concepts, but showed difficulty in integrating theories into real case applications.

Solution:

(a) ABC Life is planning to develop a set of first principles mortality assumptions for its block of Long-Term Care (LTC) business. Mortality is the only decrement for this business. LTC mortality experience and the legacy total life mortality assumption are provided in the excel workbook.

Recommend an appropriate approach ABC should use to develop its first principles LTC mortality assumption. Justify your answer.

Commentary on Question:
To earn full credit, candidates are expected to demonstrate calculating active, disabled and total life mortality and show that calculated total mortality does not compare well with legacy experience in order to make conclusion of developing implied disabled life mortality. Many candidates recommended using legacy mortality to calculate implied disabled life mortality purely based on lack of disabled life data, although correct, this would not earn full credit.

Response to part (a) can be found in the attached Excel spreadsheet.

(b) Due to the impact of the COVID-19 pandemic on insurance and economic experience, ABC Life has decided to develop stochastic mortality and interest rates models.

(i) Explain how stochastic modeling could improve ABC's understanding of its COVID-19 driven mortality risk compared to deterministic modeling.

(ii) You use the newly developed stochastic mortality model to generate loss results from 10,000 simulated scenarios (provided in the Excel workbook).

1. Continued

Prepare a report for senior management which summarizes the conclusions which can be drawn from the model with respect to ABC's risk and capital profile.

No calculations are required for this part.

Commentary on Question:
Part (i), the candidates are expected to clearly explain the distinction between stochastic and deterministic methods, and explain why COVID-19 mortality risk requires the application of stochastic modelling. Most candidates were able to list out advantages of stochastic modelling in general, yet did not provide reasons for its application particularly on pandemic risk such as COVID-19.

Part (ii) tests the candidates on the application of stochastic modelling in real practice. Many candidates were able to calculate VaR and CTE measures given the data, yet were unable to discuss the meaning of these metrics in the context of management's understanding of tail risk. Only few candidates made recommendations on setting capital levels based on calculated VaR measure.

(i) A stochastic model is able to simulate a distribution of possible outcomes that reflect the random variations in the inputs. In contrast, a deterministic method is a mathematical simplification involving a few scenario outcomes based on pre-determined input variables.

Stochastic methods are more adequate at dealing with complex risks such Covid-19 pandemic mortality, as its insufficient experience data, rapidly evolving health technology and increasing globalization could alter the outcome of the pandemic and call into question the credibility of using only a few select scenarios (deterministic) to understand its true risk exposure.

(ii) Response to part (ii) can be found in the attached Excel spreadsheet.

(c) ABC Life is developing assumptions for its stochastic interest rate model and is faced with the choice between using risk-neutral and real-world scenarios

(i) Recommend the appropriate scenario choice for each of the following situations. Justify your answer.

A. Management wants to determine the market-consistent view of liabilities under a stressed interest rate environment due to the global pandemic.

1. Continued

B. Management wants to know how much capital is needed to absorb the potential earnings loss in the current low interest rate environment.

C. To calculate the hedging cost when pricing a new Universal Life product with a minimum crediting rate guarantee.

D. When monitoring residual market risk of a Universal Life product with a minimum crediting rate guarantee.

(ii) Critique the following statement:

Short-term risk-free interest rates calculated using risk-neutral stochastic scenarios will be more conservative than those calculated using real-world scenarios.

Commentary on Question:
Part (i) was done well. Most candidates were able to differentiate the different uses of risk-neutral and real-world scenarios and recommend appropriate methods. Some candidates had difficulty providing sufficient justifications to support their choice.

For Part (ii), most candidates were able to state the existence of "term premium", however one needs to explain the concept behind the term premium in order to earn full credit. Candidates are also expected to recognize the source of this misunderstanding arising from equity securities as covered in the reading "Common Misunderstanding of Risk-neutral Valuation".

(i) A. Risk-neutral scenarios. This question is asking what the loss in cash flows due to pandemic is worth, which involves discounting liability cash flows to obtain an expected present value, a typical use of risk-neutral valuation.

B. Real-world scenarios. This is a "what-if" type of question asking for the loss impact (capital required) under a stressed scenario, a common use of real-world scenarios.

C. Risk-neutral scenarios. Calculating expected hedge cost of an insurance guarantee with embedded derivative is another typical use of risk-neutral scenarios, as it involves calculating market-consistent present value of hedge cash flows.

1. Continued

D. Real-world scenarios. This looks for the outcome of post-hedging residual risk expected from realistic market outcomes.

(ii) This statement is false. In the context of interest rate scenario generations, being conservative means lower rates.

This is a common misunderstanding that real-world scenarios would produce higher short-term returns than risk-neutral scenarios. This misunderstanding may have arisen as real-world simulations for equity returns tend to be higher than risk-neutral simulations. However, fixed-income securities are treated differently than equities.

For the risk-free yield curve, only the short-term rates are risk-free, all longer-term rates involve a "term-premium", representing the price for the risk of locking-in an interest rate in an environment where interest rates can change. Under real-world scenarios, to get the market's expectation of the path of the short-term rates, the term premiums are removed. Due to the existence of term premiums, short-term interest rates are typically higher under risk-neutral environment compared to real-world.

2. Learning Objectives:

2. The candidate will understand and be able to assess issues and concerns common to actuarial models and their development and management.

Learning Outcomes:

(2a) Describe Model Efficiency concepts and explain and apply both the representative scenarios and replicating liabilities techniques for improving Model Efficiency.

(2j) Describe and evaluate considerations around the governance of expert judgment in actuarial modelling

(2k) Describe and evaluate considerations related to modeling investments, discount rates, inflation and catastrophic mortality

Sources:

Interesting Challenges for Insurers, Product Matters, Jun 2012

LAM-116-14: Life Insurance Forecasting and Liability Models: An Examination of the Trade-Offs Involved with Certain Modeling Decisions

LAM-149-21: Application of Professional Judgment by Actuaries, 2020

Commentary on Question:

The goal of this question is to assess the candidate's understanding of interest rate risk and ways to mitigate its impacts on an insurance company, as well as professionalism in the context of assumption setting and considerations related to modeling simplifications (e.g., issue age modeling). The candidate should be able to demonstrate familiarity with the principles of professionalism that apply to actuarial work, as well as interpret and make modeling recommendations based on scenario analysis.

Solution:

(a) You are the lead pricing actuary of your company's fixed annuity product lines. Senior management has expressed concerns over the risk of a prolonged low-interest rate environment and the risk of a market correction resulting in a sudden upward spike in interest rates

(i) Analyze how your company’s profitability may be impacted under each of the two scenarios.

(ii) Propose two strategies that the company could apply to mitigate interest rate risk on its fixed annuity products.

2. Continued

Commentary on Question:
To earn full credit, the candidate should discuss the impacts on profitability under both interest rate scenarios, in the context of fixed annuity products. Partial credit may be given if the candidate's answer does not reference the annuity portfolio specifically.

A (i)
Under the sustained low interest rate scenario:

- Low interest rates reduce the returns from bonds that insurers invest in, hence negatively impacting earned rates and profitability. For fixed deferred annuities, credited rates will likely hit the minimum guarantee rate soon (if they are not already at the minimum credited rate), which puts pressure on the company's ability to earn their expected spreads. In other words, unless the company was able to reduce it's cost of borrowing (crediting rates, expenses etc.), its profit margins will decrease or erode completely, resulting in significant losses.

- Although the company's balance sheet may be strengthened on a market value basis by falling interest rates, in most instances, the corresponding market values of liabilities would have increased by at least as much as any increase of the assets, so any benefits would be minor at best.

Under the sudden spike in rates scenario:

- Writers of fixed products using a credited rate concept must often increase their credited rates or face mass lapses, since the policyholder can choose to invest their money at higher rates elsewhere. This issue is further compounded by the reduced market value of the assets held by the company that have to be sold to fund the resulting lapse or partial withdrawal (i.e., capital loss), which can result in a significant hit to profitability.
- Alternatively, if the company elects to increase the credited rate, this will typically lead to lower interest spreads and reduced profitability.

A (ii)
- Reduce minimum guaranteed rates on fixed products; employ a current credited rate mechanism to offer higher rates if interest rates rise and vice versa, and seek to pass on lower costs of interest rate risk to policyholders via enhancement of other features if the interest rate environment improves

- Move products that contain locked-in features to a design that allows them to change product features based on movements in interest rates

2. Continued

- Utilize ALM to match assets fairly closely with liabilities, especially on fixed immediate annuities (which have locked in designs)

- Highlight or push products that offer a greater potential for consumer return in a low interest rate environment (e.g., shrink the company's pure fixed annuity portfolio and expand on indexed annuities, whereas the latter offers potential for upside participation via the equity markets)

- Refine assumption setting on dynamic lapses and other policyholder behavior assumptions for its fixed annuity book

- Pursue asset strategies that are more explicitly linked to hedging interest rate risks identified in the company's repricing gap analysis

- Use reinsurance to transfer interest rate risk.

(b) As part of your periodic review of modeling assumptions and simplifications, you are assessing if a new method of defining issue age bands in your pricing model would be appropriate.

(i) Describe the principles of professionalism outlined in *Application of Professional Judgement by Actuaries*, in the context of assumption setting.

You are testing five age band definitions of varying size. The associated modeling error arising from each case is summarized below:

Case	# of Issue Age Bands	Age Error (in years)	Average Known Error	Average Unknown Error
1	15	0.1	0.5%	0.6%
2	10	0.6	3.2%	2.3%
3	8	0.7	1.2%	1.4%
4	4	0.0	0.8%	0.4%
5	3	0.4	2.4%	2.0%

Age error is defined as the weighted average difference between the weighted average issue age for each band as compared to the model issue age for that band.

(ii) Assess the implications of this analysis and recommend which case(s), if any, should be further considered for implementation.

2. Continued

(iii) Your pricing model assumes the issue age of the policy is the mid-point of the age band and all age bands are of uniform width. Your manager suggests that this approach could be applied to all pricing models in the company as it is the simplest to implement.

Critique the above suggestion.

(iv) Compare and contrast static and dynamic model validation.

Commentary on Question:
For b(i), to earn full credit, at least 4 distinct principles of professionalism should be listed.
The candidate should identify both the principle of professionalism and explain how it should be considered during assumption setting.

For b(iii), the candidate should clearly state whether they agree or disagree with both aspects of the current setup, as well as explain why there may be alternative approaches that are also valid.

For b(iv), the candidate should describe both static and dynamic validations and the relative advantages/disadvantages of each.

B(i):
Integrity (i.e., an actuary must act honestly and with highest standards of integrity): taking into consideration all relevant and appropriate data during assumption setting, even if the resulting assumption does not appear favorable; ensuring any professional judgment applied agrees with standard practices

Competence and care (i.e., an actuary must perform professional services competently and with care): ensuring the actuary has enough knowledge on sources, data quality, sample size, and any limiting factors for the choice of variables; considering consistency with similar situations applicable to this assumption; reviewing future trends and expectations for reasonableness; performing regular back-testing on assumptions

Compliance (i.e., an actuary must comply with all relevant legal, regulatory and professional requirements): reviewing appropriate ASOPs, valuation manual chapters, NAIC guidance and state regulatory requirements to ensure the assumptions are derived in accordance with these materials; ensuring assumptions are appropriately documented for regulatory purposes, such as cash flow testing memos and VM-31 reports

2. Continued

Impartiality (i.e., an actuary must not allow bias, conflict of interest or the undue influence of others to override professional judgment): ensuring the actuary is not unduly influenced by senior management or other influential stakeholders to set overly aggressive assumptions; ensuring any potential conflicts of interest are disclosed in advance and separating oneself from the assignment if warranted

Communication (i.e., an actuary must communicate in an appropriate manner and meet all applicable reporting standards): disclosing any material deficiencies with the data and/or methodology used for setting assumptions; ensuring the right people are involved in providing relevant data and feedback for each variable; compiling complete documentation on the key steps of the assumption setting process

B(ii):
The Ultimate goal of the issue age modeling is to reduce the model calculation. The issue age banding should have a good balance between the accuracy and the model complexity (i.e., reduction of data / model run time)
The scenario analysis shows that a larger number of age bands does not necessarily produce a better model; in other words, a more elaborate model is not necessarily a better model. The number of issue age bands is negatively correlated with the final three columns, but the correlation is weak, which is most likely due to the fit of the model age to the average issue age within each band. In contrast, the correlation between the age error and average known/unknown errors is much stronger. Furthermore, the table shows that any error introduced by moving to wider issue age bands can be potentially more than offset by the error removed by having more strategically placed age bands (e.g., moving from Scenario 2 to Scenario 4).

'Therefore, it appears that Scenarios 1 and 4 both produce relatively good fits overall (and hence should be further reviewed/investigated as a potential modeling enhancement), whereas the other scenarios do not fit as well.

B(iii):
Disagree. While potentially easier to implement if assuming a uniform age band width, age bands do not have to be all the same width. It is possible that modeling errors can be reduced by modeling certain ages with one age band width, then using a smaller age band width for very young or old ages. This could also be influenced by the underlying product type and mix of business.

2. Continued

Disagree. While potentially easier to implement, the actuary does not have to use the central ages of the selected age bands as the model age - in fact, the optimal model age may be something other than the central age. In practice, many actuaries will select the model age for a given age bucket by using the age that is closest to the weighted average issue age of that bucket, which can be particularly useful if the actuary chooses very wide age bands.

B(iv):
A static validation compares certain known and modeled values as of the date from which the model projects, which provides a measure of model accuracy. However, it only looks at one point in time (i.e., balance sheet validation), and only at one variable at a time (such as premiums or reserves). It also fails to capture the effect of interactions among variables, and it may hide significant distortions resulting from the modeling approach in the future.

Unlike static validations, dynamic validations can be either prospective (i.e., comparing the trend in actual historical results with the model's projected results) or retrospective/back-cast (i.e., starting with the current portfolio of business and running the model backwards through time, then comparing this against actual historical data). Dynamic validations are analogous to an income statement validation, regardless of whether the model is run forwards or backwards. Dynamic validations also involve looking at many assumptions at once and measuring the accuracy of their interaction, which can make it much more robust of a validation than static validation. However, dynamic validations are not always possible (e.g., if good historical data is not available or credible).

3. Learning Objectives:

3. The candidate will understand the principles of Asset-liability Management ("ALM"), and be able to describe and evaluate various techniques for addressing the mitigation of risk.

Learning Outcomes:

(3a) With respect to Asset-Liability Models:

- Describe and apply the fundamental elements of the theory and practice of ALM in an insurance company, including assessing the dangers of mismatched assets and liabilities.
- Describe and demonstrate how ALM can be used to identify and manage product and asset risks, including:
  - Major product risks for which ALM can be a useful tool for their management.
  - Using ALM as a means to manage interest rate risk, equity risk, and risks from optionality.
- Describe how common insurance contracts and variations generate embedded options in an insurer's balance sheet, and assess basic strategies for managing exposures created by such embedded options.
- Describe and apply the basic concepts of cash flow matching, immunization, duration/convexity matching, segmentation.
- Describe and apply Key Rate Durations (KRD) and their use in evaluating interest rate sensitivities of portfolios, including understanding the derivation of KDRs, the profiles of KDRs for selected major asset types, and assessing KRDs in a portfolio context.
- Describe and evaluate the Goldman Sachs' ALM/Strategic Asset Allocation approach for integrating ALM into an enterprise's risk and financial management framework.
- Describe and evaluate ALM modeling considerations in the context of modeling risk aggregation, dependency, correlation of risk drivers and diversification.

Sources:

LAM-130-15: Diversification: Consideration on Modelling Aspects & Related Fungibility and Transferability, CRO, Oct 2013, pp. 1-18

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a)

(i) Explain why diversifying strategies are pursued by insurance companies.

(ii) Describe two examples of how typical insurance companies pursue diversifying strategies to improve their overall risk profile.

3. Continued

Commentary on Question:
Most candidates answered part i) well. A few candidates suggested that diversification was for regulators without connecting it to a reason for the company to pursue diversification. Good candidates connected the diversification to reduced regulatory capital requirements that supported risk management and efficient use of capital.

Part ii) was looking for two examples of how companies pursue diversification strategy. Candidates had a number of good examples. It was most common for candidates to use the example of pooling dissimilar risks with annuity and life insurance or similar and independent risks. Some candidates talked only about diversifying the invested assets and needed to connect that within the context of assets backing surplus or liabilities for full credit. A few candidates provided a longer list of examples which was not required for full credit.

(i) Diversifying strategies are pursued by insurance companies to limit the company's exposure to any single risk and improve solvency. Diversifying strategies will result in smoother earnings over time and lower the need for risk capital. Diversifying strategies lower the insurer's cost of taking on each individual risk and will allow the insurer to price its products more competitively.

(ii) Companies can diversify risks by pooling similar and sufficiently independent risks such as life insurance.
Companies can diversify risks by writing a diversity of insurance products across different market segments and geographies to decrease the likelihood of experiencing adverse results for large blocks of business from a specific event or at a given time.

(b) Critique the following statements regarding methods for modelling aggregation of risk:

A. Simple summation produces an upper bound to the true risk figure.

B. It is optional to use risk diversification for the simple summation method.

C. Variance-covariance method allows for interaction across risk but requires data to develop the correlation matrix.

D. The integrated model is a tool that quantifies each risk independently and aggregates results at an appropriate level.

E. Since the integrated model accounts for all risks, it is transparent and easy to understand.

3. Continued

Commentary on Question:
Part b consisted of 5 statements for the candidate to critique. Good candidates provided a clear indication that each was correct or incorrect along with a short explanation.

The majority of candidates answered this part b) very well. Some candidates did write longer answers that were not required for full credit. In some responses it was not clear that the candidates thought the statement was correct or not. A number of candidates talked about the "true risk figure" in statement A as being something that could never be known rather than focusing on the statement as correct

A
True. Simple summation assumes no diversification at all, which is highly unlike but still possible, which represents the worst risk profile, so indeed it is an upper bound of risk figure --- but for an unlikely situation.

B
False. Simple summation assumes no risk diversification at all so it is not optional to use.

C
True. The variance-covariance model allows for interactions across risks. The challenge is having the data to develop the correlation matrix.

D
False. The integrated model assumes all risk together and does not quantify each risk independently.

E
False. The integrated model has a challenge with transparency and the communication of the results can be challenging.

(c) Calculate the aggregate risk exposure under each of the following risk aggregation approaches:

(i) Simple Summation

(ii) Variance-Covariance Matrix

3. Continued

Commentary on Question:
There are two calculation pieces for part c. All candidates were able to complete the summation of risks in the first part. Many candidates also completed the more challenging second part and received full credit. The second part required the risk aggregation to be completed using the variance-covariance matrix. Most candidates recognized that since the correlation was provided rather than the covariance it was necessary to replace the covariance term in the formula with the correlation of the risks multiplied by the Standard Deviations of the risks. Since mortality has 0 correlation with the equity and interest those terms dropped off but many candidates calculated the zero amounts.

There were two approaches to completing the calculation. Candidates that applied the formula and calculated the components correctly by squaring the four risks and adding in the correlations multiplied by the standard deviations made few errors and were able to take the square root to the correct answer. There were a few minor errors that resulted in some deductions. The second approach was to complete the calculation using spreadsheet formulas and there were various errors in the formulas used resulting in some deductions. Eg Some candidates forgot the square root, some used the wrong correlations.

(i) The calculation is a simple summation of the four risk values.

Aggregate risk = 50 + 10 + 80 + 20 = 160

(ii) The basic statistical formula for aggregating the variance of multiple correlated risks is the sum of the variances for each risk plus 2 times the sum of the covariance between individual risks.

The aggregate risk is the square root of the sum of the square of each risk plus 2 times the correlation of risks times the individual risks. Since mortality has 0 correlation with equity, and interest these terms drop off.

Interest risk squared = 50^2 = 2500

Equity risk squared = 10^2 = 100

Mortality risk squared = 80^2 = 6400

Operational risk squared = 20^2 = 400

2 times correlation of interest and equity times interest risk times equity risk =
2 x 25% x 50 x 10 = 250

3. Continued

2 times correlation of interest and operational risk times interest risk times operational risk = 2 x 50% x 50 x 20 = 1000

2 times correlation of equity and operational risk times equity risk times operational risk = 2 x 50% x 10 x 20 = 200

2 times correlation of mortality and operational risk times mortality risk times operational risk = 2 x 50% x 80 x 20 = 1600

Aggregate risk = square root of (2500 + 100 + 6400 + 400 + 250 + 1000 + 200 + 1600)
= square root of 12450 = 111.6

(d) Your company would like to approximate the variance-covariance method using a fixed diversification percentage in their modeling.

(i) Calculate a fixed diversification percentage based on the result calculated in part (c).

(ii) Explain why this approach may not be an accurate reflection of the aggregate risk exposure.

Commentary on Question:
The first part of part d asked for the fixed diversification percentage based on the result calculated in part c). Most candidates answered this question correctly. Full credit was earned for calculation of the fixed diversification percentage or the fixed percentage that would be applied to the risks provided the candidate was clear in their response.

There was no deduction for carrying forward an incorrect answer from part c. Some candidates that did not have a result for part c provided an estimated number and completed the question with that estimate for full credit.

The second part of the question asked for an explanation of why the fixed diversification might not be an accurate reflection of the aggregate risk exposure. Most candidates answered this well and included comments on the future changes in the risk exposures or changes in the correlations.

(i) The fixed diversification percentage = 1 – aggregate risk from part c divided by the simple summation = 1 – 111.6/160 = 30.3%

3. Continued

(ii) Overall, it lacks the ability to provide an accurate reflection of the aggregate risk exposure because it doesn't account for any way the risk factors may affect each other and be correlated with each other, and beyond applying a fixed percentage to the sum, it does not have any further way to model these complexities.

4. Learning Objectives:

4. The candidate will understand the basic design and function of Economic Scenario Generators and Equity Linked Insurance Models.

Learning Outcomes:

(4b) With respect to Equity-Linked models:

- Describe and apply methods for modeling long-term stock returns and certain guarantee liabilities (GMMB, GMDB, GMAB).
- Describe and evaluate the Actuarial and Hedging risk metrics for GMAB and GMDB models.
- Describe and apply methods for modeling Guaranteed annuity options and Guaranteed Minimum Income Benefits (GMIB), and EIA guarantees.

Sources:

Investment Guarantees Ch 1, Hardy, 2003

Investment Guarantees Ch 13, Hardy, 2003

LAM -139-19: Simulation of a Guaranteed Minimum Annuity Benefit, Freedman, 2019; Excel Model - Stochastic Simulation of a GMAB Option (Accompanies Simulation of a GMAB)

Commentary on Question:

Overall, most candidates did not perform well in this question. Many of them were unable to demonstrate how VA and EIA products work.
This is the last question in the exam, so quite a number of candidates were not able to complete this question.

Solution:

(a) Compare how the company's net income would differ under the following two scenarios:

- Markets are up and both products are unhedged
- Markets are down and both products are hedged

Commentary on Question:

Only a handful of candidates were able to obtain full credit in this part. Many were unable to differentiate the payout structure of EIA products. Partial credit is awarded for other reasonable answers.

Variable Annuity

The guarantee on VA is a put-option. If the market goes down, the insurer must pay (GV-MV). If the market goes up, the value to the insurer is zero

4. Continued

Equity Indexed Annuity
The guarantee on EIA is a call option. If the market goes up, the insurer must pay (GV-MV). If the market goes down, the value to the insurer is zero

Markets are up and both products are unhedged
EIA is in the money, while VA is out of the money.
Net income would be reduced by the loss on the EIA guarantee.

Markets are down and both products are unhedged
VIA is in the money and EIA is out of the money.
EIA hedge payout is 0, and VA hedge payout would offset the guarantee cost (plus/minus hedged inefficiency), but the company would incur hedging costs on both VA and EIA strategies.

(b)

(i) Calculate the value of the guaranteed minimum annual interest rate feature using the Put-Call Parity. Show all work.

(ii) Explain why the calculated result from (i) is reasonable.

Commentary on Question:
Most candidates were able list down the Put-Call Parity. Better candidates were able to complete the calculation both K and P.

S = 100,000
C = 5,000
r = 5%
t = 7
G = 4%

$K = S * (1+G)^7 = 131,593$
$P = c + Ke^{(-rt)} – S = (2,268)$

The underlying guarantee (4) is less than the risk-free rate so it is out of the money. As a result, the value of the put will always be less than the value of the call.

(c) Calculate the lifetime net income from this product if:

(i) The policyholder lapses at the end of year 3. Assume the market value of the call option at time 3 is 1,904.

(ii) The policyholder holds their product to maturity

Show all work.

4. Continued

Commentary on Question:
Candidates did poorly in part c. Most candidates were able to apply the participation rate, floor and cap rate correctly and calculate the guarantee values correctly. Only a handful were able to identify all the components in calculating the net income.

Period	Index	Return (%)	Apply Part. (%)[1]	Floor (%)	Cap (%)	Actual (%)[2]	Fund[3]	GV[4]	Max[5]
0	1,000						100,000	100,000	
1	900	-10.0	-5.0	1	10	1.0	101,000	104,000	104,000
2	800	-11.1	-5.6	1	10	1.0	102,010	108,160	108,160
3	700	-12.5	-6.3	1	10	1.0	103,030	112,486	112,486
4	800	14.3	7.1	1	10	7.1	110,389	116,986	116,986
5	1,000	25.0	12.5	1	10	10.0	121,428	121,665	121,665
6	1,100	10.0	5.0	1	10	5,0	127,500	126,532	127,500
7	1,250	13.6	6.8	1	10	6.8	136,193	131,593	136,193

Notes:
1 – Apply the 50% Floor Rate
2 – First, take the maximum between **1** above and the floor rate of 1%. Then, take the minimum of this against the cap of 10%.
Excel formula as = min (max (1%, Return * 50%), 10%)
3 – Fund value at time 0 with **2**) Actual return
4 – GV is credited 4% per year
5 – Maximum between **3**) Fund value and **4**) Guarantee Value

Lapse at time 3:

Call Cost	(5,000)
Call Payout	1,904
Zero Coupon Bond[6]	115,763
Withdrawal	(112,486)
Lapse Charge[7]	3,937
Net Income	4,117

Held to Maturity:

Call Cost	(5,000)
Call Payout[8]	4,600
Zero Coupon Bond[9]	140,710
Withdrawal	(136,193)
Lapse Charge	0
Net Income	4,117

Notes:
6 – 5% ZCB at year 3 = 100,000 * $(1+5\%)^3$
7 – Max (FV, GV) * 3.5% = 112,486 * 3.5%
8 – Max(0, FV- GV) at time 7
9 – 5% ZCB at year 7 = 100,000 * $(1+5\%)^7$

(d) Assess how the lifetime net income calculated in part (c)(ii) would change if the product was designed using the other Ratchet methods commonly sold. Justify your answer.

4. Continued

Commentary on Question:
Candidates in general were able to list out other Rachet methods. However, many cannot explain how the net income would be affected using these methods. Partial credits were given by listing out these methods with no explanations.

Point-to-point: The index payout value is determined at a certain point in the future with no annual adjustment.
High Water Mark: The payout under indexation uses the maximum equity index value over the term.
The index value is the highest at maturity so both methods would produce the same results as in part (c)(ii).

(e) Describe the residual hedging risk(s) your company is exposed to as a result of their risk mitigation strategy.

Commentary on Question:
Many candidates were able to identify credit risk, default risk or counterparty credit risk as a residual risk. Answers relating to insurance (mortality, lapse, policyholder behaviour risks) were not awarded any credits. Credits were provided to other reasonable answers.

7 years is a long duration for an equity option.
If the option writer defaults, the company would be heavily exposed to equity risk. The company is left with significant counterparty credit risk as a result of this transaction.