SOCIETY OF ACTUARIES

EXAM FM FINANCIAL MATHEMATICS

EXAM FM SAMPLE SOLUTIONS

This set of sample questions includes those published on the interest theory topic for use with previous versions of this examination. Questions from previous versions of this document that are not relevant for the syllabus effective with the October 2022 administration have been deleted. The questions have been renumbered.

Some of the questions in this study note are taken from past SOA examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Financial Mathematics (FM) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

The following model solutions are presented for educational purposes. Alternative methods of solution are acceptable.

In these solutions, s_m is the *m*-year spot rate and ${}_m f_t$ is the *m*-year forward rate, deferred *t* years.

Update history:
October 2022: Questions 208-275 were added
January 2023: Question 204 was deleted
June 2023 Questions 276-385 were added
August 2024: Questions 386-462 were added

1. Solution: C

Given the same principal invested for the same period of time yields the same accumulated value, the two measures of interest $i^{(2)} = 0.04$ and δ must be equivalent, which means:

$\left(1+\frac{i^{(2)}}{2}\right)^2 = e^{\delta}$ over a one-year period. Thus,

$$e^{\delta} = \left(1+\frac{i^{(2)}}{2}\right)^2 = 1.02^2 = 1.0404$$

$$\delta = \ln(1.0404) = 0.0396.$$

2. Solution: E

From basic principles, the accumulated values after 20 and 40 years are

$$100[(1+i)^{20} + (1+i)^{16} + \cdots + (1+i)^4] = 100\frac{(1+i)^4 - (1+i)^{24}}{1-(1+i)^4}$$

$$100[(1+i)^{40} + (1+i)^{36} + \cdots + (1+i)^4] = 100\frac{(1+i)^4 - (1+i)^{44}}{1-(1+i)^4}.$$

The ratio is 5, and thus (setting $x = (1+i)^4$)

$$5 = \frac{(1+i)^4 - (1+i)^{44}}{(1+i)^4 - (1+i)^{24}} = \frac{x - x^{11}}{x - x^6}$$

$$5x - 5x^6 = x - x^{11}$$

$$5 - 5x^5 = 1 - x^{10}$$

$$x^{10} - 5x^5 + 4 = 0$$

$$(x^5 - 1)(x^5 - 4) = 0.$$

Only the second root gives a positive solution. Thus

$$x^5 = 4$$

$$x = 1.31951$$

$$X = 100\frac{1.31951 - 1.31951^{11}}{1 - 1.31951} = 6195.$$

Annuity symbols can also be used. Using the annual interest rate, the equation is

$$100\frac{s_{\overline{40}|}}{a_{\overline{4}|}} = 5(100)\frac{s_{\overline{20}|}}{a_{\overline{4}|}}$$

$$\frac{(1+i)^{40}-1}{i} = 5\frac{(1+i)^{20}-1}{i}$$

$$(1+i)^{40} - 5(1+i)^{20} + 4 = 0$$

$$(1+i)^{20} = 4$$

and the solution proceeds as above.

3. Solution: C

Eric's (compound) interest in the last 6 months of the 8th year is $100\left(1+\frac{i}{2}\right)^{15}\frac{i}{2}$.

Mike's (simple) interest for the same period is $200\frac{i}{2}$.

Thus,

$$100\left(1+\frac{i}{2}\right)^{15}\frac{i}{2} = 200\frac{i}{2}$$

$$\left(1+\frac{i}{2}\right)^{15} = 2$$

$$1+\frac{i}{2} = 1.047294$$

$$i = 0.09459 = 9.46\%.$$

4. Solution: C

$$77.1 = v(Ia)_{\overline{n}|} + \frac{nv^{n+1}}{i}$$

$$= v\left[\frac{\ddot{a}_{\overline{n}|} - nv^n}{i}\right] + \frac{nv^{n+1}}{i}$$

$$= \frac{a_{\overline{n}|}}{i} - \frac{nv^{n+1}}{i} + \frac{nv^{n+1}}{i}$$

$$= \frac{a_{\overline{n}|}}{i} = \frac{1-v^n}{i^2} = \frac{1-v^n}{0.011025}$$

$$0.85003 = 1 - v^n$$

$$1.105^{-n} = 0.14997$$

$$n = -\frac{\ln(0.14997)}{\ln(1.105)} = 19.$$

To obtain the present value without remembering the formula for an increasing annuity, consider the payments as a perpetuity of 1 starting at time 2, a perpetuity of 1 starting at time 3, up to a perpetuity of 1 starting at time n + 1. The present value one period before the start of each perpetuity is 1/i. The total present value is $(1/i)(v + v^2 + \cdots + v^n) = (1/i)a_{\overline{n}|}$.

5. Solution: C

The interest earned is a decreasing annuity of 6, 5.4, etc. Combined with the annual deposits of 100, the accumulated value in fund Y is

$$6(Ds)_{\overline{10}|0.09} + 100s_{\overline{10}|0.09}$$

$$= 6\left(\frac{10(1.09)^{10} - s_{\overline{10}|0.09}}{0.09}\right) + 100(15.19293)$$

$$= 565.38 + 1519.29$$

$$= 2084.67.$$

6. Solution: D

For the first 10 years, each payment equals 150% of interest due. The lender charges 10%, therefore 5% of the principal outstanding will be used to reduce the principal.

At the end of 10 years, the amount outstanding is $1000(1-0.05)^{10} = 598.74$.

Thus, the equation of value for the last 10 years using a comparison date of the end of year 10 is

$$598.74 = Xa_{\overline{10}|10\%} = 6.1446X$$

$$X = 97.44.$$

7. Solution: B

The book value at time 6 is the present value of future payments:

$$BV_6 = 10,000v^4 + 800a_{\overline{4}|0.06} = 7920.94 + 2772.08 = 10,693.$$

The interest portion is 10,693(0.06) = 641.58.

8. Solution: A

The value of the perpetuity after the fifth payment is 100/0.08 = 1250. The equation to solve is:

$$1250 = X(v + 1.08v^2 + \cdots + 1.08^{24}v^{25})$$

$$= X(v + v + \cdots + v) = X(25)/1.08$$

$$X = 50(1.08) = 54.$$

9. Solution: C

Equation of value at end of 30 years:

$$10(1-d/4)^{-40}(1.03)^{40} + 20(1.03)^{30} = 100$$

$$10(1-d/4)^{-40} = [100 - 20(1.03)^{30}]/1.03^{40} = 15.7738$$

$$1 - d/4 = 1.57738^{-1/40} = 0.98867$$

$$d = 4(1 - 0.98867) = 0.0453 = 4.53\%.$$

10. Solution: E

The accumulation function is $a(t) = \exp\left[\int_0^t (s^2/100)ds\right] = \exp(t^3/300).$

The accumulated value of 100 at time 3 is $100\exp(3^3/300) = 109.41743.$

The amount of interest earned from time 3 to time 6 equals the accumulated value at time 6 minus the accumulated value at time 3. Thus

$$(109.41743 + X)[a(6)/a(3) - 1] = X$$

$$(109.41743 + X)(2.0544332/1.0941743 - 1) = X$$

$$(109.41743 + X)0.877613 = X$$

$$96.026159 = 0.122387X$$

$$X = 784.61.$$

11. Solution: A

$$167.50 = 10a_{\overline{5}|9.2\%} + 10(1.092)^{-5}\sum_{t=1}^{\infty}\left[\frac{(1+k)}{1.092}\right]^t$$

$$167.50 = 38.6955 + 6.44001\frac{(1+k)/1.092}{1-(1+k)/1.092}$$

$$(167.50-38.6955)[1-(1+k)/1.092] = 6.44001(1+k)/1.092$$

$$128.8045 = 135.24451(1+k)/1.092$$

$$1+k = 1.0400$$

$$k = 0.0400 \Rightarrow K = 4.0\%.$$

12. Solution: B

Option 1: $2000 = Pa_{\overline{10}|0.0807}$

$P = 299 \Rightarrow$ Total payments $= 2990$

Option 2: Interest needs to be $2990 - 2000 = 990$

$$990 = i[2000+1800+1600+\cdots+200]$$

$$= 11,000i$$

$$i = 0.09 = 9.00\%$$

13. Solution: B

Monthly payment at time t is $1000(0.98)^{t-1}$.

Because the loan amount is unknown, the outstanding balance must be calculated prospectively. The value at time 40 months is the present value of payments from time 41 to time 60:

$$OB_{40} = 1000[0.98^{40}v^1 + \cdots + 0.98^{59}v^{20}]$$

$$= 1000\frac{0.98^{40}v^1 - 0.98^{60}v^{21}}{1-0.98v}, \; v = 1/(1.0075)$$

$$= 1000\frac{0.44238 - 0.25434}{1-0.97270} = 6888.$$

14. Solution: C

The equation of value is

$$98S_{\overline{3n|}} + 98S_{\overline{2n|}} = 8000$$

$$\frac{(1+i)^{3n}-1}{i} + \frac{(1+i)^{2n}-1}{i} = 81.63$$

$$\left(1+i\right)^{n} = 2$$

$$\frac{8-1}{i} + \frac{4-1}{i} = 81.63$$

$$\frac{10}{i} = 81.63$$

$$i = 12.25\%$$

15. Solution: B

Convert 9% convertible quarterly to an effective rate of j per month:

$(1+j)^{3} = \left(1+\frac{0.09}{4}\right)$ or j = 0.00744.

Then

$$2(Ia)_{\overline{60|}0.00744} = 2\frac{\ddot{a}_{\overline{60|}0.00744} - 60v^{60}}{0.00744} = 2\frac{48.6136-38.4592}{0.00744} = 2729.7.$$

16. Solution: A

Equating present values:

$$100 + 200v^{n} + 300v^{2n} = 600v^{10}$$

$$100 + 200(0.76) + 300(0.76)^{2} = 600v^{10}$$

$$425.28 = 600v^{10}$$

$$0.7088 = v^{10}$$

$$0.96617 = v$$

$$1.03501 = 1+i$$

$$i = 0.035 = 3.5\%.$$

17. Solution: A

The accumulation function is:

$$a(t) = e^{\int_0^t \frac{1}{8+r}dr} = e^{\ln(8+r)|_0^t} = \frac{8+t}{8}.$$

Using the equation of value at end of 10 years:

$$20,000 = \int_0^{10} (8k + tk) \frac{a(10)}{a(t)} dt = k\int_0^{10} (8+t) \frac{18/8}{(8+t)/8} dt = k\int_0^{10} 18 dt$$

$$= 180k \Rightarrow k = \frac{20,000}{180} = 111.$$

18. Solution: D

Let C be the redemption value and $v = 1/(1+i)$. Then

$$X = 1000 r a_{\overline{2n}|i} + Cv^{2n}$$

$$= 1000r \frac{1-v^{2n}}{i} + 381.50$$

$$= 1000(1.03125)(1-0.5889^2) + 381.50$$

$$= 1055.11.$$

19. Solution: D

Equate net present values:

$$-4000 + 2000v + 4000v^2 = 2000 + 4000v - Xv^2$$

$$\frac{4000 + X}{1.21} = 6000 + \frac{2000}{1.1}$$

$$X = 5460.$$

20. Solution: D

The present value of the perpetuity = X/i. Let B be the present value of Brian's payments.

$$B = Xa_{\overline{n}|} = 0.4\frac{X}{i}$$

$$a_{\overline{n}|} = \frac{0.4}{i} \Rightarrow 0.4 = 1 - v^n \Rightarrow v^n = 0.6$$

$$K = v^{2n}\frac{X}{i}$$

$$K = 0.36\frac{X}{i},$$

Thus the charity's share is 36% of the perpetuity's present value.

21. Solution: D

The given information yields the following amounts of interest paid:

$$\text{Seth} = 5000\left(\left(1+\frac{0.12}{2}\right)^{10} - 1\right) = 8954.24 - 5000 = 3954.24$$

$$\text{Janice} = 5000(0.06)(10) = 3000.00$$

$$\text{Lori} = P(10) - 5000 = 1793.40 \text{ where } P = \frac{5000}{a_{\overline{10}|6\%}} = 679.35$$

The sum is 8747.64.

22. Solution: E

For Bruce, $X = 100[(1+i)^{11} - (1+i)^{10}] = 100(1+\mathrm{i})^{10} i$. Similarly, for Robbie, $X = 50(1+i)^{16} i$ .Dividing the second equation by the first gives $1 = 0.5(1+i)^6$ which implies $i = 2^{1/6} - 1 = 0.122462$. Thus $X = 100(1.122462)^{10}(0.122462) = 38.879.$

23. Solution: D

Year t interest is $ia_{\overline{n-t+1}|i} = 1 - v^{n-t+1}$.

Year t+1 principal repaid is $1-(1-v^{n-t}) = v^{n-t}$.

$X = 1 - v^{n-t+1} + v^{n-t} = 1 + v^{n-t}(1-v) = 1 + v^{n-t}d.$

24. Solution: B

For the first perpetuity,

$32 = 10(v^3 + v^6 + \cdots) = 10v^3 / (1-v^3)$

$32 - 32v^3 = 10v^3$

$v^3 = 32/42.$

For the second perpetuity,

$X = v^{1/3} + v^{2/3} + \cdots = v^{1/3} / (1 - v^{1/3}) = (32/42)^{1/9} / [1 - (32/42)^{1/9}] = 32.599.$

25 Solution: D

Under either scenario, the company will have 822,703(0.05) = 41,135 to invest at the end of each of the four years. Under Scenario A these payments will be invested at 4.5% and accumulate to $41{,}135 s_{\overline{4}|0.045} = 41{,}135(4.2782) = 175{,}984.$ Adding the maturity value produces 998,687 for a loss of 1,313. Note that only answer D has this value.

The Scenario B calculation is

$41{,}135 s_{\overline{4}|0.055} = 41{,}135(4.3423) = 178{,}621 + 822{,}703 - 1{,}000{,}000 = 1{,}324.$

26. Solution: D.

The present value is

$$5000[1.07v+1.07^2v^2+\cdots+1.07^{20}v^{20}]$$

$$=5000\frac{1.07v-1.07^{21}v^{21}}{1-1.07v}=5000\frac{1.01905-1.48622}{1-1.01905}=122,617.$$

27. Solution: C.

The first cash flow of 60,000 at time 3 earns 2400 in interest for a time 4 receipt of 62,400. Combined with the final payment, the investment returns 122,400 at time 4. The present value is $122,400(1.05)^{-4}=100,699.$ The net present value is 699.

28. Solution: B.

Using spot rates, the value of the bond is:

$60/1.07+60/1.08^2+1060/1.09^3=926.03.$

29. Solution: E.

Using spot rates, the value of the bond is:

$60/1.07+60/1.08^2+1060/1.09^3=926.03.$ The annual effective rate is the solution to $926.03=60a_{\overline{3}|i}+1000(1+i)^{-3}$. Using a calculator, the solution is 8.9%.

30. Solution: C.

Duration is the negative derivative of the price multiplied by one plus the interest rate and divided by the price. Hence, the duration is –(–700)(1.08)/100 = 7.56.

31. Solution: C

The size of the dividend does not matter, so assume it is 1. Then the duration is

$$\frac{\sum_{t=1}^{\infty}tv^t}{\sum_{t=1}^{\infty}v^t}=\frac{(Ia)_{\overline{\infty}|}}{a_{\overline{\infty}|}}=\frac{\ddot{a}_{\overline{\infty}|}/i}{1/i}=\frac{1/(di)}{1/i}=\frac{1}{d}=\frac{1.1}{0.1}=11.$$

32. Solution: B

$$\text{Duration} = \frac{\sum_{t=1}^{\infty} tv^t R_t}{\sum_{t=1}^{\infty} v^t R_t} = \frac{\sum_{t=1}^{\infty} tv^t 1.02^t}{\sum_{t=1}^{\infty} v^t 1.02^t} = \frac{(Ia)_{\overline{\infty}|j}}{a_{\overline{\infty}|j}} = \frac{\ddot{a}_{\overline{\infty}|j} / j}{1/j} = \frac{1}{d}.$$

The interest rate *j* is such that $(1+j)^{-1} = 1.02v = 1.02/1.05 \Rightarrow j = 0.03/1.02.$ Then the duration is $1/d = (1+j)/j = (1.05/1.02)/(0.03/1.02) = 1.05/0.03 = 35.$

33. Solution: A

The outstanding balance is the present value of future payments. With only one future payment, that payment must be 559.12(1.08) = 603.85. The amount borrowed is $603.85a_{\overline{4}|0.08} = 2000.$ The first payment has 2000(0.08) = 160 in interest, thus the principal repaid is 603.85 – 160 = 443.85.

Alternatively, observe that the principal repaid in the final payment is the outstanding loan balance at the previous payment, or 559.12. Principal repayments form a geometrically decreasing sequence, so the principal repaid in the first payment is $559.12/1.08^3 = 443.85.$

34. Solution: B

Because the yield rate equals the coupon rate, Bill paid 1000 for the bond. In return he receives 30 every six months, which accumulates to $30s_{\overline{20}|j}$ where *j* is the semi-annual interest rate. The equation of value is $1000(1.07)^{10} = 30s_{\overline{20}|j} + 1000 \Rightarrow s_{\overline{20}|j} = 32.238.$ Using a calculator to solve for the interest rate produces *j* = 0.0476 and so $i = 1.0476^2 - 1 = 0.0975 = 9.75\%.$

35. Solution: A

To receive 3000 per month at age 65 the fund must accumulate to 3,000(1,000/9.65) = 310,880.83. The equation of value is $310,880.83 = X\ddot{s}_{\overline{300}|0.08/12} = 957.36657X \Rightarrow 324.72.$

36. Solution: D

(A) The left-hand side evaluates the deposits at age 0, while the right-hand side evaluates the withdrawals at age 17.

(B) The left-hand side has 16 deposits, not 17.

(C) The left-hand side has 18 deposits, not 17.

(D) The left-hand side evaluates the deposits at age 18 and the right-hand side evaluates the withdrawals at age 18.

(E) The left-hand side has 18 deposits, not 17 and 5 withdrawals, not 4.

37. Solution: D

Because only Bond II provides a cash flow at time 1, it must be considered first. The bond provides 1025 at time 1 and thus 1000/1025 = 0.97561 units of this bond provides the required cash. This bond then also provides 0.97561(25) = 24.39025 at time 0.5. Thus Bond I must provide 1000 – 24.39025 = 975.60975 at time 0.5. The bond provides 1040 and thus 975.60975/1040 = 0.93809 units must be purchased.

38. Solution: C

Because only Mortgage II provides a cash flow at time two, it must be considered first. The mortgage provides $Y / a_{\overline{2}|0.07} = 0.553092Y$ at times one and two. Therefore, 0.553092Y = 1000 for Y = 1808.02. Mortgage I must provide 2000 – 1000 = 1000 at time one and thus X = 1000/1.06 = 943.40. The sum is 2751.42.

39. Solution: A

Bond I provides the cash flow at time one. Because 1000 is needed, one unit of the bond should be purchased, at a cost of 1000/1.06 = 943.40.

Bond II must provide 2000 at time three. Therefore, the amount to be reinvested at time two is 2000/1.065 = 1877.93. The purchase price of the two-year bond is $1877.93/1.07^2 = 1640.26$. The total price is 2583.66.

40. Solution: C

Given the coupon rate is greater than the yield rate, the bond sells at a premium. Thus, the minimum yield rate for this callable bond is calculated based on a call at the earliest possible date because that is most disadvantageous to the bond holder (earliest time at which a loss occurs). Thus, X, the par value, which equals the redemption value because the bond is a par value bond, must satisfy

Price = $1722.25 = 0.04Xa_{\overline{30}|0.03} + Xv_{0.03}^{30} = 1.196X \Rightarrow X = 1440.$

41. Solution: B

Because 40/1200 is greater than 0.03, for early redemption the earliest redemption should be evaluated. If redeemed after 15 years, the price is $40a_{\overline{30}|0.03} + 1200/1.03^{30} = 1278.40$. If the bond is redeemed at maturity, the price is $40a_{\overline{40}|0.03} + 1100/1.03^{40} = 1261.80$. The smallest value should be selected, which is 1261.80. (When working with callable bonds, the maximum a buyer will pay is the smallest price over the various call dates. Paying more may not earn the desired yield.)

42. Solution: E

Given the coupon rate is less than the yield rate, the bond sells at a discount. Thus, the minimum yield rate for this callable bond is calculated based on a call at the latest possible date because that is most disadvantageous to the bond holder (latest time at which a gain occurs). Thus, X, the par value, which equals the redemption value because the bond is a par value bond, must satisfy

Price = $1021.50 = 0.02Xa_{\overline{20|}0.03} + Xv_{0.03}^{20} = 0.851225X \Rightarrow X = 1200.$

43. Solution: B

Given the price is less than the amount paid for an early call, the minimum yield rate for this callable bond is calculated based on a call at the latest possible date. Thus, for an early call, the effective yield rate per coupon period, *j*, must satisfy Price = $1021.50 = 22a_{\overline{19|}j} + 1200v_j^{19}$. Using the calculator, *j* = 2.86%. We also must check the yield if the bond is redeemed at maturity. The equation is $1021.50 = 22a_{\overline{20|}j} + 1100v_j^{20}$. The solution is *j* = 2.46% Thus, the yield, expressed as a nominal annual rate of interest convertible semiannually, is twice the smaller of the two values, or 4.92%.

44. Solution: C

First, the present value of the liability is $PV = 35,000a_{\overline{15|}6.2\%} = 335,530.30.$

The duration of the liability is:

$$\bar{d} = \frac{\sum tv^t R_t}{\sum v^t R_t} = \frac{35,000v + 2(35,000)v^2 + \cdots + 15(35,000)v^{15}}{335,530.30} = \frac{2,312,521.95}{335,530.30} = 6.89214.$$

Let X denote the amount invested in the 5 year bond.

Then, $\frac{X}{335,530.30}(5) + \left(1 - \frac{X}{335,530.30}\right)(10) = 6.89214 \Rightarrow X = 208,556.$

45 . Solution: A

The present value of the first eight payments is:

$$PV = 2000v + 2000(1.03)v^2 + ... + 2000(1.03)^7 v^8 = \frac{2000v - 2000(1.03)^8 v^9}{1 - 1.03v} = 13,136.41.$$

The present value of the last eight payments is:

$$PV = 2000(1.03)^7 0.97v^9 + 2000(1.03)^7 (0.97)^2 v^{10} + \cdots + 2000(1.03)^7 (0.97^8) v^{16}$$

$$= \frac{2000(1.03)^7 0.97v^9 - 2000(1.03)^7 (0.97)^9 v^{17}}{1 - 0.97v} = 7,552.22.$$

Therefore, the total loan amount is *L* = 20,688.63.

46. Solution: E

$$2000 = 500\exp\left(\int_0^t \frac{\frac{r^2}{100}}{3+\frac{r^3}{150}}\,dr\right)$$

$$4 = \exp\left(0.5\int_0^t \frac{\frac{r^2}{50}}{3+\frac{r^3}{150}}\,dr\right) = \exp\left[0.5\ln\left(3+\frac{r^3}{150}\right)\Big|_0^t\right]$$

$$4 = \exp\left[0.5\ln\left(1+\frac{t^3}{450}\right)\right] = \left(1+\frac{t^3}{450}\right)^{\frac{1}{2}}$$

$$16 = \left(1+\frac{t^3}{450}\right)$$

$$t = 18.8988$$

47. Solution: E

Let *F*, *C*, r, and *i* have their usual interpretations. The discount is $(Ci - Fr)a_{\overline{n|}}$ and the discount in the coupon at time t is $(Ci - Fr)v^{n-t+1}$. Then,

$$194.82 = (Ci - Fr)v^{26}$$

$$306.69 = (Ci - Fr)v^{21}$$

$$0.63523 = v^5 \Rightarrow v = 0.91324 \Rightarrow i = 0.095$$

$$(Ci - Fr) = 194.82(1.095)^{26} = 2062.53$$

$$\text{Discount} = 2062.53a_{\overline{40|}0.095} = 21{,}135$$

48. Solution: A

$$699.68 = Pv^{8-5+1}$$

$$P = 842.39 \text{ (annual payment)}$$

$$P_1 = \frac{699.68}{1.0475^4} = 581.14$$

$$I_1 = 842.39 - 581.14 = 261.25$$

$$L = \frac{261.25}{0.0475} = 5500 \text{ (loan amount)}$$

$$\text{Total interest} = 842.39(8) - 5500 = 1239.12$$

49. Solution: D

$$OB_{18} = 22,000(1.007)^{18} - 450.30 s_{\overline{18|}0.007} = 16,337.10$$

$$16,337.10 = Pa_{\overline{24|}0.004}$$

$$P = 715.27$$

50. Solution: C

If the bond has no premium or discount, it was bought at par so the yield rate equals the coupon rate, 0.038.

$$d = \frac{\frac{1}{2}\left(1(190)v + 2(190)v^2 + \cdots + 14(190)v^{14} + 14(5000)v^{14}\right)}{190v + 190v^2 + \cdots + 190v^{14} + 5000v^{14}}$$

$$d = \frac{95\left(Ia\right)_{\overline{14|}} + 7(5000)v^{14}}{190a_{\overline{14|}} + 5000v^{14}}$$

$$d = 5.5554$$

Or, taking advantage of a shortcut:

$d = \ddot{a}_{\overline{14|}0.038} = 11.1107.$ This is in half years, so dividing by two, $d = \frac{11.1107}{2} = 5.5554$.

51. Solution: A

$$\bar{v} = \frac{7.959}{1.072} = 7.425$$

$$P(0.08) = P(0.072)\left[1 - (\Delta i)\bar{v}\right]$$

$$P(0.08) = 1000\left[1 - (0.008)(7.425)\right] = 940.60$$

52. Solution: E

$$(1+s_3)^3 = (1+s_2)^2(1+{}_1f_2)$$

$$0.85892 = \frac{1}{(1+s_3)^3}, s_3 = 0.052$$

$$0.90703 = \frac{1}{(1+s_2)^2}, s_2 = 0.050$$

$$1.052^3 = 1.050^2(1+{}_1f_2)$$

$${}_1f_2 = 0.056$$

53. Solution: C

Let d_0 be the Macaulay duration at time 0.

$d_0 = \ddot{a}_{\overline{8}|0.05} = 6.7864$

$d_1 = d_0 - 1 = 5.7864$

$d_2 = \ddot{a}_{\overline{7}|0.05} = 6.0757$

$\frac{d_1}{d_2} = \frac{5.7864}{6.0757} = 0.9524$

This solution employs the fact that when a coupon bond sells at par the duration equals the present value of an annuity-due. For the duration just before the first coupon the cash flows are the same as for the original bond, but all occur one year sooner. Hence the duration is one year less.

Alternatively, note that the numerators for d_1 and d_2 are identical. That is because they differ only with respect to the coupon at time 1 (which is time 0 for this calculation) and so the payment does not add anything. The denominator for d_2 is the present value of the same bond, but with 7 years, which is 5000. The denominator for d_1 has the extra coupon of 250 and so is 5250. The desired ratio is then 5000/5250 = 0.9524.

54. Solution: A

Let *N* be the number of shares bought of the bond as indicated by the subscript.

$N_C(105) = 100, N_C = 0.9524$

$N_B(100) = 102 - 0.9524(5), N_B = 0.9724$

$N_A(107) = 99 - 0.9524(5), N_A = 0.8807$

55. Solution: B

All are true except B. Immunization requires frequent rebalancing.

56. Solution: D

Set up the following two equations in the two unknowns:

$A(1.05)^2 + B(1.05)^{-2} = 6000$

$2A(1.05)^1 - 2B(1.05)^{-3} = 0.$

Solving simultaneously gives:

$A = 2721.09$

$B = 3307.50$

$|A - B| = 586.41.$

57. Solution: A

Set up the following two equations in the two unknowns.

(1) $5000(1.03)^3 + B(1.03)^{-b} = 12,000 \Rightarrow$

$5463.635 + B(1.03)^{-b} = 12,000 \Rightarrow B(1.03)^{-b} = 6536.365$

(2) $3(5000)(1.03)^3 - bB(1.03)^{-b} = 0 \Rightarrow 16,390.905 - b6536.365 = 0$

$b = 2.5076$

$B = 7039.27$

$\frac{B}{b} = 2807.12$

58. Solution: D

$P_A = A(1+i)^{-2} + B(1+i)^{-9}$

$P_L = 95,000(1+i)^{-5}$

$P_A' = -2A(1+i)^{-3} - 9B(1+i)^{-10}$

$P_L' = -5(95,000)(1+i)^{-6}$

Set the present values and derivatives equal and solve simultaneously.

$0.92456A + 0.70259B = 78,083$

$-1.7780A - 6.0801B = -375,400$

$$B = \frac{78,083(1.7780/0.92456) - 375,400}{0.70259(1.7780/0.92456) - 6.0801} = 47,630$$

$A = [78,083 - 0.70259(47,630)]/0.92456 = 48,259$

$\frac{A}{B} = 1.0132$

59. Solution: D

Throughout the solution, let $j = i/2$.

For bond A, the coupon rate is $(i + 0.04)/2 = j + 0.02$.

For bond B, the coupon rate is $(i - 0.04)/2 = j - 0.02$.

The price of bond A is $P_A = 10,000(j + 0.02)a_{\overline{20}|j} + 10,000(1+j)^{-20}$.

The price of bond B is $P_B = 10,000(j - 0.02)a_{\overline{20}|j} + 10,000(1+j)^{-20}$.

Thus,

$P_A - P_B = 5,341.12 = [200 - (-200)]a_{\overline{20}|j} = 400a_{\overline{20}|j}$

$a_{\overline{20}|j} = 5,341.12/400 = 13.3528.$

Using the financial calculator, $j = 0.042$ and $i = 2(0.042) = 0.084$.

60. Solution: D

The initial level monthly payment is

$$R = \frac{400,000}{a_{\overline{15\times12}|0.09/12}} = \frac{400,000}{a_{\overline{180}|0.0075}} = 4,057.07.$$

The outstanding loan balance after the 36th payment is

$$B_{36} = Ra_{\overline{180-36}|0.0075} = 4,057.07a_{\overline{144}|0.0075} = 4,057.07(87.8711) = 356,499.17.$$

The revised payment is 4,057.07 – 409.88 = 3,647.19.

Thus,

$$356,499.17 = 3,647.19a_{\overline{144}|j/12}$$

$$a_{\overline{144}|j/12} = 356,499.17 / 3,647.19 = 97.7463.$$

Using the financial calculator, *j*/12 = 0.575%, for *j* = 6.9%.

61. Solution: D

The price of the first bond is

$$1000(0.05/2)a_{\overline{30\times2}|0.05/2} + 1200(1+0.05/2)^{-30\times2} = 25a_{\overline{60}|0.025} + 1200(1.025)^{-60}$$

$$= 772.72 + 272.74 = 1,045.46.$$

The price of the second bond is also 1,045.46. The equation to solve is

$$1,045.46 = 25a_{\overline{60}|j/2} + 800(1+ j/2)^{-60}.$$

The financial calculator can be used to solve for *j*/2 = 2.2% for *j* = 4.4%.

62. Solution: E

Let *n* = years. The equation to solve is

$$1000(1.03)^{2n} = 2(1000)(1.0025)^{12n}$$

$$2n\ln 1.03 + \ln 1000 = 12n\ln 1.0025 + \ln 2000$$

$$0.029155n = 0.69315$$

$$n = 23.775.$$

This is 285.3 months. The next interest payment to Lucas is at a multiple of 6, which is 288 months.

63. Solution: A

Equating the accumulated values after 4 years provides an equation in *K*.

$$10\left(1+\frac{K}{25}\right)^4 = 10\exp\left(\int_0^4 \frac{1}{K+0.25t}\,dt\right)$$

$$4\ln(1+0.04K) = \int_0^4 \frac{1}{K+0.25t}\,dt = 4\ln(\mathrm{K}+0.25\,\mathrm{t})\Big|_0^4 = 4\ln(K+1) - 4\ln(K) = 4\ln\frac{K+1}{K}$$

$$1+0.04K = \frac{K+1}{K}$$

$$0.04K^2 = 1$$

$$K = 5.$$

Therefore, $X = 10(1+5/25)^4 = 20.74.$

64. Solution: D

The outstanding balance at time 25 is $100(Da)_{\overline{25}|} = 100\frac{25-a_{\overline{25}|}}{i}$. The principle repaid in the 26th payment is $X = 2500 - i(100)\frac{25-a_{\overline{25}|}}{i} = 2500 - 2500 + 100a_{\overline{25}|} = 100a_{\overline{25}|}$. The amount borrowed is the present value of all 50 payments, $2500a_{\overline{25}|} + v^{25}100(Da)_{\overline{25}|}$. Interest paid in the first payment is then

$$i\left[2500a_{\overline{25}|} + v^{25}100(Da)_{\overline{25}|}\right]$$
$$= 2500(1-v^{25}) + 100v^{25}(25-a_{\overline{25}|})$$
$$= 2500 - 2500v^{25} + 2500v^{25} - v^{25}100a_{\overline{25}|}$$
$$= 2500 - Xv^{25}.$$

65. Solution: C

The accumulated value is $1000\ddot{s}_{\overline{20}|0.0816} = 50{,}382.16$. This must provide a semi-annual annuity-due of 3000. Let *n* be the number of payments. Then solve $3000\ddot{a}_{\overline{n}|0.04} = 50{,}382.16$ for *n* = 26.47. Therefore, there will be 26 full payments plus one final, smaller, payment. The equation is $50{,}382.16 = 3000\ddot{a}_{\overline{26}|0.04} + X(1.04)^{-26}$ with solution *X* = 1430. Note that the while the final payment is the 27th payment, because this is an annuity-due, it takes place 26 periods after the annuity begins.

66. Solution: D

For the first perpetuity,

$$\frac{1}{(1+i)^2-1}+1=7.21$$

$$\frac{1}{6.21}=(1+i)^2-1$$

$i=0.0775.$

For the second perpetuity,

$$R\left[\frac{1}{(1.0775+0.01)^3-1}+1\right](1.0875)^{-1}=7.21$$

$$1.286139R=7.21(1.0875)(0.286139)$$

$R=1.74.$

67. Solution: E

$$10,000=100(Ia)_{\overline{5}|}+Xv^5a_{\overline{15}|}=100\left(\frac{\ddot{a}_{\overline{5}|}-5v^5}{0.05}\right)+Xv^5a_{\overline{15}|}$$

$$10,000=1256.64+8.13273X$$

$$1075=X$$

68. Solution: C

$$5000=Xs_{\overline{10}|0.06}(1.05)^5$$

$$X=\frac{5000}{13.1808(1.2763)}=297.22$$

69. Solution: E

The monthly payment on the original loan is $\frac{65,000}{a_{\overline{180}|8/12\%}}=621.17$. After 12 payments the outstanding balance is $621.17a_{\overline{168}|8/12\%}=62,661.40$. The revised payment is $\frac{62,661.40}{a_{\overline{168}|6/12\%}}=552.19.$

70. Solution: E

At the time of the final deposit the fund has $750s_{\overline{18}|0.07} = 25,499.27.$ This is an immediate annuity because the evaluation is done at the time the last payments is made (which is the end of the final year). A tuition payment of $6000(1.05)^{17} = 13,752.11$ is made, leaving 11,747.16. It earns 7%, so a year later the fund has 11,747.16(1.07) = 12,569.46. Tuition has grown to 13,752.11(1.05) = 14,439.72. The amount needed is 14,439.72 – 12,569.46 = 1,870.26

71. Solution: B

The coupons are 1000(0.09)/2 = 45. The present value of the coupons and redemption value at 5% per semiannual period is $P = 45a_{\overline{40}|0.05} + 1200(1.05)^{-40} = 942.61.$

72. Solution: A

For a bond bought at discount, the minimum price will occur at the latest possible redemption date. $P = 50a_{\overline{20}|0.06} + 1000(1.06)^{-20} = 885.30.$ (When working with callable bonds, the maximum a buyer will pay is the smallest price over the various call dates. Paying more may not earn the desired yield.)

73. Solution: C

$$\frac{1.095^5}{1.090^4} - 1 = 11.5\%$$

74. Solution: D

The accumulated value of the first year of payments is $2000s_{\overline{12}|0.005} = 24,671.12.$ This amount increases at 2% per year. The effective annual interest rate is $1.005^{12} - 1 = 0.061678.$ The present value is then

$$P = 24,671.12\sum_{k=1}^{25}1.02^{k-1}(1.061678)^{-k} = 24,671.12\frac{1}{1.02}\sum_{k=1}^{25}\left(\frac{1.02}{1.061678}\right)^k$$

$$= 24,187.37\frac{0.960743 - 0.960743^{26}}{1 - 0.960743} = 374,444.$$

This is 56 less than the lump sum amount.

75. Solution: A

The monthly interest rate is 0.072/12 = 0.006. 6500 five years from today has value $6500(1.006)^{-60} = 4539.77$. The equation of value is

$$4539.77 = 1700(1.006)^{-n} + 3400(1.006)^{-2n}.$$

Let $x = 1.006^{-n}$. Then, solve the quadratic equation

$$3400x^2 + 1700x - 4539.77 = 0$$

$$x = \frac{-1700 + \sqrt{1700^2 - 4(3400)(-4539.77)}}{2(3400)} = 0.93225.$$

Then,

$$1.006^{-n} = 0.9325 \Rightarrow -n\ln(1.006) = \ln(0.93225) \Rightarrow n = 11.73.$$

To ensure there is 6500 in five years, the deposits must be made earlier and thus the maximum integral value is 11.

76. Solution: C

$$\frac{(1-d/2)^{-4}}{(1-d/4)^{-4}} = \left(\frac{39}{38}\right)^4 \Rightarrow \frac{1-d/2}{1-d/4} = \frac{38}{39} \Rightarrow 39 - 39(d/2) = 38 - 38(d/4)$$

$$d\left(39/2 - 38/4\right) = 39 - 38$$

$$d = 1/(19.5 - 9.5) = 0.1$$

$$1 + i = (1 - d/2)^{-2} = .95^{-2} = 1.108 \Rightarrow i = 10.8\%.$$

77. Solution: C

The monthly interest rate is 0.042/12 = 0.0035. The quarterly interest rate is $1.0035^3 - 1 = 0.0105$. The investor makes 41 quarterly deposits and the ending date is 124 months from the start. Using January 1 of year *y* as the comparison date produces the following equation:

$$X + \sum_{k=1}^{41} \frac{100}{1.0105^k} = \frac{1.9X}{1.0035^{124}}$$

Substituting $1.0105 = 1.0035^3$ gives answer (C).

78. Solution: D

Convert the two annual rates, 4% and 5%, to two-year rates as $1.04^2 - 1 = 0.0816$ and $1,05^2 - 1 = 0.1025$.

The accumulated value is

$$100\ddot{s}_{\overline{3}|0.0816}(1.05)^4 + 100\ddot{s}_{\overline{2}|0.1025} = 100(3.51678)(1.21551) + 100(2.31801) = 659.269.$$

With only five payments, an alternative approach is to accumulate each one to time ten and add them up.

The two-year yield rate is the solution to $100\ddot{s}_{\overline{5}|i} = 659.269$. Using the calculator, the two-year rate is 0.093637. The annual rate is $1.093637^{0.5} - 1 = 0.04577$ which is 4.58%.

79. Solution: C

$$(1.08)^{1/12} - 1 = 0.006434$$

$$\frac{1}{1.08^{15}} 25,000\ddot{a}_{\overline{4}|8\%} = X\ddot{a}_{\overline{216}|0.6434\%}$$

$$X = \frac{25,000(3.57710)}{3.17217(117.2790)} = 240.38$$

80. Solution: B

$$\text{PV}_{\text{perp.}} = \left[\frac{1}{0.1} + \frac{\frac{1}{0.08} - \frac{1}{0.1}}{1.1^{10}}\right](15,000) + 15,000$$

$$= 164,457.87 + 15,000 = 179,457.87$$

$$X\left(\ddot{a}_{\overline{10}|0.10} + \frac{\ddot{a}_{\overline{15}|0.08}}{1.10^{10}}\right) = 179,458$$

$$X\left(6.759 + \frac{9.244}{1.10^{10}}\right) = 179,458$$

$$X = 17,384$$

81. Solution: A

$$1050.50 = 22.50a_{\overline{14}|0.03} + X(Ia)_{\overline{14}|0.03} + 300(1.03)^{-14}$$

$$= 22.50a_{\overline{14}|0.03} + X\left(\frac{\ddot{a}_{\overline{14}|0.03} - 14(1.03)^{-14}}{0.03}\right) + 300(1.03)^{-14}$$

$$= 22.50(11.2961) + X(79.3102) + 198.3353$$

$$X = 7.54$$

82. Solution: D

The amount of the loan is the present value of the deferred increasing annuity:

$$(1.05)^{-10}\left[500\ddot{a}_{\overline{30}|0.05}+500(I\ddot{a})_{\overline{30}|0.05}\right]=(1.05^{-10})(500)\left[\ddot{a}_{\overline{30}|0.05}+\frac{\ddot{a}_{\overline{30}|0.05}-30(1.05)^{-30}}{0.05/1.05}\right]=64,257.$$

83. Solution: C

$$50,000\left[\frac{(1+i)^{30}-(1.03)^{30}}{(1+i)^{30}(i-0.03)}\right](1+i)=5,000\left[\frac{(1+i)^{30}-(1.03)^{30}}{i-0.03}\right]$$

$$50,000/(1+i)^{29}=5,000$$

$$(1+i)^{29}=10$$

$$i=10^{1/29}-1=0.082637$$

The accumulated amount is

$$50,000\left[\frac{(1.082637)^{30}-(1.03)^{30}}{(1.082637)^{30}(0.082637-0.03)}\right](1.082637)=797,836.82$$

84. Solution: D

The first payment is 2,000, and the second payment of 2,010 is 1.005 times the first payment. Since we are given that the series of quarterly payments is geometric, the payments multiply by 1.005 every quarter.

Based on the quarterly interest rate, the equation of value is

$$100,000=2,000+2,000(1.005)v+2,000(1.005)^2v^2+2,000(1.005)^3v^3+\cdots=\frac{2,000}{1-1.005v}$$

$$1-1.005v=2,000/100,000\Rightarrow v=0.98/1.005.$$

The annual effective rate is $v^{-4}-1=\left(0.98/1.005\right)^{-4}-1=0.10601=10.6\%$.

85. Solution: A

Present value for the first 10 years is $\dfrac{1-\left(1.06\right)^{-10}}{\ln\left(1.06\right)}=7.58$

Present value of the payments after 10 years is

$$\left(1.06\right)^{-10}\int_0^\infty\left(1.03\right)^s\left(1.06\right)^{-s}ds=\frac{0.5584}{\ln\left(1.06\right)-\ln\left(1.03\right)}=19.45$$

Total present value = 27.03

86. Solution: C

$$\left[10,000(1.06)^5 + X(1.06)^2\right]e^{\int_5^{10}\frac{1}{t+1}dt} = 75,000$$

$$(13,382.26 + 1.1236X)\frac{11}{6} = 75,000$$

$$1.1236X = 27,526.83$$

$$X = 24,498.78$$

87. Solution: D

The effective annual interest rate is $i = (1-d)^{-1} - 1 = (1-0.055)^{-1} - 1 = 5.82\%$

The balance on the loan at time 2 is $15,000,000(1.0582)^2 = 16,796,809$.

The number of payments is given by $1,200,000a_{\overline{n}|} = 16,796,809$ which gives n = 29.795 => 29 payments of 1,200,000. The final equation of value is

$$1,200,000a_{\overline{29}|} + X(1.0582)^{-30} = 16,796,809$$

$$X = (16,796,809 - 16,621,012)(5.45799) = 959,490.$$

88. Solution: C

$$1 - v^2 = 0.525(1 - v^4) \Rightarrow 1 = 0.525(1 + v^2) \Rightarrow v^2 = 0.90476 \Rightarrow v = 0.95119$$

$$1 - v^2 = 0.1427(1 - v^n) \Rightarrow 1 - \mathrm{v}^n = (1 - 0.90476)/0.1427 = 0.667414 \Rightarrow v^n = 0.332596$$

$$n = \ln(0.332596)/\ln(0.95119) = 22$$

89. Solution: C

The monthly payment is $200,000 / a_{\overline{360}|0.005} = 1199.10$. Using the equivalent annual effective rate of 6.17%, the present value (at time 0) of the five extra payments is 41,929.54 which reduces the original loan amount to 200,000 – 41,929.54 = 158,070.46. The number of months required is the solution to $158,070.46 = 1199.10a_{\overline{n}|0.005}$. Using calculator, n = 215.78 months are needed to pay off this amount. So there are 215 full payments plus one fractional payment at the end of the 216th month, which is December 31, 2020.

90. Solution: D

The annual effective interest rate is 0.08/(1 – 0.08) = 0.08696. The level payments are $500,000 / a_{\overline{5}|0.08696} = 500,000 / 3.9205 = 127,535.$ This rounds up to 128,000. The equation of value for X is

$$128,000a_{\overline{4}|0.08696} + X(1.08696)^{-5} = 500,000$$

$$X = (500,000 - 417,466.36)(1.51729) = 125,227.$$

91. Solution: B

The accumulated value is the reciprocal of the price. The equation is
X[(1/0.94)+(1/0.95)+(1/0.96)+(1/0.97)+(1/0.98)+(1/0.99)] = 100,000.
X= 16,078

92. Solution: D

Let P be the annual payment. The fifth line is obtained by solving a quadratic equation.

$$P(1-v^{10}) = 3600$$

$$Pv^{10-6+1} = 4871$$

$$\frac{1-v^{10}}{v^5} = \frac{3600}{4871}$$

$$1-v^{10} = 0.739068v^5$$

$$v^5 = 0.69656$$

$$v^{10} = 0.485195$$

$$i = 0.485195^{-1/10} - 1 = 0.075$$

$$X = P\frac{1-v^{10}}{i} = \frac{3600}{0.075} = 48,000$$

93. Solution: A

Let j = periodic yield rate, r = periodic coupon rate, F = redemption (face) value, P = price, n = number of time periods, and $v_j = \frac{1}{1+j}$. In this problem, $j = (1.0705)^{\frac{1}{2}} - 1 = 0.03465$, r = 0.035, P = 10,000, and n = 50.

The present value equation for a bond is $P = Fv_j^{\,n} + Fra_{\overline{n|}j}$; solving for the redemption value F yields

$$F = \frac{P}{v_j^{\,n} + ra_{\overline{n|}j}} = \frac{10,000}{(1.03465)^{-50} + 0.035a_{\overline{50|}0.03465}} = \frac{10,000}{0.18211 + 0.035(23.6044)} = 9,918.$$

.

94. Solution: B

Jeff’s monthly cash flows are coupons of 10,000(0.09)/12 = 75 less loan payments of 2000(0.08)/12 = 13.33 for a net income of 61.67. At the end of the ten years (in addition to the 61.67) he receives 10,000 for the bond less a 2,000 loan repayment. The equation is

$$8000 = 61.67a_{\overline{120|}i^{(12)}/12} + 8000(1+i^{(12)}/12)^{-120}$$

$$i^{(12)}/12 = 0.00770875$$

$$i = 1.00770875^{12} - 1 = 0.0965 = 9.65\%.$$

95. Solution: B

The present value equation for a par-valued annual coupon bond is $P = Fv_i^n + Fra_{\overline{n}|i}$; solving for the coupon rate r yields $r = \frac{P - Fv_i^n}{Fa_{\overline{n}|i}} = \frac{P}{a_{\overline{n}|i}}\left(\frac{1}{F}\right) - \frac{v_i^n}{a_{\overline{n}|i}}$.

All three bonds have the same values except for F. We can write $r = x(1/F) + y$. From the first two bonds:

$0.0528 = x/1000 + y$ and $0.0440 = x/1100 + y$. Then,

$0.0528 – 0.044 = x(1/1000 – 1/1100)$ for $x = 96.8$ and $y = 0.0528 – 96.8/1000 = –0.044$. For the third bond, $r = 96.8/1320 – 0.044 = 0.2933 = 2.93\%$.

96. Solution: A

The effective semi-annual yield rate is $1.04 = \left(1 + \frac{i^{(2)}}{2}\right)^2 \Rightarrow \frac{i^{(2)}}{2} = 1.9804\%$. Then,

$$582.53 = c(1.02)v + c(1.02v)^2 + \cdots + c(1.02v)^{12} + 250v^{12}$$

$$= c\frac{1.02v - (1.02v)^{13}}{1 - 1.02v} + 250v^{12} = 12.015c + 197.579 \Rightarrow c = 32.04.$$

$$582.53 = c\frac{1.02v - (1.02v)^{13}}{1 - 1.02v} + 250v^{12} = 12.015c + 197.579 \Rightarrow c = 32.04$$

97. Solution: E

Book values are linked by BV3(1 + i) – Fr = BV4. Thus 1254.87(1.06) – Fr = 1277.38. Therefore, the coupon is Fr = 52.7822. The prospective formula for the book value at time 3 is

$$1254.87 = 52.7822\frac{1 - 1.06^{-(n-3)}}{0.06} + 1890(1.06)^{-(n-3)}$$

$$375.1667 = 1010.297(1.06)^{-(n-3)}$$

$$n - 3 = \frac{\ln(375.1667/1010.297)}{-\ln(1.06)} = 17.$$

Thus, n = 20. Note that the financial calculator can be used to solve for n – 3.

98. Solution: A

Book values are linked by BV3(1 + *i*) – *Fr* = BV4. Thus BV3(1.04) – 2500(0.035) = BV3 + 8.44. Therefore, BV3 = [2500(0.035) + 8.44]/0.04 = 2398.5. The prospective formula for the book value at time 3 is, where *m* is the number of six-month periods.

$$2398.5 = 2500(0.035)\frac{1-1.04^{-(m-3)}}{0.04} + 2500(1.04)^{-(m-3)}$$

$$211 = 312.5(1.04)^{-(m-3)}$$

$$m-3 = \frac{\ln(211/312.5)}{-\ln(1.04)} = 10.$$

Thus, m = 13 and $n = m/2 = 6.5$. Note that the financial calculator can be used to solve for m – 3.

99. Solution: C

$$s_1 = {}_1f_0 = 0.04$$

$${}_1f_1 = 0.06 = \frac{(1+s_2)^2}{(1+s_1)} - 1 \Rightarrow s_2 = \sqrt{(1.06)(1.04)} - 1 = 0.04995$$

$${}_1f_2 = 0.08 = \frac{(1+s_3)^3}{(1+s_2)^2} - 1 \Rightarrow s_3 = [(1.08)(1.04995)^2]^{1/3} - 1 = 0.05987 = 6\%.$$

100. Solution: B

The Macaulay duration of Annuity A is $0.93 = \frac{0(1)+1(v)+2(v^2)}{1+v+v^2} = \frac{v+2v^2}{1+v+v^2}$, which leads to the quadratic equation $1.07v^2 + 0.07v - 0.93 = 0$. The unique positive solution is v = 0.9.

The Macaulay duration of Annuity B is $\frac{0(1)+1(v)+2(v^2)+3(v^3)}{1+v+v^2+v^3} = 1.369.$

101. Solution: D

With v =1/1.07,

$$D = \frac{2(40,000)v^2 + 3(25,000)v^3 + 4(100,000)v^4}{40,000v^2 + 25,000v^3 + 100,000v^4} = 3.314.$$

102. Solution: C

$$30 = MacD = \frac{\sum_{n=0}^{\infty} nv^n}{\sum_{n=0}^{\infty} v^n} = \frac{Ia_{\overline{\infty|}}}{\ddot{a}_{\overline{\infty|}}} = \frac{1/(di)}{1/d} = \frac{(1+i)/i^2}{(1+i)/i} = \frac{1}{i}$$ and so $i = 1/30$.

$$ModD = \frac{MacD}{1+i} = \frac{30}{1+\frac{1}{30}} = 29.032.$$

103. Solution: B

I) False. The yield curve structure is not relevant.

II) True.

III) False. Matching the present values is not sufficient when interest rates change.

104. Solution: A

The present value function and its derivatives are

$$P(i) = X + Y(1+i)^{-3} - 500(1+i)^{-1} - 1000(1+i)^{-4}$$

$$P'(i) = -3Y(1+i)^{-4} + 500(1+i)^{-2} + 4000(1+i)^{-5}$$

$$P''(i) = 12Y(1+i)^{-5} - 1000(1+i)^{-3} - 20,000(1+i)^{-6}.$$

The equations to solve for matching present values and duration (at $i = 0.10$) and their solution are

$$P(0.1) = X + 0.7513Y - 1137.56 = 0$$

$$P'(0.1) = -2.0490Y + 2896.91 = 0$$

$$Y = 2896.91/2.0490 = 1413.82$$

$$X = 1137.56 - 0.7513(1413.82) = 75.36.$$

The second derivative is

$$P''(0.1) = 12(1413.82)(1.1)^{-5} - 1000(1.1)^{-3} - 20,000(1.1)^{-6} = -1506.34.$$

Redington immunization requires a positive value for the second derivative, so the condition is not satisfied.

105. Solution: D

This solution uses time 8 as the valuation time. The two equations to solve are

$$P(i) = 300,000(1+i)^2 + X(1+i)^{8-y} - 1,000,000 = 0$$

$$P'(i) = 600,000(1+i) + (8-y)X(1+i)^{7-y} = 0.$$

Inserting the interest rate of 4% and solving:

$$300,000(1.04)^2 + X(1.04)^{8-y} - 1,000,000 = 0$$

$$600,000(1.04) + (8-y)X(1.04)^{7-y} = 0$$

$$X(1.04)^{-y} = [1,000,000 - 300,000(1.04)^2]/1.04^8 = 493,595.85$$

$$624,000 + (8-y)(1.04)^7(493,595.85) = 0$$

$$y = 8 + 624,000/[493,595.85(1.04)^7] = 8.9607$$

$$\text{X} = 493,595.85(1.04)^{8.9607} = 701,459.$$

106. Solution: A

This solution uses Macaulay duration and convexity. The same conclusion would result had modified duration and convexity been used.

The liabilities have present value $573/1.07^2 + 701/1.07^5 = 1000.$ Only portfolios A, B, and E have a present value of 1000.

The duration of the liabilities is $[2(573)/1.07^2 + 5(701)/1.07^5]/1000 = 3.5.$ The duration of a zero coupon bond is its term. The portfolio duration is the weighted average of the terms. For portfolio A the duration is [500(1) + 500(6)]/1000 = 3.5. For portfolio B it is [572(1) + 428(6)]/1000 = 3.14. For portfolio E it is 3.5. This eliminates portfolio B.

The convexity of the liabilities is $[4(573)/1.07^2 + 25(701)/1.07^5]/1000 = 14.5.$ The convexity of a zero-coupon bond is the square of its term. For portfolio A the convexity is [500(1) + 500(36)]/1000 = 18.5 which is greater than the convexity of the liabilities. Hence portfolio A provides Redington immunization. As a check, the convexity of portfolio E is 12.25, which is less than the liability convexity.

107. Solution: D

The present value of the liabilities is 1000, so that requirement is met. The duration of the liabilities is $402.11[1.1^{-1} + 2(1.1)^{-2} + 3(1.1)^{-3}]/1000 = 1.9365.$ Let X be the investment in the one-year bond. The duration of a zero-coupon is its term. The duration of the two bonds is then [X + (1000 – X)(3)]/1000 = 3 – 0.002X. Setting this equal to 1.9365 and solving yields X = 531.75.

108. Solution: A

Let x, y, and z represent the amounts invested in the 5-year, 15-year, and 20-year zero-coupon bonds, respectively. Note that in this problem, one of these three variables is 0.

The present value, Macaulay duration, and Macaulay convexity of the assets are, respectively,

$$x+y+z, \frac{5x+15y+20z}{x+y+z}, \frac{5^2x+15^2y+20^2z}{x+y+z}.$$

We are given that the present value, Macaulay duration, and Macaulay convexity of the liabilities are, respectively, 9697, 15.24, and 242.47.

Since present values and Macaulay durations need to match for the assets and liabilities, we have the two equations

$$x+y+z=9697, \quad \frac{5x+15y+20z}{x+y+z}=15.24.$$

Note that 5 and 15 are both less than the desired Macaulay duration 15.24, so z cannot be zero. So try either the 5-year and 20-year bonds (i.e. y = 0), or the 15-year and 20-year bonds (i.e. x = 0).

In the former case, substituting y = 0 and solving for x and z yields

$$x=\frac{(20-15.24)9697}{20-5}=3077.18 \text{ and } z=\frac{(15.24-5)9697}{20-5}=6619.82.$$

We need to check if the Macaulay convexity of the assets exceeds that of the liabilities.

The Macaulay convexity of the assets is $\frac{5^2(3077.18)+20^2(6619.82)}{9697}=281.00$, which exceeds the Macaulay convexity of the liabilities, 242.47. The company should invest 3077 for the 5-year bond and 6620 for the 20-year bond.

Note that setting x = 0 produces y = 9231.54 and z = 465.46 and the convexity is 233.40, which is less than that of the liabilities.

109. Solution: E

The correct answer is the lowest cost portfolio that provides for \$11,000 at the end of year one and provides for \$12,100 at the end of year two. Let H, I, and J represent the face amount of each purchased bond. The time one payment can be exactly matched with H + 0.12J = 11,000. The time two payment can be matched with I + 1.12J = 12,100. The cost of the three bonds is H/1.1 + I/1.2321 + J. This function is to be minimized under the two constraints. Substituting for H and I gives (11,000 – 0.12J)/1.1 + (12,100 – 1.12J)/1.2321 + J = 19,820 – 0.0181J. This is minimized by purchasing the largest possible amount of J. This is 12,100/1.12 = 10,803.57. Then, H = 11,000 – 0.12(10,803.57) = 9703.57. The cost of Bond H is 9703.57/1.1 = 8,821.43.

110. Solution: C

The strategy is to use the two highest yielding assets: the one-year bond and the two-year zero-coupon bond. The cost of these bonds is $25,000/1.0675+20,000/1.05^2=41,560.$

111. Solution: E

Let P be the annual interest paid. The present value of John's payments is $Pa_{\overline{X}|0.05}$. The present value of Karen's payments is $P(1.05)^{-X}a_{\overline{\infty}|0.05}=P(1.05)^{-X}/0.05$. Then,

$$P(1.05)^{-X}/0.05=1.59Pa_{\overline{X}|0.05}$$

$$\frac{1.05^{-X}}{0.05}=1.59\frac{1-1.05^{-X}}{0.05}$$

$$1.59=2.59(1.05)^{-X}$$

$$\ln 1.59=\ln 2.59-X\ln 1.05$$

$$X=10.$$

112. Solution: A

Cheryl's force of interest at all times is ln(1.07) = 0.06766. Gomer's accumulation function is from time 3 is 1 + yt and the force of interest is $y/(1+yt)$. To be equal at time 2, the equation is 0.06766 = $y/(1+2y)$, which implies 0.06766 + 0.13532y = y for y = 0.07825. Gomer's account value is 1000(1 + 2x0.07825) = 1156.5.

113. Solution: D

One way to view these payments is as a sequence of level immediate perpetuities of 1 that are deferred n-1, n, n+1,… years. The present value is then

$$v^{n-1}/i+v^n/i+v^{n+1}/i+\cdots=(v^{n-2}/i)(v+v^2+v^3+\cdots)=v^{n-2}/i^2.$$

Noting that only answers C, D, and E have this form and all have the same numerator,

$$v^{n-2}/i^2=v^n/(vi)^2=v^n/d^2.$$

114. Solution: B

The monthly interest rate is $j=(1.08)^{1/12}-1=0.643\%.$ Then,

$$20,000s_{\overline{4}|0.08}=X\ddot{s}_{\overline{252}|0.00643},\quad 90,122.24=630.99X,\quad X=142.83.$$

115. Solution: D

$\bar{a}_{\overline{20|}} = 1.5\bar{a}_{\overline{10|}}$, $\dfrac{1-e^{-20\delta}}{\delta} = 1.5\dfrac{1-e^{-10\delta}}{\delta}$, $e^{-20\delta} - 1.5e^{-10\delta} + 0.5 = 0$. Let $X = e^{-10\delta}$. We then have the quadratic equation $X^2 - 1.5X + 0.5 = 0$ with solution $X = 0.5$ for $\delta = \ln 0.5/(-10) = 0.069315$.

Then, the accumulated value of a 7-year continuous annuity of 1 is $\bar{s}_{\overline{7|}} = \dfrac{e^{7(0.069315)} - 1}{0.069315} = 9.01$.

116. Solution: B

The present value is

$$v^3 + v^{10} + v^{17} + \cdots + v^{-4+7n}$$

$$= \frac{v^3 - v^{3+7n}}{1-v^7} = \frac{(1-v^{3+7n}) - (1-v^3)}{1-v^7} = \frac{a_{\overline{3+7n|}} - a_{\overline{3|}}}{a_{\overline{7|}}}.$$

117. Solution: C

From the first annuity, $X = 21.8s_{\overline{n|}0.109} = 21.8 \cdot \dfrac{1.109^n - 1}{0.109} = 200[1.109^n - 1]$.

From the second annuity, $X = 19{,}208(v^n + v^{2n} + \cdots) = 19{,}208\dfrac{v^n}{1-v^n} = 19{,}208\dfrac{1}{1.109^n - 1}$.

Hence,

$$200[1.109^n - 1] = 19{,}208\frac{1}{1.109^n - 1}$$

$$[1.109^n - 1]^2 = 19{,}208/200 = 96.04$$

$$1.109^n - 1 = 9.8$$

$$X = 200(9.8) = 1960.$$

118. Solution: C

$$2(Ia)_{\overline{60|}1\%} = 2\frac{\ddot{a}_{\overline{60|}} - 60v^{60}}{0.01} = 2\frac{45.4 - 33.03}{0.01} = 2{,}474.60.$$

119. Solution: E

Let j be the semi-annual interest rate. Then,

$$475,000 = 300 + 300a_{\overline{\infty}|j} + (1+j)^{-1}200(Ia)_{\overline{\infty}|j} = 300 + 300/j + 200/j^2$$

$$474,700j^2 - 300j - 200 = 0$$

$$j = \frac{300 + \sqrt{300^2 - 4(474,700)(-200)}}{2(474,700)} = 0.02084$$

$$i = (1+j)^2 - 1 = 0.04212 = 4.21\%.$$

120. Solution: B

The present value is

$$4a_{\overline{\infty}|0.06} + 2(Ia)_{\overline{\infty}|0.06} = 4/0.06 + 2(1.06)/0.06^2 = 655.56.$$

121. Solution: A

The present value of the income is $100a_{\overline{\infty}|0.1025} = 100/0.1025 = 975.61.$ The present value of the investment is

$$X\left[1 + 1.05/1.1025 + (1.05/1.1025)^2 + (1.05/1.1025)^3 + (1.05/1.1025)^4 + (1.05/1.1025)^5\right]$$

$$= X[1 + 1.05^{-1} + 1.05^{-2} + 1.05^{-3} + 1.05^{-4} + 1.05^{-5}] = X\frac{1 - 1.05^{-6}}{1 - 1.05^{-1}} = 5.3295X.$$

Then 975.61=5.3295*X* for *X* = 183.06.

122. Solution: A

The present value of the ten level payments is $X\ddot{a}_{\overline{10}|0.05} = 8.10782X.$ The present value of the remaining payments is

$$X(v^{10}1.015 + v^{11}1.015^2 + \cdots) = X\frac{v^{10}1.015}{1 - v1.015} = X\frac{1.015/1.05^{10}}{1 - 1.015/1.05} = 18.69366X.$$

Then, 45,000 = 8.10782*X* + 18.69366*X* = 26.80148*X* for *X* = 1679.

123. Solution: D

The equation of value is

$$10,000 = X(v + v^2 0.996 + v^3 0.996^2 + \cdots) = X\frac{v}{1 - v0.996} = X\frac{e^{-0.06}}{1 - e^{-0.06}0.996} = 15.189X.$$ The solution is *X* = 10,000/15.189 = 658.37.

124. Solution: D

Discounting at 10%, the net present values are 4.59, –2.36, and –9.54 for Projects A, B, and C respectively. Hence, only Project A should be funded. Note that Project C's net present value need not be calculated. Its cash flows are the same as Project B except being 50 less at time 2 and 50 more at time 4. This indicates Project C must have a lower net present value and therefore be negative.

125. Solution: D

The loan balance after 10 years is still 100,000. For the next 10 payments, the interest paid is 10% of the outstanding balance and therefore the principal repaid is 5% of the outstanding balance. After 10 years the oustanding balance is $100,000(0.95)^{10} = 59,874.$ Then,

$$X = 59,874 / a_{\overline{10}|0.1} = 59,874 / 6.14457 = 9,744.$$

126. Solution: B

First determine number of regular payments:

$4000 = 600v^4 a_{\overline{n}|0.06}, a_{\overline{n}|0.06} = (4000/600)1.06^4 = 8.4165.$ Using the calculator, n = 12.07 and thus there are 11 regular payments. The equation for the balloon payment, X, is:

$$4000 = 600v^4 a_{\overline{11}|0.06} + Xv^{16} = 3748.29 + 0.39365X,\ X = 639.43.$$

127. Solution: C

$$20,000 = X\left(a_{\overline{5}|0.11} + 1.11^{-5} a_{\overline{5}|0.12}\right) = X(3.69590 + 3.60478/1.68506) = 5.83516\text{X}$$

$$\text{X} = 20,000/5.83516 = 3427.50.$$

128. Solution: A

The principal repaid in the first payment is 100 – iL. The outstanding principal is L – 100 + iL = L + 25. Hence, iL = 125. Also,

$$L = 300a_{\overline{16}|} - 200a_{\overline{8}|} = \frac{300(1-v^{16}) - 200(1-v^8)}{i}$$

$$125 = iL = 100 + 200v^8 - 300v^{16}$$

$$300v^{16} - 200v^8 + 25 = 0$$

$$v^8 = \frac{200 \pm \sqrt{200^2 - 4(300)(25)}}{600} = \frac{200 \pm 100}{600} = 0.5.$$

The larger of the two values is used due to the value being known to exceed 0.3. The outstanding valance at time eight is the present value of the remaining payments:

$$300a_{\overline{8}|} = 300\frac{1-0.5}{2^{1/8}-1} = 1657.$$

129. Solution: E

Let j be the monthly rate and X be the level monthly payment. The principal repaid in the first payment is 1400 = X – 60,000j. The principal repaid in the second payment is 1414 = X – (60,000 – 1400)j. Substituting X = 1400 + 60,000j from the first equation gives 1414 = 1400 + 60,000j – 58,600j or 14 = 1400j and thus j = 0.01 and X = 2000. Let n be the number of payments. Then $60,000 = 2000a_{\overline{n}|0.01}$ and the calculator (or algebra) gives n = 35.8455. The equation for the drop payment, P, is $60,000 = 2000a_{\overline{35}|0.01} + Pv^{36} = 58,817.16 + 0.698925P$ for P = 1692.

130. Solution: C

The accumulated value is

$$1000\left(s_{\overline{24}|0.06/12}(1+0.08/12)^{24} + s_{\overline{24}|0.08/12}\right) = 1000(25.4320(1.1729) + 25.9332) = 55,762.$$

131. Solution: C

Each month the principal paid increases by $1.1^{1/12}$. Thus, the amount of principal paid increases to $500(1.1^{1/12})^{30-6} = 500(1.1)^2 = 605.$

132. Solution: C

$$\text{Int}_{11} = i \cdot \left[900 \cdot a_{\overline{20}|i} + 300a_{\overline{10}|i}\right] = 900(1-v^{20}) + 300(1-v^{10}) = 1200 - 300v^{10} - 900v^{20}$$

$$\text{Int}_{21} = i\left[900 \cdot a_{\overline{10}|i}\right] = 900(1-v^{10})$$

$$\text{Int}_{11} = 2\text{Int}_{21} \Rightarrow 1200 - 300v^{10} - 900v^{20} = 1800 - 1800v^{10}$$

$$\Rightarrow 9v^{20} - 15v^{10} + 6 = 0 \Rightarrow v^{10} = 2/3$$

$$\text{Int}_{21} = 900(1-v^{10}) = 300$$

133. Solution: C

The original monthly payment is $85,000 / a_{\overline{240}|0.005} = 85,000/139.5808 = 608.97$. On July 1, 2009 there has been 4 years of payments, hence 16x12 = 192 remaining payments. The outstanding balance is $608.97a_{\overline{192}|0.005} = 608.97(123.2380) = 75,048.24$. The number of remaining payments after refinancing is determined as

$$75,048.24 = 500a_{\overline{n}|0.0045} = 500\frac{1-1.0045^{-n}}{0.0045}$$

$$0.67543 = 1 - 1.0045^{-n}$$

$$n = -\ln(0.32457)/\ln(1.0045) = 250.62.$$

Thus the final payment will be 251 months from June 30, 2009. This is 20 years and 11 months and so the final payment is May 31, 2030.

134. Solution: B
Just prior to the extra payment at time 5, the outstand balance is
$1300a_{\overline{20}|0.07} = 1300(10.5940) = 13,772.20$. After the extra payment it is 11,172.20. Paying this off in 15 years requires annual payments of $11,172.20 / a_{\overline{15}|0.07} = 11,172.20 / 9.1079 = 1226.65$.

135. Solution: C
During the first redemption period the modified coupon rate is 1000(0.035)/1250 = 2.80% which is larger than the desired yield rate. If redeemed during this period, bond sells at a premium and so the worst case for the buyer is the earliest redemption. The price if called at that time is $35a_{\overline{20}|0.025} + 1250(1.025)^{-20} = 35(15.5892) + 762.84 = 1308.46$. During the second redemption period the modified coupon rate is 1000(0.035)/1125 = 3.11% which is also larger than the desired yield rate and the worst case for the buyer is again the earliest redemption. The price if called at that time is $35a_{\overline{40}|0.025} + 1125(1.025)^{-40} = 35(25.1028) + 418.98 = 1297.58$. Finally, if the bond is not called, its value is $35a_{\overline{60}|0.025} + 1000(1.025)^{-60} = 35(30.9087) + 227.28 = 1309.08$.
The appropriate price is the lowest of these three, which relates to the bond being called after the 40th coupon is paid.

136. Solution: B
Because the yield is less than the coupon rate, the bond sells at a premium and the worst case for the buyer is an early call. Hence the price should be calculated based on the bond being called at time 16. The price is $100a_{\overline{16}|0.05} + 1000(1.05)^{-16} = 100(10.0378) + 458.11 = 1542$. (When working with callable bonds, the maximum a buyer will pay is the smallest price over the various call dates. Paying more may not earn the desired yield.)

137. Solution: A
All calculations are in millions. For the ten-year bond, at time ten it is redeemed for $2(1.08)^{10} = 4.31785$. After being reinvested at 12% it matures at time twenty for $4.31785(1.12)^{10} = 13.4106$. The thirty-year bond has a redemption value of $4(1.08)^{30} = 40.2506$. For the buyer to earn 10%, it is sold for $40.2506(1.1)^{-10} = 15.5184$. The gain is 13.4106 + 15.5184 – 6 = 22.9290.

138. Solution: A
The book value after the third coupon is
$7500(0.037)a_{\overline{37}|0.0265} + C(1.0265)^{-37} = 6493.05 + 0.379943C$ and after the fourth coupon it is
$7500(0.037)a_{\overline{36}|0.0265} + C(1.0265)^{-36} = 6387.61 + 0.390012C$. Then,

$$6493.05 + 0.379943C - (6387.61 + 0.390012C) = 28.31$$
$$105.44 - 0.010069C = 28.31$$
$$C = 7660.15.$$

139. Solution: C

The semiannual yield rate is $1.1^{1/2} - 1 = 0.0488$. Assuming the bond is called for 2900 after four years, the purchase price is $150a_{\overline{8}|0.0488} + 2900(1.0488)^{-8} = 150(6.4947) + 1980.87 = 2955.08$. With a call after the first coupon, the equation to solve for the semi-annual yield rate (*j*) and then the annual effective rate (*i*) is

$2955.08 = (150 + 2960)/(1 + j)$

$1 + j = 1.05242$

$i = 1.05242^2 - 1 = 0.10759.$

140. Solution: C

The book value after the sixth coupon is $1000(r/2)a_{\overline{34}|0.036} + 1000(1.036)^{-34} = 9716.01r + 300.45$.

After the seventh coupon it is $1000(r/2)a_{\overline{33}|0.036} + 1000(1.036)^{-33} = 9565.79r + 311.26$. Then,

$4.36 = 9565.79r + 311.26 - (9716.01r + 300.45) = 10.81 - 150.22r$

$r = (10.81 - 4.36)/150.22 = 0.0429.$

141. Solution: B

The two equations are:

$P = (10,000r)a_{\overline{5}|0.04} + 9,000(1.04)^{-5} = 44,518.22r + 7,397.34$

$1.2P = [10,000(r + 0.01)]a_{\overline{5}|0.04} + 11,000(1.04)^{-5} = 44,518.22r + 9,486.38.$

Subtracting the first equation from the second gives 0.2*P* = 2089.04 for *P* = 10,445.20. Inserting this in the first equation gives *r* = (10,445.20 – 7,397.34)/44,518.22 = 0.0685.

142. Solution: C

When the yield is 6.8% < 8%, the bond is sold at a premium and hence an early call is most disadvantageous. Therefore, $P = 40a_{\overline{10}|0.034} + 1000(1.034)^{-10} = 1050.15$. When the yield is 8.8% > 8%, the bond is sold at discount. Hence, *Q* < 1000 < *P*. and thus *Q* = 1050.15 – 123.36 = 926.79. Also, because the bond is sold at a discount, the latest call is the most disadvantageous. Thus,

$$926.79 = 40a_{\overline{2n}|0.044} + 1000(1.044)^{-2n} = \frac{40}{0.044} + (1.044)^{-2n}\left(1000 - \frac{40}{.044}\right) = 909.09 + 90.90(1.044)^{-2n}$$

$17.70 = 90.90(1.044)^{-2n}$

$2n = -\ln(17.70/90.90)/\ln(1.044) = 38$

$n = 19.$

143. Solution: B

The fund will have $500(1.05)^4 - 100s_{\overline{4|}0.05} = 176.74$ after four years. After returning 75% to the insured, the insurer receives 0.25(176.74) = 44.19. So the insurer's cash flows are to pay 100 at time 0, receive 125 at time 2, and receive 44.19 at time four. The equation of value and the solution are:

$$100(1+i)^4 - 125(1+i)^2 - 44.19 = 0$$

$$(1+i)^2 = \frac{125 \pm \sqrt{(-125)^2 - 4(100)(-44.19)}}{200} = 1.5374$$

$$1+i = 1.2399$$

$$i = 24\%.$$

144. Solution: B

The Macaulay duration of the perpetuity is $\frac{\sum_{n=1}^{\infty} nv^n}{\sum_{n=1}^{\infty} v^n} = \frac{(Ia)_{\overline{\infty|}}}{a_{\overline{\infty|}}} = \frac{(1+i)/i^2}{1/i} = \frac{1+i}{i} = 1 + 1/i = 17.6.$

This implies that i = 1/16.6. With $i = 2i$ = 2/16.6, the duration is 1 + 16.6/2 = 9.3.

145. Solution: A

Because the interest rate is greater than zero, the Macaulay duration of each bond is greater than its modified duration. Therefore, the bond with a Macaulay duration of c must be the bond with a modified duration of a and $a = c/(1 + i)$ which implies $1 + i = c/a$. The Macaulay duration of the other bond is $b(1 + i) = bc/a$.

146. Solution: B

$P(0.1025) \approx P(0.10)\left(\frac{1.10}{1.1025}\right)^{11} = 0.97534P(0.10).$ Therefore, the approximate percentage price change is 100(0.97534 – 1) = –2.47%.

147. Solution: B

Cash-flow matching limits the number of investment choices available to the portfolio manager to a subset of the choices available for immunization.

148. Solution: C
Options for full immunization are:
2J (cost is 3000), K+2L (cost is 2500), and M (cost is 4000). The lowest possible cost is 2500. Another way to view this is that the prices divided by total cash flows are 0.6, 0.5, 0.5, and 0.8. The cheapest option will be to use K and L, if possible.

149. Solution: B
The present value of the assets is 15,000 + 45,000 = 60,000 which is also the present value of the liability. The modified duration of the assets is the weighted average, or 0.25(1.80) + 0.75*D*mod. The modified duration of the liability is 3/1.1 and so *D*mod = (3/1.1 – 0.45)/0.75 = 3.04.

150. Solution: C
Let *A* be the redemption value of the zero-coupon bonds purchased and *B* the number of two-year bonds purchased. The total present value is:
$1783.76 = A/1.05 + B(100/1.06 + 1100/1.06^2) = 0.95238A + 1073.3357B.$
To exactly match the cash flow at time one, *A* + 100*B* = 1000. Substituting *B* = 10 – 0.01*A* in the first equation gives 1783.76 = 0.95238*A* + 10733.357 – 10.733357*A* for *A* = 8949.597/9.780977 = 915. The amount invested is then 915/1.05 = 871.

151. Solution: B
The company must purchase 4000 in one-year bonds and 6000 in two-year bonds. The total purchase price is $4000/1.08 + 6000/1.11^2 = 8573.$

152. Solution: C
The modified duration is 11/1.10 = 10. Then,
$P(0.1025) \approx P(0.10)[1-(0.1025-0.10)10] = 0.975P(0.10).$ Therefore, the approximate percentage price change is 100(0.975 – 1) = –2.50%.

153. Solution: B
$$P(0.08) \approx 1000\left(\frac{1.072}{1.08}\right)^{7.959} = 942.54.$$

154. Solution: E
Modified duration = (Macaulay duration)/(1 + *i*) and so Macaulay duration = 8(1.064) = 8.512.
$E_{MAC} = 112,955\left(\frac{1.064}{1.07}\right)^{8.512} = 107,676$ and $E_{MOD} = 112,955[1-(0.07-0.064)(8)] = 107,533.$
Then, $E_{MAC} - E_{MOD} = 107,676 - 107,533 = 143.$

155. Solution: C

The Macaulay duration of the portfolio is $\frac{35,000(7.28)+65,000(12.74)}{35,000+65,000}=10.829.$ Then,

$$105,000=100,000\left(\frac{1.0432}{1+i}\right)^{10.829} \Rightarrow \frac{1.0432}{1+i}=\left(\frac{105,000}{100,000}\right)^{1/10.829}=1.004516 \Rightarrow i=0.0385.$$

156. Solution: A

$$121,212=123,000\left(\frac{1.05}{1.054}\right)^{D_{MAC}} \Rightarrow D_{MAC}=\frac{\ln(121,212/123,000)}{\ln(1.05/1.054)}=3.8512.$$ Then,

$D_{MOD}=3.8512/1.05=3.67.$

157. Solution: A

I provides cash flows that exactly matches the liabilities. II only has PV(A) = PV(B), which is not sufficient for exact matching. III describes Redington immunization, not exact matching.

158. Solution: D

Let F be the face amount of Bond X. Then,

$2695.39=200a_{\overline{15|}}+Fv^{15}$ and $3490.78=200a_{\overline{15|}}+2Fv^{15}$.

Subtract the first equation from the second to obtain $795.39=Fv^{15.}$

Then for bond X, $2695.39=200a_{\overline{15|}}+795.39 \Rightarrow a_{\overline{15|}}=(2695.39-795.39)/200=9.5$. This implies i =0.0634. Then $9.5=(1-v^{15})/0.0634 \Rightarrow v^{15}=1-0.0634(9.5)=0.3977$ and $F=795.39/0.3977=2000.$ The coupon rate is 200/2000 = 10.0%.

159. Solution: D

The value of 1 invested with bank P after three years is $1.04^3+0.02=1.144864.$ The yield from Bank Q satisfies $1.144864=(1+i)^3 \Rightarrow i=1.144864^{1/3}-1=0.04613=4.6\%.$

160. Solution: D

With a continuously compounded annual interest rate of 6%, $v = e^{-0.06}$. The value of the first annuity is

$$600,000 = X(I\ddot{a})_{\overline{20}|} = X\frac{\ddot{a}_{\overline{20}|} - 20v^{20}}{d} = X\frac{\frac{1-e^{-1.2}}{1-e^{-0.06}} - 20e^{-1.2}}{1-e^{-0.06}} = 102.614X.$$ Hence,

X = 600,000/102.614 = 5847.155. Then the value of the second annuity is

$$5847.155\frac{\ddot{a}_{\overline{25}|} - 25v^{25}}{d} = 5847.155\frac{\frac{1-e^{-1.5}}{1-e^{-0.06}} - 25e^{-1.5}}{1-e^{-0.06}} = 779,366.$$

161. Solution: B

The amount of principal repaid at payment 15 is (where R is the quarterly payment)

$$10,030.27 = Rv^{40-15+1} = R(1.03)^{-26} \Rightarrow R = 10,030.27(1.03)^{26} = 21,631.19.$$

The amount of interest in payment 25 is

$$21,631.19(1 - v^{40-25+1}) = 21,631.19(1 - 1.03^{-16}) = 8,151.35.$$

162. Solution: E

The present value of the payments (4000 at month 36 plus the payments of X) must match the present value of the present value of the amounts borrowed (4000 at month 0 plus the payments of 800).

The quarterly interest rate is 0.264/4 and all payment times should be in quarters of a year. On that time scale, the 4000 at month 36 is at time 12. The payments of 4000 are at times 1/6, 3/6, 5/6, …, 71/6 and there are 36 such payments. One way to write the present value of these payments is

$$\frac{4000}{\left(1+\frac{0.264}{4}\right)^{12}} + \sum_{n=1}^{36}\frac{X}{\left(1+\frac{0.264}{4}\right)^{\frac{n-0.5}{3}}}.$$

The payments of 800 are at times 1, 3, 5, 7, 9, and 11,, in quarters. One way to write the present value of these payments plus the initial debt of 4000 is

$$4000 + \sum_{n=1}^{6}\frac{800}{\left(1+\frac{0.264}{12}\right)^{2n-1}}.$$

These are the two sides of equation in answer choice E.

163. Solution: B

Let r be the coupon rate for Bond A. The coupon rate for Bond B is then r + 0.01. Then,

$$1600 = 1000\left[\frac{1}{(1.1)^{20}} + ra_{\overline{20}|0.1} + \frac{1}{(1.1)^{20}} + (r+0.01)a_{\overline{20}|0.1}\right]$$

$$1.6 = \frac{2}{(1.1)^{20}} + 2ra_{\overline{20}|0.1} + 0.01a_{\overline{20}|0.1} = 0.29729 + 17.02713r + 0.08514$$

$$r = \frac{1.6 - 0.29729 - 0.08514}{17.02713} = 0.0715 = 7.15\%.$$

164. Solution: E

Let n be the number of payments and let j be the interest rate per half-year. Because the given values are n – 1 half-years apart, $7,968.89(1+j)^{n-1} = 19,549.25$. Also,

$$7,968.89 = 1,000\ddot{a}_{\overline{n}|} = 1,000(a_{\overline{n-1}|} + 1) = 1,000\left(\frac{1-v^{n-1}}{j} + 1\right) = 1,000\left(\frac{1 - 7,968.89/19,549.25}{j} + 1\right).$$

Then,

$$j = \frac{1 - 7,968.89/19,549.25}{\frac{7,968.89}{1,000} - 1} = 0.085$$

for $i = (1.085)^2 - 1 = 0.1772 = 17.7\%.$

165. Solution: B

The denominator of the duration is the present value of the annuity:

$$X\ddot{a}_{\overline{20}|0.02} + 4Xv^{20}\ddot{a}_{\overline{30}|0.02} = 78.1729X.$$

The numerator is the time-weighted present value of the annuity. In units of X we need the present value of 0, 1, …, 19, 80, 84, …, 196. One way to view this is as four times a 49-year increasing immediate annuity (so payments of 4, 8, …, 76, 80, 84, …, 196) less three times a 19-year increasing immediate annuity (so payments of 3, 6, …, 57). The present value is:

$$4X(Ia)_{\overline{49}|0.02} - 3X(Ia)_{\overline{19}|0.02} = [4(655.2078) - 3(147.4923)]X = 2,178.3542X.$$

The duration is the ratio, 2,178.3542/78.1729 = 27.87.

166. Solution: A

$$100\left(1+\frac{i}{2}\right)^{22}\left[\left(1+\frac{i}{2}\right)^{2} - 1\right] = 2(100)\left(1+\frac{i}{2}\right)^{8}\left[\left(1+\frac{i}{2}\right)^{2} - 1\right]$$

$$2 = \left(1+\frac{i}{2}\right)^{14}$$

$$i = (2^{1/14} - 1)2 = 0.1015 = 10.15\%.$$

167. Solution: D

Let C be the amount of the semiannual coupon for bond B.

$X = 40a_{\overline{10}|0.03} + 1000(1.03)^{-10} = 1085.30$

$X = 1085.30 = Ca_{\overline{10}|0.035} + 1000(1.035)^{-10} = 8.3166C + 708.9188$

$C = (1085.30 - 708.9188)/8.3166 = 45.2566$

$y = \dfrac{45.2566 \times 2}{1000} = 0.0905 = 9.05\%.$

168. Solution: C

Let X be the original loan value. From the original loan terms, $X = 50a_{\overline{15}|}$. Under the revised repayment plan, $X = 50a_{\overline{10}|} + 30v^5 a_{\overline{5}|}$. Equating the two gives $50a_{\overline{15}|} = 50a_{\overline{10}|} + 30v^5 a_{\overline{5}|}$ which does not match answer A. All the other choices use s. Multiplying both sides by $(1+i)^{10}$ gives $50v^5 s_{\overline{15}|} = 50s_{\overline{10}|} + 30s_{\overline{5}|}$, which is answer C. This can also be obtained by equating the values of the two payment streams at time 10 rather than time 0.

169. Solution: A

The effective monthly rate is $1.065^{1/12} - 1 = 0.0052617$. The accumulated value is

$$1097s_{\overline{180}|0.0052617} + 5(Is)_{\overline{180}|0.0052617}$$
$$= 1097(298.733) + 5\frac{\ddot{s}_{\overline{180}|0.0052617} - 180}{0.0052617}$$
$$= 327,710 + 5\frac{300.3049 - 180}{0.0052617} = 442,031.$$

170. Solution D

Fund K receives 1000 at the end of each year and also receives interest payments of 1300, 1235, 1170, …, 65. The accumulated value is

$$1000s_{\overline{20}|0.0825} + 65(Ds)_{\overline{20}|0.0825}$$
$$= 1000(47.0491) + 65\frac{20(1.0825)^{20} - s_{\overline{20}|0.0825}}{0.0825}$$
$$= 47,049.1 + 65\frac{97.6311 - 47.0491}{0.0825} = 86,902.$$

171. Solution: D

For Q the accumulated value is $Xs_{\overline{25}|0.09} = 84.7009X$. For R the accumulated value is

$100(25) + 9(Is)_{\overline{24}|0.08} = 2500 + 9\dfrac{\ddot{s}_{\overline{24}|0.08} - 24}{0.08} = 2500 + 9\dfrac{72.1059 - 24}{0.08} = 7911.91.$ Then

X = 7911.91/84.7009 = 93.41.

172. Solution: E

The accumulated values for Funds X and Y are $1000\left(1+\dfrac{k}{2}\right)^{10}$ and $921.90\left(1-\dfrac{k}{2}\right)^{-10}$

respectively. Equating them and solving for k:

$$1000\left(1+\frac{k}{2}\right)^{10} = 921.90\left(1-\frac{k}{2}\right)^{-10}$$

$$0.9219 = \left[\left(1+\frac{k}{2}\right)\left(1-\frac{k}{2}\right)\right]^{10} = \left(1-\frac{k^2}{4}\right)^{10}$$

$$1-\frac{k^2}{4} = 0.9919$$

$$k^2 = 0.0324$$

$$k = 0.18.$$

$$P = 1000\left(1+\frac{0.18}{2}\right)^{10} = 2367.36.$$

173. Solution: A

The 3-year interest rate is $1.07^3 - 1 = 0.225043$. Then,

$735 = X\ddot{a}_{\overline{\infty}|0.225043} = X\dfrac{1.225043}{0.225043} = 5.443595X$ and X = 735/5.443595 = 135.02.

174. Solution: B

$$2600 = Pa_{\overline{\infty}|0.06} + v9(Ia)_{\overline{\infty}|0.06}$$

$$= P\frac{1}{0.06} + \frac{1}{1.06}9\frac{1.06}{0.06^2}$$

$$= \frac{P}{0.06} + 2500$$

$$P = (2600 - 2500)(0.06) = 6.$$

175. Solution: B

$475a_{\overline{10}|i} = 400\left(a_{\overline{5}|i} + v^{10}a_{\overline{\infty}|}\right)$

$475\dfrac{1-v^{10}}{i} = 400\dfrac{1-v^5+v^{10}}{i}$

$475\left(1-v^{10}\right) = 400\left(1-v^5+v^{10}\right)$

$875v^{10} - 400v^5 - 75 = 0$

$v^5 = \dfrac{400 \pm \sqrt{400^2 + 4(875)(75)}}{2(875)} = 0.6$

$i = (1/0.6)^{1/5} - 1 = 0.1076 = 10.76\%$

176. Solution: E

Based on the effective yield rate, $100 = X/1.1 + 2X/1.1^2 \Rightarrow X = 39.03$. After one year, the outstanding loan balance is 100 + 8 – 39.03 = 68.97. For the balance to be zero after two years, $68.97(1+i) - 2(39.03) = 0 \Rightarrow i = 78.06/68.97 - 1 = 0.1318 = 13.2\%$.

177. Solution: D

The amount borrowed is $1000a_{\overline{5}|0.1} + 2000v^5a_{\overline{5}|0.1} = 8498.35$. The outstanding balance after five years is $2000a_{\overline{5}|0.1} = 7581.57$. The principal repaid is 8498.35 – 7581.57 = 916.78. The interest paid is 5000 – 916.78 = 4083.22.

178. Solution: B

The accumulation is

$$10X + 0.12X(Is)_{\overline{10}|0.08} = 10X + 0.12X\frac{\ddot{s}_{\overline{10}|0.08} - 10}{0.08} = 10X + 0.12X\frac{15.6455 - 10}{0.08} = 18.4683X.$$

Then, X = 10,000/18.4683 = 541.47

179. Solution: C

The equation of value is

$$10,000 = (X+5)a_{\overline{30}|0.05} - 5(Ia)_{\overline{30}|0.05} = (X+5)a_{\overline{30}|0.05} - 5\frac{\ddot{a}_{\overline{30}|0.05} - 30v^{30}}{0.05}$$

$$= (X+5)(15.37245) - 5\frac{16.14107 - 6.94132}{0.05} = 15.37245X - 843.11275$$

$X = (10,000 + 843.11275)/15.37245 = 705.36.$

180. Solution: A

$$NPV = -600 + \frac{150-100}{1.15} + \frac{150-50}{1.15^2} + \frac{150}{1.15^3} + \frac{150}{1.15^4} + \frac{150}{1.15^5} = -221.94$$

181. Solution: A

I is true.

II is false, the price sensitivity of assets and liabilities must be equal.

III is false, the convexity of assets should be greater than the convexity of liabilities.

182. Solution: D

The effective annual rate of interest is $(1.005)^{12} - 1 = 0.06168$. The present value of the tuition payments six months before the first payment is $25,000(1.005)^{-6}\ddot{a}_{\overline{4}|0.06168} = 24,262.95(3.66473) = 88,917.16$. The accumulated value of the deposits at that time is $1000 s_{\overline{n}|0.005}$. Equating the two amounts:

$$88,917.16 = 1000\frac{1.005^n - 1}{0.005}$$

$$1.44459 = 1.005^n$$

$$n = \ln(1.44459)/\ln(1.005) = 73.75.$$

Therefore, at least 74 payments will be required.

183. Solution: E

Let x be the annual payment amount. Macaulay duration is

$$\frac{\frac{1x}{1.1} + \frac{2x}{1.1^2} + \frac{3x}{1.1^3} + \cdots + \frac{7x}{1.1^7}}{\frac{x}{1.1} + \frac{x}{1.1^2} + \frac{x}{1.1^3} + \cdots + \frac{x}{1.1^7}} = \frac{17.6315}{4.8684} = 3.62$$

.

Alternatively, the duration can be calculated as $(Ia)_{\overline{7}|0.1} / a_{\overline{7}|0.1}$.

184. Solution: E

PV of liabilities is $402.11(1/1.1 + 1/1.1^2 + 1/1.1^3) = 1000$. Duration of liabilities is $402.11(1/1.1 + 2/1.1^2 + 3/1.1^3)/1000 = 1.93653$. Let X be the investment in one-year bonds. To match duration, since zero-coupon bonds have duration = maturity, 1.93653 = [X + 3(1000 – X)]/1000. Then, 2X = 3000 – 1936.53 = 1063.47 and X = 532.

185. Solution: E

A change in face value multiplies all cash flows by the same amount. Therefore, there is no change in the duration. If the coupon rate increases, the coupons become larger, but the redemption value stays the same. This causes payments prior to redemption to receive more weight relative to the payment at redemption and thus the duration will decrease.

186. Solution: A

We are given $i^{(4)} = 8\%$ and want to determine $i^{(12)}/12$. The equation that links the two and its solution is:

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(1+\frac{i^{(4)}}{4}\right)^{4} = \left(1+\frac{8\%}{4}\right)^{4}$$

$$\left(1+\frac{i^{(12)}}{12}\right) = \left(1+\frac{8\%}{4}\right)^{4/12}$$

$$\frac{i^{(12)}}{12} = \left(1+\frac{8\%}{4}\right)^{1/3} - 1.$$

187. Solution: E

Let m be the monthly payment and i be the monthly interest rate. The interest in the first payment is 125,000i and the principal repaid is 125,000 – 124,750 = 250. Thus m = 125,000i + 250. Similarly, for the second payment, m = 124,750i +252. Thus, 250i = 2 for i = 2/250 = 0.008 and then m = 1250. To obtain the number of payments, the equation to solve is

$$125,000 = 1250 a_{\overline{n}|0.008}$$

$$100 = \frac{1-1.008^{-n}}{0.008}$$

$$0.2 = 1.008^{-n}$$

$$n = -\ln(0.2)/\ln(1.008) = 202.$$

188. Solution: C

Let x and y be the amount invested in the five and twenty year bonds respectively. To match the present values: $x + y = 500,000e^{-0.07(10)} + 500,000e^{-0.07(15)} = 423,262.$ To match the durations, noting that the denominators of the durations for assets and liabilities are the same, $5x + 20y = 500,000(10)e^{-0.07(10)} + 500,000(15)e^{-0.07(15)} = 5,107,460.$ Subtracting five times the first equation from the second one gives 15y = 2,991,150 for y = 199,410 and x = 423,262 – 199,410 = 223,852.

189.	Solution: C

Let *n* be the term of the bond in half-years. We know that $601 = 1080v^n$ and thus $v^n = 601/1080$. Then $a_{\overline{n}|0.05} = \dfrac{1-v^n}{0.05} = \dfrac{1-601/1080}{0.05} = 8.87037.$ The purchase price of the bond is $40a_{\overline{n}|0.05} + 1080v^n = 40(8.87037) + 610 = 956.$

190.	Solution: B

Principal repaid in the first payment is 1000 – 10,000(0.04) = 600. Therefore, the principle repaid in the tenth payment is $600(1.04)^9 = 854$ and the interest paid is 1000 – 854 = 146.

191.	Solution: C

After one year the outstanding balance is $500a_{\overline{48}|0.025} = 13,886.58$. This must match the present value of the revised payments:

$13,886.58 = Xv^6a_{\overline{6}|0.025} + 500v^{12}a_{\overline{36}|0.025} = 4.74964X + 8,757.69$

$X = (13,886.58 - 8,757.69) / 4.74964 = 1,079.85.$

Alternatively, each missing payment is being replaced with a larger payment six months later. The larger payment should be the payment due plus the missed payment with interest, or $500 + 500(1.025)^6 = 1,079.85.$

192.	Solution: A

Only Bond III can match the liability at time 3. The bond must mature for 1000. Only Bond II can match the liability at time 2. The face value and coupon must total 1000. If *X* is the face value, then *X* + 0.02*X* = 1000 and thus *X* = 980.39. Only Answer A has these to values. To check, Bond II also provides a coupon of 0.02(980.39) = 19.61 at time 1. Therefore, Bond I must provide the remaining 980.39 from its coupon and redemption value. If *Y* is the face value, then *Y* + 0.01*Y* = 980.39 for *Y* = 970.68.

193.	Solution: A

Cash flows (in thousands) are 12, 12, 12, 12, and 162. The first bond provides payments of 10, 10, 10, 10, and 110. Therefore, the second bond must provide 2, 2, 2, 2, and 52. This implies a coupon rate of 2/50 = 4% and a face amount of 50. Only Answers A and B provide these. At an 8% yield, the price of this bond is 42.015 (or 42,015).

194. Solution: C

Let i be the yield rate. Then,

$$3609.29 = 2000(2i)a_{\overline{30}|i} + 2250(1+i)^{-30}$$

$$= 4000[1-(1+i)^{-30}] + 2250(1+i)^{-30}$$

$$(1+i)^{-30} = (4000-3609.29)/(4000-2250) = 0.22326$$

$$i = 0.22326^{-1/30} - 1 = 0.051251.$$

Modified duration is Macaulay duration divided by one plus the yield rate: 14.14/1.051251 = 13.71.

195. Solution: A

The amount of the dividends does not matter, so they will be assumed to be 1. First, calculate the Macaulay duration. The present value of the dividends is $v^4 a_{\overline{\infty}|} = 1.1^{-4}(1/0.1) = 6.83013.$ The numerator is the present value of "payments" of 5, 6, 7, … starting five years from now. This can be decomposed as a level of annuity of 4 and an increasing annuity of 1, 2, 3, … . The present value is $v^4[4a_{\overline{\infty}|} + (Ia)_{\overline{\infty}|}] = v^4\left[\frac{4}{i} + \frac{1+i}{i^2}\right] = \frac{1}{1.1^4}\left[\frac{4}{0.1} + \frac{1.1}{0.1^2}\right] = 102.452.$ The Macaulay duration is 102.452/6.83013 = 15. The modified duration is 15/1.1 = 13.64.

196. Solution: A

Let r be the semiannual coupon rate. For the original bond,

$P = 1000ra_{\overline{6}|0.05} + 1000v^6 = 5075.692r + 746.215.$ For the modified bond,

$P - 49 = 1000ra_{\overline{12}|0.05} + 1000v^{12} = 8863.252r + 556.837.$ Subtracting the second equation from the first gives 49 = –3787.56r + 189.378. The solution is r = 0.037 and the coupon is 37.

197. Solution: A

Let $h(i)$ be the present value of the cash flows. For Redington immunization, the value of the function and its first derivative at 25% must be zero and the second derivative must be positive.

X is immunized because:

$$h(0.25) = 102,400 - 192,000/1.25 + 100,000/1.25^3 = 0$$

$$h'(0.25) = 192,000/1.25^2 - 100,000(3)/1.25^4 = 0$$

$$h''(0.25) = -192,000(2)/1.25^3 + 100,000(3)(4)/1.25^5 = 196,608 > 0$$

Y is not immunized because:

$$h(0.25) = 158,400 - 342,000/1.25 + 100,000/1.25^2 + 100,000/1.25^3 = 0$$

$$h'(0.25) = 342,000/1.25^2 - 100,000(2)/1.25^3 - 100,000(3)/1.25^4 = -6,400 \neq 0$$

Z is not immunized because

$$h(0.25) = -89,600 + 288,000/1.25 + 100,000/1.25^2 - 300,000/1.25^3 = 51,200 \neq 0$$

198. Solution: E
The bond sells at a premium, so the worst-case scenario is redemption at time six. Then,

$$1023 = 1000\frac{i}{0.96}a_{\overline{6}|i} + 1000(1+i)^{-6}$$

$$= \frac{1000}{0.96}[1-(1+i)^{-6}] + 1000(1+i)^{-6}$$

$$(1+i)^{-6} = 0.448$$

$$i = 0.448^{-1/6} - 1 = 0.1432$$

14.32%.

199. Solution: E
Answer E is false because the convexity of the assets must be greater than the convexity of the liabilities.

200. Solution: E
The accumulated value to time 4 is

$$\int_1^3 100e^{0.5t}e^{0.08(4-t)}dt = 100e^{0.32}\int_1^3 e^{0.42t}dt = 100e^{0.32}\left.\frac{e^{0.42t}}{0.42}\right|_1^3 = \frac{100e^{0.32}(e^{1.26}-e^{0.42})}{0.42} = 657.$$

201. Solution: E
The value at time 17 of the payments beginning at time 18 is

$$2500\left(\frac{1+k}{1.035}+\frac{(1+k)^2}{0.035^2}+\cdots\right) = 2500\frac{\frac{1+k}{1.035}}{1-\frac{1+k}{1.035}} = 2500\frac{1+k}{0.035-k}.$$ The total present value is

$$115,000 = 2500(1.035^{-2})a_{\overline{15}|0.035} + 2500v^{17}\frac{1+k}{0.035-k}$$

$$46(0.035-k) = 10.7516(0.035-k) + 0.55720(1+k)$$

$$k = \frac{1.61-0.37631-0.55720}{46-10.7516+0.55720} = 0.01889 = 1.89\%.$$

202. Solution: B

The initial payment, X is

$$200,000 = X\left(\frac{1}{1.03}+\frac{1.02}{1.03^2}+\cdots+\frac{1.02^{19}}{1.03^{20}}\right) = X\,\frac{1/1.03-1.02^{20}\,/1.03^{21}}{1-1.02/1.03} = 17.7267$$

$X = 11,282.42.$

The final payment is $11,282.42(1.02)^{19} = 16,436.36.$

203. Solution: E

The annual payment is $10,000/a_{\overline{10}|0.1} = 10,000/6.14457 = 1627.45.$ The balance at time 3 is $1627.45a_{\overline{7}|0.1} = 1627.45(4.8684) = 7923.08.$ With one-half year at simple interest, the balance at time 3.5 is 7923.08(1.05) = 8319.23.

204.

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205. Solution: E

Let x be the amount invested in Bond A and y the amount invested in Bond B. Then $2y$ is invested in Bond C. To match the present value of the assets and liabilities:

$$x+y+2y = 190,000(1.07)^{-20.5}$$

$$x+3y = 47,466.39.$$

To match the Macauley durations, $20.5 = \dfrac{10x+15y+30(2y)}{47,466.39}$. Then,

$$20.5(47,466.39) = 10(47,466.39-3y)+75y$$

$$y = \frac{20.5(47,366.39)-10(47,466.39)}{75-30} = 11,075.49$$ and

X = 47,466.39 – 3(11,075.49) = 14,239.92.

206. Solution: B

Let X be the price of Bond X. Then, for the two bonds:

$$X = 10,000(0.03)a_{\overline{2n}|0.035} + c(1.035)^{-2n}$$

$$X - 969.52 = 10,000(0.025)a_{\overline{2n}|0.035} + (c+50)(1.035)^{-2n}.$$

Subtracting the second equation from the first gives

$$969.52 = 50a_{\overline{2n}|0.035} - 50(1.035)^{-2n}$$

$$969.52 = \frac{50}{0.035}[1-(1.035)^{-2n}] - 50(1.035)^{-2n}$$

$$1.035^{-2n} = 459.05 / 1478.57 = 0.310469$$

$$n = -(0.5)\ln(0.310469)/\ln(1.035) = 17.$$

207. Solution: B

With simple interest, the deposit in Bank X earns 1000(0.07) = 70 in year 8.
With compound interest, the earning in Bank Y in year 8 is
$1000(1.0125)^{32} - 1000(1.0125)^{28} = 72.14.$ The absolute difference is 2.14.

208. Solution: E

Split this into two perpetuities. One starts at time 0.5 at 500 increasing by 10 every year. The other starts at time 1 at 500 with payments increasing by 10 every year. The semiannual interest rate is $1.075^{0.5}$-1=0.0368221. The present value of an increasing perpetuity immediate is found using the formula: $\frac{P}{i} + \frac{Q}{i^2}$, where P is the initial amount and Q is the increase amount.

The first perpetuity, valued at time 0: $\left(\frac{500}{0.075} + \frac{10}{0.075^2}\right)(1.0368221) = 8755.39$

The second perpetuity, valued at time 0: $\left(\frac{500}{0.075} + \frac{10}{0.075^2}\right) = 8444.44$

The total is 8755.39 + 8444.44 = 17,199.83.

209. Solution: B

$$5000(10) + 5000i(Is)_{\overline{10}|0.05} = 100,000$$

$$5000i\left\{\frac{\ddot{s}_{\overline{10}|0.05} - 10}{0.05}\right\} = 50,000$$

$$i\left\{\frac{13.206787 - 10}{0.05}\right\} = 10$$

$$i = 0.15592$$

210. Solution: C

Payment equals: $\left(10,000/a_{\overline{10}|0.08}\right)=1,490.29$

Accumulated total equals: $1,490.29s_{\overline{10}|0.10}=23,751.46$

$23,751.46-10,000=13,751.46$

211. Solution: D

Value of fund after 20 years: $500\ddot{s}_{\overline{120}|0.01}(1.01)^{120}=383,404.42$

$$383,404.42=\frac{X}{d}=\frac{X}{i}(1+i)=\frac{X}{0.01}(1.01)$$

$X=3796.08$

212. Solution: D

Using the retrospective method, $\text{OB}_{12}=12,000(1.10)^{12}-1000s_{\overline{12}|0.10}$

$=37,661.14-1000(21.38428)=16,276.86$

213. Solution: E

Using the BAII Plus calculator:
n = 12
PV = 911.37
PMT = -40
FV = -1000

CPT I/Y and you get 5.0% is the half-year rate: $\frac{i^{(2)}}{2}$. $(1+i)=(1+0.05)^2; i=0.1025$.

214. Solution: B

$$100\left(1-\frac{0.12}{4}\right)^{-4n}=200e^{0.08n}$$

$\left(-4n\right)\ln 0.97=\ln(200/100)+\left(0.08n\right)$

$0.0418n=\ln 2$

$n=16.57$

215. Solution: D

Equating present values gives

$$\frac{5000}{1.04^{10}} = X\left[\frac{1-\left(\frac{1.092}{1.04}\right)^{10}}{.04-.092}\right]$$

$$3377.82 = X(12.094127)$$

$$X = 279.29$$

216. Solution: E

$$P = C + (Fr - Ci)a_{\overline{n}|}$$

$$962.92 = C + (0.04C - 0.05C)a_{\overline{20}|0.05}$$

$$962.92 = C + (-0.01C)12.46221$$

$$962.92 = 0.875378C$$

$$C = 1100$$

The discount is 1100 – 962.92 = 137.08.

217. Solution: D

Equate the accumulated value of the deposits to the present value of the perpetuity:

$$(Is)_{\overline{10}|} = \frac{\ddot{s}_{\overline{10}|} - 10}{i} = \frac{10}{i}$$

$$\ddot{s}_{\overline{10}|} - 10 = 10$$

$$\ddot{s}_{\overline{10}|} = 20 \text{ (using the BAII Plus)} \Rightarrow i = 12.3\%$$

The PV of the perpetuity is 10/0.123 = 81.30.

218. Solution: C

$$1300a_{\overline{20}|0.08} - 4000 = Xa_{\overline{10}|0.08}$$

$$12,763.59 - 4000 = X(6.71008) = 1306.03$$

219. Solution: C

$$\bar{d}_L = \frac{2v^3 3 + 1v^2 2}{2v^3 + 1v^2} = \frac{6.16078}{2.32908} = 2.6452$$

$$\bar{d}_A = \frac{Xv^5 5 + Xv}{Xv^5 + Xv} = \frac{4.0137}{1.5300} = 2.6233$$

$$2.6452 - 2.6233 = 0.0219$$

220. Solution: D

The bank's accumulated value at the end of 30 years is:

$$\frac{100,000}{a_{\overline{30}|\,0.05}} s_{\overline{30}|\,0.04} = 364,841$$

$$100,000(1+i)^{30} = 364,841$$

$$i = 0.044$$

221. Solution: E

999.35 x 1.06 = 1059.31 will be available to make the first payment of 1000, leaving 59.31 to be reinvested at *X*%.

817.65 x 1.07^2 = 936.13 will be available from the second bond to make the second payment of 1000, leaving 63.87 to come from the reinvestment of 59.31.

X = 100(63.87 / 59.31 – 1) = 7.69.

222. Solution: E

$$(1+s_4)^4(1+{}_1f_4) = (1+s_5)^5$$

$$(1.09)^4(1+{}_1f_4) = (1.095)^5$$

$${}_1f_4 = 0.1152$$

223. Solution: C

$$D_{\text{mod}} = -\frac{\frac{d}{di}(1+i)^{-n}}{(1+i)^{-n}} = \frac{n(1+i)^{-n-1}}{(1+i)^{-n}} = n(1+i)^{-1}$$

224. Solution: E

The amount invested in three-year bond is equal to the PV of the third year's payout,

$$1000\Big/\left(1.1^3\right) = 751.31$$

The amount invested in one-year bond is equal to the PV of the first year's payout,

$$1000/1.08 = 925.93$$

$$925.93 - 751.31 = 174.61$$

225. Solution: B

$$\text{PV} = \frac{10}{1.04} + \frac{12}{(1.045)^2} + \frac{15}{(1.055)^3} + \frac{20}{(1.07)^4} = 9.615 + 10.989 + 12.774 + 15.258 = 48.64$$

226. Solution: D

Let i = yield rate, r = coupon rate (if any), F = face value, P = price, n = # of years.

For the first bond:

$$P = 0.8F = Fv^{36}$$

$$0.8 = v^{36}$$

$$i = 0.006218$$

For the second bond:

$$P = 0.8F = Fv^n + \frac{4}{9}(0.006218)Fa_{\overline{n}|0.006218}$$

$$0.8 = v^n + (0.0027634)a_{\overline{n}|0.006218}$$

Using the BAII Plus, where PV=0.8, I/Y=.6218, PMT=0.0027634, FV=1

CPT N results in n=72.

227. Solution: B

Since the bond has no coupons, the Macaulay duration is the same as the amount of time until maturity, namely 4 years.

Thus, the effective annual yield rate, y, is $\left(\frac{1200}{1000}\right)^{1/4} - 1 = 0.046635$.

The modified duration equals the Macaulay duration divided by (1 + y). Thus the modified duration is $\frac{4}{1.046635} = 3.82177$ years.

228. Solution: C

Using the general Macaulay duration formula: $\frac{\sum R_t v^t t}{\sum R_t v^t}$ where R is the cashflow:

Period	Cashflow	PV at 8%	Period ×PV
1	10	9.26	9.26
2	12	10.29	20.58
3	15	11.91	35.73
4	20	14.70	58.80
5	30	20.42	102.10
Total		66.58	226.47

Macaulay duration = 226.47/66.58 = 3.401472 years

229. Solution: C

When a company's position is Redington immunized, its position is definitely protected from sufficiently small changes in yield rate, in either direction. However, its position may or may not be protected from large changes in yield rate.

230. Solution: D

Amount of loan = L

Initial expected yield rate = 10.00%

Annual payment = $L / a_{\overline{10}|10\%}$

Accumulated value at time 10 = $(L / a_{\overline{10}|10\%})(s_{\overline{4}|10\%}1.07^6 + s_{\overline{6}|7\%})$

$$\text{Yield rate} = \left(\frac{\text{Accum Value}}{L}\right)^{1/10} - 1$$

$$= \left(\frac{s_{\overline{4}|10\%}1.07^6 + s_{\overline{6}|7\%}}{a_{\overline{10}|10\%}}\right)^{1/10} - 1$$

$$= \left(\frac{4.6410(1.5007) + 7.1533}{6.1446}\right)^{1/10} - 1$$

$$= 8.67\%$$

231. Solution: E

Let L = the loan amount. Note that $1+i = (1+j)^5$. The equation of value is

$$P \cdot a_{\overline{k}|\,i} = L = 120 \cdot a_{\overline{5k}|\,j}$$

so that

$$P = \frac{120 a_{\overline{5k}|\,j}}{a_{\overline{k}|\,i}}$$

$$= 120 \frac{1-(1+j)^{-5k}}{j} \frac{i}{1-(1+i)^{-k}}$$

$$= 120 \frac{1-(1+i)^{-k}}{j} \frac{i}{1-(1+i)^{-k}}$$

$$= 120 \frac{i}{j}$$

$$= 120 \frac{(1+j)^5 - 1}{j}$$

Next, using the fact that $0 < j < 0.04$, we get

$5 < \frac{(1+j)^5 - 1}{j} < 5.41633$ by plugging in a small value like 0.000000001 and 0.04 resulting in P equaling more than 600 but less than 650.

232. Solution: B

Using the retrospective method:

$$4000(1.05)^6 - 250s_{\overline{6}|0.05}$$

$$5360.38 - 1700.48 = 3659.90$$

233. Solution: D

Let t represent the number of years since the beginning of year 1. Since the annual effective interest rate is 3% in each of years 1 through 10, and 2% each year thereafter, the present value of an amount is calculated by multiplying it by a discounting factor of $\frac{1}{(1.03)^t}$ if $0 \le t \le 10$, and $\frac{1}{(1.03)^{10}(1.02)^{t-10}}$ if $t > 10$.

The balance is initially 0 (the account is new before the first deposit). Deposits of X are made at times t = 0, 1, 2, 3,…, 25, or equivalently at time $t = k-1$ for each whole number k from 1 to 26 inclusive.

For the final balance to become 0, a withdrawal of 100,000 at time t = 25 would be needed. Since the net present value of the cash flows (withdrawals minus deposits) must be zero, in a time period from a zero balance to another zero balance, we have

$$\frac{100,000}{(1.03)^{10}(1.02)^{25-10}} - X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}} - X\sum_{k=12}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-1-10}} = 0$$

$$\frac{100,000}{(1.03)^{10}(1.02)^{15}} = X\sum_{k=1}^{11}\frac{1}{(1.03)^{k-1}} + X\sum_{k=12}^{26}\frac{1}{(1.03)^{10}(1.02)^{k-11}}.$$

234. Solution: A

For the first bond: $P = (0.076)(6000)a_{\overline{20}|0.065} + 6000v^{20}$

P = 6727.22.

For the second bond: $6727.22 = (r)(7500)a_{\overline{20}|0.065} + 7500v^{20}$

$7500r$ = 417.37, so r = 0.0556.

235. Solution: E

$$P_X = Fra_{\overline{40}|} + Cv^{40} = 75a_{\overline{40}|} + Cv^{40}$$

$$P_Y = P_X - 257.18 = 60a_{\overline{40}|} + (C+K)v^{40}$$

$$P_X - P_Y = 257.18 = 15a_{\overline{40}|} - Kv^{40}$$

$$Kv^{40} = 15a_{\overline{40}|} - 257.18$$

$$K(0.252572) = 320.33 - 257.18$$

$$K = 250.01$$

236. Solution: C

The PV of the first twenty payments is: $14,000\left(\dfrac{1-\left(\dfrac{1.04}{1.10}\right)^{20}}{0.10-0.04}\right) = 157,337.48$

The PV of the remaining payments starting at time 21 is:

$$14,000(1.04)^{19}(1.01)\left(\frac{1}{0.10-0.01}\right)(1.10)^{-20} = 49,202.44$$

Total equals 206,539.92.

237. Solution: B

$$16,000 = 1,000\left(\frac{1}{0.057-(-r/100)}\right)$$

$$16 = \frac{1}{0.057 + r/100}$$

$$0.057 + r/100 = \frac{1}{16}$$

$$r = 0.55$$

238. Solution: E

Let j equal the five-year interest rate.

$$(1+j) = (1.09)^5$$

$$j = 0.538624$$

$$PV = \frac{2}{0.538624} + \frac{10}{(0.538624)^2} = 38.18$$

239. Solution: A

From the first bond: $P = 25a_{\overline{8}|0.03} + Cv^8$

From the second bond: $0.93P = 25a_{\overline{4}|0.03} + Cv^4$

Multiply the first equation by 0.93 and plug into the second equation:

$0.93(25)a_{\overline{8}|0.03} + 0.93C = 25a_{\overline{4}|0.03} + Cv^4$

$163.2078 + 0.73415C = 92.9275 + 0.88849C$

$70.2804 = 0.15434C$

$C = 455.37$

240. Solution: D

The PV of the liability is $\frac{600,000}{1.046^2} = 548,387.92$ and its Macaulay duration is 2.

Then, equating present values:

$\frac{x}{1.046} + \frac{y}{1.046^4} = 548,387.92$

And equating durations:

$\frac{(x/1.046)}{548,387.92}(1) + \frac{(y/1.046^4)}{548,387.92}(4) = 2$

Solving the system of equations results in x = 382,409

241. Solution: E

Solution: $4000(1.04^3) = 1400[1 + (1+i) + (1+i)^2]$, or i solves the quadratic equation:
$i^2 + 3i - 0.2139 = 0$. Thus, i = 6.97% because the other root is negative.

242. Solution: B

$100,000 = Xa_{\overline{6}|0.01}v_{0.01}^4 + \frac{X}{0.05}v_{0.01}^{10}$

$100,000 = X(5.79548)(0.96098) + X(20)(0.90529)$

$100,000 = X(23.6751)$

$X = 4223.85$

243. Solution: D

Using BA II Plus:

$$60,000 = Xa_{\overline{180}|0.075/12}$$

$$X = 556.21$$

$$49,893 = 556.21a_{\overline{m}|0.075/12}$$

$$m = 132$$

So, (180 – 132) = 48 payments have been made so far:

$$49,893 = 556.21\,a_{\overline{n}|0.6/12}$$

$$n = 119.3$$

Use 120 future payments including the smaller one.

48+120=168.

244. Solution: C

$$2000 = 110a_{\overline{20}|0.10} + X\left[\frac{a_{\overline{20}|0.10} - 20v^{20}}{0.10}\right]$$

$$2000 = 110(8.51356) + X(55.40691)$$

$$1063.51 = X(55.40691)$$

$$X = 19.1945$$

245. Solution: E

$$PV = 100\left[\frac{1-\left(\frac{1.15}{1.05}\right)^{10}}{0.05-0.15}\right] = 1483.62$$

246. Solution: D

$$OB_{n-1} = Ra_{\overline{1}|} = 1144.5 = Rv$$

$$P_{n-4} = Rv^{n-(n-4)+1} = Rv^5 = 865$$

$$\frac{Rv^5}{Rv} = v^4 = \frac{865}{1144.5} = 0.75579$$

$$v^4 = 0.75579, \quad v = 0.93240, \quad i = 0.07251$$

$$I_1 = X(0.07251) = 797.50$$

$$X = 10,999.02$$

247. Solution: C

Given that the problem states that the inequality is true for all interest rates from 0% to 10% and all values of Y, it is sufficient to determine it for one set of values. Select i = 7% and Y = 121. Then,

$$Q = 121/(1+3(0.07)) = 100$$

$$R = 121/(1.07)^3 = 98/77$$

$$S = 121(1-0.07(3)) = 95.59$$

$$T = 121(0.93)^3 = 97.33$$

Hence,

S < T < R < Q

248. Solution: C

The yield rate on Kate's bond is

$$(1000-100) = 25a_{\overline{10}|\frac{i^{(2)}}{2}} + 1000v^{10}$$

$$\frac{i^{(2)}}{2} = 0.0371551$$

The discount on Wallace's bond is

$$(1000-D) = 25a_{\overline{8}|0.05} + 1000v^{8}$$

$$1000-D = 838.42, \qquad D = 161.58$$

The book value of Kate's bond at time 1 is

$$B = 25a_{\overline{8}|0.0371551} + 1000v^{8}$$

$$B = 917.19$$

The difference is $B - D$ =917.19 – 161.58 = 755.61

249. Solution: B

$P_L = 1000(1+i)^{-2} + 300(1+i)^{-4}$

$P_L = 1153.84$

$P_L' = -2000(1+i)^{-3} - 1200(1+i)^{-5}$

$P_L' = -2667.91$

$P_A = X(1+i)^{-1} + Y(1+i)^{-3}$

$1153.84 = 0.95238X + 0.86384Y$

$P_L' = -X(1+i)^{-2} - 3Y(1+i)^{-4}$

$-2667.91 = -0.90703X - 2.46811Y$

So, we have two equations and two unknowns. Solving simultaneously, we get:

$Y = 953.57, \quad X = 346.61, \quad \frac{Y}{X} = 2.75.$

250. Solution: C

$1000 = Pa_{\overline{20}|0.03}$

$P = 67.22$

$67.22(20) = 1344.31$

$1344.31 = 1000 + i(1000 + 950 + 900 + \cdots + 50)$

$344.31 = i(10,500)$

$i = 0.03279$

251. Solution: B

$\frac{i^{(12)}}{12} = 1.08^{1/12} - 1 = 0.006434$

$600 = P(1/1.006434)^{120-6+1}$

$P = 1254.47$

$P_{24} = 1254.47v^{120-24+1} = 673.42$

252. Solution: B

$e^{\int_0^{20} \frac{2}{1+2t}dt} = e^{\ln(1+2t)\big|_0^{20}} = 41$

$41 = (1+i)^{20}$

$i = 0.204035$

$(1+0.204035)^5 = 2.53$

253. Solution: B

$$1000 \cdot a_{\overline{20}|i} = \frac{600}{i} + \frac{600v^{10}}{i}$$

$$5\left(\frac{1-v^{20}}{i}\right) = \frac{3}{i}\left(1+v^{10}\right)$$

$$5-5v^{20} = 3+3v^{10}$$

$$0 = 5v^{20} + 3v^{10} - 2$$

$$\text{Let } x = v^{10} = \frac{-3 \pm \sqrt{9+4(5)(2)}}{2(5)} = \frac{-3 \pm 7}{10} = 0.4 \Rightarrow i = 9.59582\%$$

$$X = \frac{600}{0.0959582}\left(1+0.4\right) = 8753.8$$

254. Solution: B

$$5000 = \frac{150}{i} + \frac{10}{i^2}$$

$$5000i^2 - 150i - 10 = 0$$

$$i = \frac{150 \pm \sqrt{(-150)^2 - 4(5000)(-10)}}{10,000}$$

$$i = 0.06217$$

255. Solution: D

If paid in one lump sum, the total interest paid is $X(1.05^{20} - 1) = 1.65330X$.

With level payments for 10 years, the total interest paid is

$$10\left(\frac{X}{a_{\overline{10}|0.05}}\right) - X = 0.29505X.$$

Then,

$$1.65330X = 1000 + 0.29505X$$

$$X = 736.24.$$

256. Solution: B

$P_t = I_t$

$v^{20-t+1} = (1 - v^{20-t+1})$

$2v^{20-t+1} = 1$

$v^{21-t} = 0.5$

$1.05^{21-t} = 2$

$(21-t)\ln(1.05) = \ln 2$

$t = 7$

257. Solution: B

$10,000 + 10,815v^2 = 20,800v$

$$v = \frac{20,800 \pm \sqrt{20,800^2 - 4(10,815)(10,000)}}{21,630}$$

$v = 0.970873 \text{ or } 0.952381$

$i = 0.03 \text{ or } 0.05$

$|0.03 - 0.05| = 0.02$

258. Solution: A

$P_A = 45a_{\overline{30|}0.042} + 1200v^{30}$

$P_A = 1108.85$

$1108.85 = 20a_{\overline{4n|}0.021} + 1376.69v^{4n}$

Using the BA II Plus:
PV = 1108.85
PMT = 20
FV = 1376.69
I/Y = 2.1
Solve for 4*n* and get 4*n* = 48, *n* = 12.
Or solve the equation to get $v^{4n} = 0.36876$ and then solve for *n*.

259. Solution: C

Statement I should have an $\ddot{s}_{\overline{60|}}$ on the left.

Statement II has an annuity-due rather than an annuity-immediate on the right.
Statement III is correct.

260. Solution: D

$$\frac{15(961.54)(1)+20(966.14)(2)+30(878.41)(3)}{15(961.54)+20(966.14)+30(878.41)}=2.198495$$

261. Solution: B

$$1,000,000(1.10)^{17}-Ps_{\overline{18}|0.10}=1,000,000$$

$$5,054,470.285-P(45.59917)=1,000,000$$

$$4,054,470.285=P(45.59917)$$

$$P=88,915.43$$

262. Solution: D

Using time 5 as the first reference point, then bringing that value back to time 0:

$$v^5\left[500\ddot{a}_{\overline{5}|i}+500\left(I\ddot{a}\right)_{\overline{5}|i}\right]$$

This combines a five-year level annuity-due of 500 plus an increasing annuity-due starting with 500 and increasing by 500.

263. Solution: B

Let face amount equal 1.

$$1.61=2.25i\left(\frac{1-v^{18}}{i}\right)+v^{18}$$

$$1.61=2.25(1-v^{18})+v^{18}$$

$$1.25v^{18}=0.64$$

$$v=0.96342$$

$$1.45=2.25i\left(\frac{1-v^{n}}{i}\right)+v^{n}$$

$$1.45=2.25\left(1-v^{n}\right)+v^{n}$$

$$1.25v^{n}=0.8$$

$$v^{n}=0.64$$

$$n\ln 0.963492=\ln 0.64$$

$$n=12$$

264. Solution: C

$450 = Xa_{\overline{10}|0.10} + 1000v^{10}$

Using BA II Plus calculator:

$X = 10.50$

$1.10 = (1+j)^2,\ j = 0.048881$

$P = 5.25a_{\overline{20}|0.048881} + 1000v^{20}$

$P = 451.64$

265. Solution: E

Let I be the amount of interest in the first month.

$P_0 - m + I = P_1, I = P_1 - (P_0 - m)$

In the first month, the interest $P_1 - (P_0 - m)$ was charged on a principal of P_0, so the effective monthly interest rate (expressed as a decimal) of the first loan is $\frac{P_1 - (P_0 - m)}{P_0} = \frac{P_1 - P_0 + m}{P_0}$.

The nominal annual interest rate (expressed as a decimal) for both loans is therefore $12\left(\frac{P_1 - P_0 + m}{P_0}\right)$, so the effective daily rate (expressed as a decimal) for the second loan is $\frac{12}{365}\left(\frac{P_1 - P_0 + m}{P_0}\right)$.

Finally, the effective monthly rate (expressed as a decimal) for the second loan is

$$\left[1 + \frac{12}{365}\left(\frac{P_1 - P_0 + m}{P_0}\right)\right]^{365/12} - 1.$$

266. Solution: D.

$P(0,m) = (1+i)^{-m}$

$P(0,n) = (1+i)^{-n}$

$$X = \frac{(1+i)^{-m}}{(1+i)^{-n}} = (1+i)^{-m+n}$$

$$X = \frac{P(0,m)}{P(0,n)}$$

267. Solution: A

$OB_{180} = 2000a_{\overline{120}|0.005} = 180{,}146.91$

$180{,}146.91 = Pa_{\overline{180}|0.005}$

$P = 1520.18$

$L = 1520.18a_{\overline{360}|0.005} = 253{,}553.61$

268. Solution: E

The conditions
1) assets and liabilities have equal present values and equal modified durations, and
2) the convexity of its assets exceeds the convexity of its liabilities
are precisely what is required for Redington immunization.

269. Solution: E

$90.17 = 4a_{\overline{6}|j} + Xv^6$

$132.47 = 4a_{\overline{6}|j} + 1.6Xv^6$

Multiply the first equation by 1.6:

$144.272 = 6.4a_{\overline{6}|j} + 1.6Xv^6$

Subtract the second equation:

$11.802 = 2.4a_{\overline{6}|j}$

Use annuity calculation on BA II Plus:

$j = 6\% = \frac{i^{(2)}}{2}, i^{(2)} = 12\%$

270. Solution: B

$9297 = Pa_{\overline{2}|0.05}$

$P = 5000$

$\frac{5000}{1.05^2} = 4535.12$

271. Solution: B

Using the BA II Plus calculator:

$$1.04=\left(1+\frac{i^{(2)}}{2}\right)^2$$

$$\frac{i^{(2)}}{2}=0.019804$$

$$P_A=25a_{\overline{10}|0.019804}+1000v^{10}$$

$$P_A=1046.72$$

$$1046.72-100=30a_{\overline{5}|j}+1000v_j^{\ 5}$$

$$j=4.2036\%$$

272. Solution: E

Value at time 5 years:

$$100,000(1.02)^{20}-2500s_{\overline{20}|0.02}$$

$$87,851.32=5000a_{\overline{m}|0.02}$$

m = 21.86, using the BA II Plus. Since we want a balloon payment, use m = 21.

$$87,851.32=5000a_{\overline{21}|0.02}+Bv^{21}$$

B = 4236.70, so balloon equals 5000 + 4236.70 = 9236.70

273. Solution: B

$$900=1000ra_{\overline{40}|0.05}+900v^{40}$$

Using BA II Plus:

$$1000r=45$$

$$P=45a_{\overline{20}|0.04}+900v^{20}$$

$$P=1022.31$$

$$1022.31=Fra_{\overline{20}|0.04}+1100v^{20}$$

$$Fr=38.28$$

274. Solution: C

Let Y indicate the nominal value of the two-year bond, then:

$$9465=\frac{0.05Y}{1.08}+\frac{1.05Y}{1.08^2}, \text{ so } Y=10,000.$$

Thus, the amount of liability at the end of the second year is 10,500.
Hence, the liability at the end of the first year is:

$$\frac{10,500}{2}=5250.$$

So, the amount invested in the one-year bond is:

$$\frac{5250-10,000(0.05)}{1.06}=4481.$$

275. Solution: C

Macaulay duration of the liability is 3. Asset duration must equal 3.
Let P_1 and P_4 be the present values of the two assets.

$$\frac{P_1 \bullet 1+P_4 \bullet 4}{P_1+P_4}=3, \text{ then } P_4=2P_1$$

$$P_1=\frac{20,000}{1+i}, P_4=\frac{50,000}{(1+i)^4}, \ \frac{P_1}{P_4}=\frac{P_1}{2P_1}=\frac{1}{2}=\frac{\frac{20,000}{1+i}}{\frac{50,000}{(1+i)^4}}: \ 1=0.8(1+i)^3:$$

$$(1+i)^3=1.25; \ (1+i)=1.077217$$

PV of assets must equal PV of liabilities. So, PV of assets equals:

$$P_1+2P_1=3P_1=3\frac{20,000}{1.077217}=55,699.07.$$

Amount of liability equals: $55,699.07(1.077217)^3 = 69,623.83$.

276. Solution: D

$$20a_{\overline{10}|0.06}+v^{10}\left(Da\right)_{\overline{19}|0.06}+v^{29}\left(\frac{1}{0.06}\right)$$

$$=147.20+v^{10}130.70+v^{29}16.67$$

$$=147.20+72.98+3.08=223.26$$

277. Solution: C

$$2000 = X(1.08)^{14} + X(1.03)(1.08)^{13} + X(1.03)^2(1.08)^{12} + \cdots + X(1.03)^{14}$$
$$= X\frac{1.08^{14} - 1.03^{15}/1.08}{1 - 1.03/1.08}$$
$$= 32.284X$$
$$X = 61.95$$

278. Solution: E

The present value of annuity X is $1.0331a_{\overline{10|}} = 1.0331\frac{1-v^{10}}{i}$.

The present value of annuity Y is $P(v^2 + \cdots + v^{10}) = P\frac{v^2 - v^{12}}{1-v^2} = P\frac{1-v^{10}}{(1+i)^2 - 1}$.

Equating the present values and solving,

$$P = 1.0331\frac{(1+i)^2 - 1}{i} = 1.0331s_{\overline{2|}} = 1.0331(2.075) = 2.14.$$

279. Solution: D

$$1.12 = \left[1 + \frac{i^{(12)}}{12}\right]^{12}$$
$$\frac{i^{(12)}}{12} = 0.00948879$$
$$900 = P\left(\frac{1}{1.00948879}\right)^{60-3+1}$$
$$P = 1556.43$$
$$P_{33} = 1556.43\left(\frac{1}{1.00948879}\right)^{60-33+1} = 1194.78$$

280. Solution: C

$$X = 5000\left[10 + 0.08\left(Is\right)_{\overline{10|}0.05}\right]$$
$$X = 5000\left[10 + 0.08\left(\frac{\ddot{s}_{\overline{10|}0.05} - 10}{0.05}\right)\right]$$
$$X = 75,654.30$$

281. Solution: B

$BV_{12} = 38a_{\overline{8}|0.03} + 1000v^8$

$= 1056.16$

282. Solution: D

$I_3 + I_4 = (1+i)^2(P_1 + P_2)$

$(1-v^2) + (1-v) = (1+i)^2(v^4 + v^3)$

$(1-v^2 + 1 - v) = (1+i)^2(v^4 + v^3)$

$v^2(1-v^2 + 1 - v) = (v^4 + v^3)$

$2v^2 - v^2(v^2 + v) = v^2(v^2 + v)$

$2v^2 = 2v^2(v^2 + v)$

$1 = v^2 + v$

$v^2 + v - 1 = 0$

$v = 0.61803$

$1 + i = 1.61803$

$i = 0.61803$

283. Solution: C

$500(1+X)^2 = 600$

$X = 0.095445$

$100(1+Y)^2 + 100(1+Y) = Z$

$600(1.10)^2 + 100(1.10) = 600 + Z$

$Z = 236$

$100(1+Y)^2 + 100(1+Y) = 236$

$(1+Y)^2 + (1+Y) - 2.36 = 0$

$(1+Y) = \frac{-1 \pm \sqrt{1 + 9.44}}{2}$

$Y = 0.115549$

$Y - X = 0.0201$

284. Solution: B

Assuming the loss of interest is the loss of the last 9 months interest:

$(1.08)^{3-0.75} = (1.08)^{2.25} = 1.189 = (1+j)^3$

$j = 5.94\%$

285. Solution: C

$$1000\,\ddot{s}_{\overline{20|}0.0925} = 57,485.26$$

$$500\,\ddot{a}_{\overline{360|}\frac{10}{12}\%} = 57,450.21$$

$$57,485.26 - 57,450.21 = 35.05$$

286. Solution: B

Let j equal the quarterly rate and let k equal the four-year rate.

$$300 = \frac{1}{j}(1+j)$$

$$j = \frac{1}{299}$$

$$\left(1+\frac{1}{299}\right)^4 = (1+k)^{\frac{1}{4}}$$

$$k = 0.054875$$

$$300 = \frac{X(1+k)}{k}$$

$$X = 15.61$$

287. Solution: A

The outstanding loan balance at any point in time is equal to the present value (at that same point in time) of the remaining installment payments.

$$B_{40} = 1000(0.98)^{40}\left[\frac{1-\left(\frac{0.98}{1.0075}\right)^{20}}{0.0075+0.02}\right] = 6889.11$$

288. Solution: C

$$OB_5 = 400a_{\overline{10|}0.06} - 1000 = 1944.03$$

$$1944.03 = Xa_{\overline{5|}}0.06$$

$$X = 461.51$$

289. Solution: C

Option 1

$$1500 = Pa_{\overline{20}|0.05}$$

$$P = 120.3639$$

$$\text{Total Payment} = 120.3639(20) = 2407.28$$

Option 2

$$\begin{aligned}\text{Total Payment} &= 75*20 + 1500i + \left(1500-75\right)i + \\ &\quad \left(1500-2*75\right)\cdot i + \cdots + (1500-19*75)i \\ &= 1500 + 1500i(20) - 75i\cdot\sum_{k=0}^{19} k \\ &= 1500 + 30,000i - 75i\cdot\left(\frac{19\left(20\right)}{2}\right) \\ &= 1500 + 15,750i\end{aligned}$$

$$2407.28 = 1500 + 15,750i$$

$$i = 0.0576$$

290. Solution: C

$$MV_5 = 1,100,000 = 1,000,000v^{15} + 40,000a_{\overline{15}|i^{(2)}/2}$$

$$\frac{i^{(2)}}{2} = .03153$$

$$i^{(2)} = 0.06306$$

291. Solution: B

$$150 = 100v + 80v^2$$

$$80v^2 + 100v - 150 = 0$$

$$v = \frac{-100 \pm \sqrt{100^2 - 4(80)(-150)}}{160}$$

$$v = 0.880199$$

$$i = 0.136106$$

$$1.136106 = \left[1 + \frac{i^{(4)}}{4}\right]^4$$

$$i^{(4)} = 0.129664$$

292. Solution: B

Since the bond is bought at a premium and redemption will occur to the investor's greatest disadvantage, assume the bond is called at the earliest possible redemption date, or simultaneous with the coupon payment occurring at the end of the 15th year. The cash flows then become a bond paying semiannual coupons of 40 for 15 years and returning 1000 at the end of the 15th year. At a yield of 7% effective annually:

$$1.07 = \left[1 + \frac{i^{(2)}}{2}\right]^2$$

$$\frac{i^{(2)}}{2} = 0.034408$$

$$P = 40a_{\overline{30|}} + 1000v^{30}$$

$$P = 1103.61$$

293. Solution: A

$$-\frac{P'(i)}{P(i)} = \frac{1500(1+i)^{-4} - 4000(1+i)^{-5}}{100 + 500(1+i)^{-3} - 1000(1+i)^{-4}}$$

294. Solution: E

$$1.05^4 = 1.04^2(1+f)^2$$

$$f = 0.0601$$

295. Solution: B

Besides present values of assets and liabilities matching, 1) their modified durations must also match, and 2) the convexity of the assets must exceed the convexity of the liabilities, in order for the company's position to be immunized against small changes in interest rate. Only company V satisfies all these conditions.

296. Solution: B

Let j be the monthly interest rate.

$$50,000(1+j) - 800 = 49,800$$

$$j = 0.012$$

$$50,000 = 800a_{\overline{n|}0.012}$$

$$n = 116.22$$

Drop payment at payment number 117.

297. Solution: C

$$0.75F = \frac{21}{37} iFa_{\overline{27}|i} + Fv^{27}$$

$$0.75 = \frac{21}{37}(1 - v^{27}) + v^{27}$$

$$0.18243 = \frac{16}{37} v^{27}$$

$$i = 0.03248$$

$$0.75F = Fv^n$$

$$(1+i)^n = \frac{4}{3}$$

$$n = 9$$

298. Solution: A

$$\left[\frac{1 - \left(\frac{1.04}{1.12} \right)^5}{0.12 - 0.04} \right] + \left[\left(\frac{1.04}{1.12} \right)^5 \left(\frac{1}{0.12 - 0.08} \right) \right] = 3.87 + 17.26 = 21.13$$

299. Solution: B

Let x be the amount invested in the one-year bond and y the amount invested in the four-year bond. First match the present value of assets and liabilities:

$$PV_A = PV_L$$

$$x + y = 2000$$

Second, the durations of assets and liabilities should also match:

$$D_A = \frac{1x + 4y}{x + y}$$

$$D_A = \frac{1x + 4(2000 - x)}{2000} = D_L = 3$$

$$x = 666.67$$

Convexity of the assets is:

$$\frac{666.67(1^2) + 1333.33(4^2)}{2000} = 11$$

Convexity of the liability is: $3^2 = 9$.

Convexity of assets is greater than convexity of liabilities so Reddington immunization is met.

300. Solution: C

$5{,}000{,}000 = Xa_{\overline{40}|0.02}$

$X = 182{,}778.74$

$OB_{20} = 5{,}000{,}000(1.02)^{20} - 183{,}000 s_{\overline{20}|0.02}$

$OB_{20} = 2{,}983{,}318.31$

$2{,}983{,}318.31 = 200{,}000 a_{\overline{n}|0.02}$

$n = 17.89$

20 original payments plus 18 with the drop payment equals 38 total payments.

301. Solution: A

Since the coupon rate per coupon payment period 4% is greater than the effective rate of interest per coupon payment period 2.9563%, it is to the disadvantage of the bond holder to have the bond redeemed at an early date. Hence, we only need to calculate the present value of such a bond at the worst-case scenario, which is that the bond is called at the end of the 5th year.

$P = 4a_{\overline{10}|0.029563} + 100v^{10}$

$P = 108.92$

302. Solution: D

$$FV = 100\left[\frac{1-\left(\frac{0.9}{1.03}\right)^{20}}{0.03+0.10}\right](1.03)^{20}$$

$FV = 1295.80$

Alternatively,

$$FV = 100[(1.03)^{19} + 0.9(1.03)^{18} + \cdots + (0.9)^{19}(1.03)^{0}] = 100\frac{(1.03)^{19} - (0.9)^{20}(1.03)^{-1}}{1-0.9(1.03)^{-1}} = 1205.80.$$

303. Solution: D

First, find $a(t)$

$$a(t) = \exp\left[\int_0^t \delta_r dr\right] = \exp\left[\int_0^t \frac{r}{50} dr\right] = \exp\left[t^2/100\right]$$

The balance in the account at time 10 is: $300a(10) + X\dfrac{a(10)}{a(4)} = 815.48 + 2.31637X$

The total interest earned from $t = 0$ *to* $t = 10$ is:

$815.48 + 2.31637X - (300 + X) = 4X \Rightarrow X = 192.08.$

304. Solution: A

Since Asset Y provides a cash flow at the same time that the liability is due (t = 4), we can apply its 250,000 value to reducing the liability amount from 750,000 to 500,000. Then, we can establish the following two equations, both using t = 4 as the reference point for all cash flows.

$$500,000 = A_X v^{-2} + A_Z v = A_X (0.95)^{-2} + A_Z (0.95)$$

Second, taking the derivative (with respect to v) of both sides of the first equation, we have:

$$0 = -2A_X v^{-3} + A_Z = -2A_X (0.95)^{-3} + A_Z$$

$$0 = -2A_X (0.95)^{-2} + A_Z (0.95)$$

Then, subtracting the second equation from the first equation yields:

$$500,000 = 3A_X (0.95)^{-2}$$

$$A_X = 150,416.67$$

305. Solution: D

$$1.06 = \left[1 + \frac{i^{(4)}}{4}\right]^4$$

$$\frac{i^{(4)}}{4} = 0.014674$$

$$50,000(1.014674)^5 = 2,125 a_{\overline{n}|}$$

$$n = 31.86$$

There will be 31 payments of 2125.

306. Solution: A

The PV and duration of the liability payments using 7% rate are

$PV = 1,750,000 v^{12} = 777,021$ and duration 12.

The amount invested in the 5-year bond is $\dfrac{242,180}{1.07^5} = 172,671$,

Thus, the amount invested in the 14 year bond is 777,021-172,671=604,350.

The maturity value of the 14-year bond is $604,350(1.07)^{14} = 1,558,337$.

The surplus if the interest rate moves to 4% is:

$$PV_A - PV_L = \left(\frac{242,180}{1.04^5} + \frac{1,558,337}{1.04^{14}}\right) - \frac{1,750,000}{1.04^{12}} = 5,910$$

.

307. Solution: A

Let $h(i) = PV_A(i) - PV_L(i)$.

Full immunization of a single liability requires both equations:

$$h(i) = 0, h'(i) = 0$$

$$A_1 v + A_3 v^3 - 20,000v^2 = 0$$
$$A_1 v^2 + 3A_3 v^4 - 40,000v^3 = 0$$
$$v = \frac{1}{1.055}.$$

Solve these two equations in two unknowns to get A_1 = 9478.67

308. Solution: D

Let F_1, F_2, F_3 be the redemption amounts of each bond for purchase.

To exactly match the liabilities with cash income:

$$1000 = F_3$$
$$F_3 = 1000$$
$$1000 = F_2(1.02)$$
$$F_2 = 980.39$$
$$1000 - 980.39(0.02) = F_1(1.01)$$
$$F_1 = 970.69$$

The total purchase price is

$$\frac{970.69(1.01)}{1.14} + \frac{980.39(0.02)}{1.15} + \frac{980.39(1.02)}{1.15^2} + \frac{1000}{1.18^3} = 2241.82.$$

309. Solution: B

The PV of the annuity following the 11th payment is:

$10a_{\overline{9}|0.06} = 68.0169.$

The effective semi-annual rate is $j = \frac{i^{(2)}}{2} = 1.06^{1/2} - 1 = 0.02956301.$

Next,

$$PV = K\left[\frac{1}{0.02956301 - 0.005}\right] = 68.0169$$
$$K = 1.67$$

310. Solution: B

Split this into three perpetuities with payments 3 years apart. Find the three-year interest rate:

$1.125 = (1+j)^{\frac{1}{3}}$, $j = 0.423828$.

The present value of the three perpetuities, starting at times 0, 1, and 2 is:

$$\frac{100(1.423828)}{0.423828} + \frac{X(1.423828)}{0.423828(1.125)} + \frac{100(1.423828)}{0.423828(1.125)^2} = 9450$$

$X = 2963.19$.

311. Solution: E

Note that had the borrower 1) charged X at the end of month 0, and 2) paid off the remaining 3000 at the beginning of month 16, then the initial and final balances would have become 0. In that situation, the present value of the amounts charged to the credit card, minus the present values of the payments, would have been 0 in the 15-month period.

The amounts charged to the card were 79.99 at each of times $t = \frac{n-0.5}{12}$, for each whole number of n from 1 to 15.

The monthly payments were 250 at each of times $t = \frac{n}{12}$, for each whole number n from 1 to 15 inclusive.

Then to make the final balance 0, the final (additional) payment would have been 3000 at time $t = \frac{15}{12} = \frac{5}{4}$.

Therefore, we have

$$X + \sum_{n=1}^{15} \frac{79.99}{(1.168)^{\frac{n-0.5}{12}}} - \sum_{n=1}^{15} \frac{250}{(1.168)^{\frac{n}{12}}} - \frac{3000}{(1.168)^{\frac{5}{4}}} = 0$$

$$X + \sum_{n=1}^{15} \frac{79.99}{(1.168)^{\frac{n-0.5}{12}}} = \frac{3000}{(1.168)^{\frac{5}{4}}} + \sum_{n=1}^{15} \frac{250}{(1.168)^{\frac{n}{12}}}.$$

312. Solution: A

The accumulated value is $a(t) = \exp\left[\int_0^t 0.03 + 0.005r\,dr\right] = \exp[0.03t + 0.0025t^2]$, The account balance is $100a(2) + 100\frac{a(2)}{a(1)} = 100(1.0725) + 100\frac{1.0725}{1.0330} = 211.07$.

313. Solution: E

Break this into two parts – the first 30 increasing payments and the remaining level perpetuity.

$$350\left[\frac{1-\left(\frac{1.03}{1.07}\right)^{30}}{0.07-0.03}\right](1.07)+v^{29}\left(\frac{1}{0.07}\right)350(1.03)^{29}=8033.38$$

314. Solution: E

Let n index the payment times in months. Then for a payment at time n, the discount factor is $\frac{1}{(1.045)^{n/6}}$. The end-of-month payment at month n is be $500+(n-1)X$.

Therefore, we have $30,000=\sum_{n=1}^{60}\frac{500+(n-1)X}{(1.045)^{n/6}}$.

315. Solution: A

Let i be the yield rate, v = 1/(1 + i), and let n be the term. For Bond A, $20,000v^n=10,000$ and so $v^n=0.5$. For Bond B, $10,835.58(v^n+0.04a_{\overline{n}|i})=10,000$ and so

$$a_{\overline{n}|i}=\left(\frac{10,000}{10,835.58}-0.5\right)/\,0.04=10.5721.$$

For Bond C, $10,000=X(v^n+0.03a_{\overline{n}|i})=0.81716X\Rightarrow X=12,237.51.$

316. Solution: C

$$50,000=ka_{\overline{10}|0.052}+v^{10}\left[(k+200)\left(\frac{1}{0.052}\right)+200\left(\frac{1}{0.052^2}\right)\right]$$

$$50,000=k(7.647284)+(0.602341)\left[19.230769k+3846.153846+73964.50\right]$$

$$50,000=19.230765k+46,868.54$$

$$k=162.83$$

317. Solution: B

Let r_A represent the coupon rate of bond A. The coupon rate of bond B is then $r_A + 0.005$.

From the given information,

$$3000 = 1000\left[\frac{1}{(1.07)^{30}} + r_A a_{\overline{30}|0.07} + \frac{1}{(1.07)^{30}} + \left(r_A + 0.005\right) a_{\overline{30}|0.07}\right]$$

$$3 = \frac{2}{(1.07)^{30}} + 2r_A a_{\overline{30}|0.07} + 0.005 a_{\overline{30}|0.07}$$

$$3 = 0.26273 + 24.81808 r_A + 0.06205$$

$$r_A = \frac{3 - 0.26273 - 0.06205}{24.81808} = 0.1078 = 10.78\%$$

.

318. Solution: C

$$7 = P_{12} = C(i-g)v^{20-12+1}$$

$$7 = \left[Ci - Cg\right]\left(\frac{1}{0.0375}\right)^9$$

$$9.7497 = Ci - Fr$$

$$9.7497 = C(0.0375) - 35$$

$$C = 1193.33$$

319. Solution: C

$$P_A = 350 a_{\overline{40}|0.025} + 1.1 P_A v^{40}$$

$$P_A = 8785.97 + P_A(0.409674)$$

$$P_A = 14{,}883.24$$

$$R = 1.1 P_A = 16{,}371.57$$

$$14{,}724.91 = 350 a_{\overline{m}|0.025} + 16{,}371.57 v^m$$

$$m = 48$$

$$n = 24$$

320. Solution: A

Let i represent the common yield rate of the two bonds.

Since the modified duration is the Macaulay duration divided by $(1 + i)$ and $i > 0$, the Macaulay duration of each bond is greater than its modified duration. Since $a < d < b$, the Macaulay duration of d years must be associated with the bond with modified duration a years.

Since the bonds have the same yield rate, the ratio of the two types of duration is the same for each bond. So if x represents the Macaulay duration of the other bond in years, we have

d / a = x / b implies ax = bd implies x = bd / a.

The Macaulay duration of the other bond is *bd / a* years.

321. Solution: C

$$1000\left(1+\frac{0.04}{4}\right)^{3x4}\left(e^{0.05(3)}\right)\left(1-\frac{0.06}{2}\right)^{-2x4}$$
$$=1670.42$$

322. Solution: C

$$7s_{\overline{45}|i}=\frac{75.6}{1.2i}$$
$$\frac{7\left[\left(1+i\right)^{45}-1\right]}{i}=\frac{75.6}{1.2i}$$
$$7\left(1+i\right)^{45}-7=63$$
$$\left(1+i\right)^{45}=10$$
$$i=0.0525$$
$$X=7s_{\overline{45}|0.0525}=1200$$

323. Solution: C

$$20,000 = X\left[\frac{1-\left(\frac{1.06}{1.10}\right)^{10}}{0.10-0.06}\right]$$

$$X = 2584.39$$

$$LB_8 = X\left(\frac{1.06^8}{1.10}+\frac{1.06^9}{1.10^2}\right) = 7353$$

324. Solution: D

$$a(t) = \exp\left(\int_0^t \frac{1}{r+8}\,dr\right)$$

$$= \exp\left[\ln(t+8)\Big|_0^t\right]$$

$$= \exp\left[\ln(t+8)-\ln(8)\right]$$

$$= \exp\left(\ln\frac{t+8}{8}\right) = \frac{t+8}{8}$$

$$\frac{a(5)}{a(4)} = \frac{13/8}{12/8} = 1.08333$$

$$i_5 = 0.08333$$

325. Solution: D

$$98.5 = 3a_{\overline{2}|j/2} + 100v^2$$

$$\frac{j}{2} = 0.037929$$

$$j = 0.07586$$

326. Solution: D

$$\text{Adjustment in book value} = \left(g-i\right)v_i^{n-t+1}F$$

$$= \left(0.08-0.04\right)v_{0.04}^{13}F = 43.24$$

$$F = 1800$$

$$P = 1800\left(0.08\right)a_{\overline{20}|\,0.04} + 1800v_4^{20}$$

$$= 144a_{\overline{20}|\,0.04} + 1800v_4^{20} = 2778.50$$

327. Solution: A

$$5.5554 = \frac{1-\left(\frac{1}{1.0614}\right)^n}{0.0614}$$

$n = 7$

328. Solution: C

$15,000 - \frac{8000}{1.01^{36}} = 1000 + Xa_{\overline{36}|0.01}$

$8408.60 = Xa_{\overline{36}|0.01}$

$8408.60 = X(30.10751)$

$X = 279.29$

329. Solution: B

$$R_1 = \frac{20000}{\frac{1-\left(\frac{1.1}{1.09}\right)^{10}}{0.09-0.1}} = \frac{20000}{9.562}$$

$= 2091.61$

The fourth payment is $2091.61(1.1) = 2300.71$

The principal outstanding before the fourth payment is

$$2300.71\frac{1-\left(\frac{1.1}{1.09}\right)^9}{0.09-0.1} = 19708.94$$

Interest in the fourth payment is $= 19708.94(0.09) = 1773.80$

X = Principal repaid $= 2300.71 - 1773.80 = 526.91$

330. Solution: E

Interest earned on the first account is $3\left(e^{\int_0^3 \frac{dt}{t+1}} - e^{\int_0^2 \frac{dt}{t+1}}\right) = 3\left[(1+3)-(1+2)\right] = 3.$

Interest on the second account is $X\left(1.05^3 - 1.05^2\right) = 0.055125X$.

$3 = 0.055125X \Rightarrow X = 54.42$.

331. Solution: D

$1.0725^5 = 1.0575^3(1+f)^2$

$f = 0.0954$

332. Solution: C

$150,000 = \left(14,000\,\ddot{s}_{\overline{5}|0.08} + X\ddot{s}_{\overline{2}|0.08}\right)1.08^5$

$150,000 = (88,703.01 + 2.2464X)1.46933$

$X = 5,958$

333. Solution: A

I is false as it is true for small parallel changes in interest rates only.
II is false, as the durations will be equal.
III is false as the convexity of assets should be greater than the convexity of liabilities.

334. Solution: B

$9550(1+i)^5 = 756.97\ddot{a}_{\overline{20}|0.04}$

$9550(1+i)^5 = 10,698.97$

$(1+i)^5 = 1.12031$

$i = 0.02298$

335. Solution: B

$600(1+i)^n = 1000$

$(1+i)^n = \frac{10}{6}$

$v^n = 0.6$

$600 = F\frac{i}{2}a_{\overline{n}|i} + Fv^n$

$600 = F\left[\frac{i}{2}\frac{1-v^n}{i} + v^n\right]$

$600 = F\left[\frac{1}{2}\left(1-0.6\right) + 0.6\right]$

$F = 750$

336. Solution: B

$$\left[40(Is)_{\overline{29}|0.03}+30(1000)\right]=\left[40\frac{\ddot{s}_{\overline{29}|0.03}-29}{0.03}+30(1000)\right]=53,433.89$$

337. Solution: C

$5020=360a_{\overline{n}|0.0625}$

$n=33.85$

Use $n=33$

$5020=360a_{\overline{33}|0.0625}+Xv^{33}$

$5020=4980.95+Xv^{33}$

$39.05=Xv^{33}$

$X=288.75$

$B=360+288.75=648.75$

338. Solution: A

For a 7.5% yield rate, the present value and Macaulay duration of the assets are, respectively,

30,000 + 20,000 = 50,000 and $\frac{30,000(28)+20,000(35)}{30,000+20,000}=30.8$.

The present value and Macaulay duration, of the liabilities are, respectively,

$\frac{50,000(1.075)^y}{(1.075)^y}=50,000$ and y.

Note that the present values of assets and liabilities already match. Since Macaulay durations must match, y = 30.8.

339. Solution: B

Use the full immunization equations and let N be the maturity value of the asset maturing in n years.

$242{,}180(1.07)^7+N(1.07)^{-(n-12)}-1,750,000=0$

$242{,}180(7)(1.07)^7-N(n-12)(1.07)^{-(n-12)}=0$

From the first equation:

$N(1.07)^{-(n-12)}=1,750,000-242{,}180(1.07)^7=1,361{,}112.$

Substituting this in the second equation:

$n-12=242{,}180(7)(1.07)^7/1,361{,}112=2$ and so n = 14.

340. Solution: B

$$2895.28 = Rv + R(1.01)v^2 + R(1.01)^2 v^3 + ... + R(1.01)^{29} v^{30} + 1000v^{30}$$

$$=> R\left[\frac{1-\left(\frac{1.01}{1.031}\right)^{30}}{0.031-0.01}\right] + 1000v^{30} = 2895.28$$

$$=> R = 113.75$$

341. Solution: B

$$i = e^{0.12} - 1$$

$$i = 0.12749685$$

$$3000\left[\frac{1}{0.12749685-0.07}\right](1.12749685)$$

$$= 58,829.14$$

342. Solution: E

If accumulating the payments, they accumulate for 0, 1, …, 39 periods. This accumulation is reflected in the left-hand sides of (A) and (D). Since the accumulated value is *X*, neither right-hand side is correct. If discounting the payments to time zero, the payments are discounted for 1, 2, …, 40 periods as reflected in the left-hand side of (E). The right-hand side of (E) discounts the accumulated value of *X* for 40 periods and hence is the correct value. Answers (B) and (C) do not reflect the time-zero present value.

343. Solution: D

The cost to purchase the bond at the end of year 3 is $2000 \times (1.03)^{-2} = 1885.19$. Subtracting this cost from 2260.19 we get 375, the amount of interest paid which is 3.75% of 10000. Thus the interest rate is 3.75%.

344. Solution: A

Let *a*, *b*, and *c* represent the face values of the three bonds. One, two, three, and four years from now, respectively:

the 1-year bond provides payments of 1.01*a*, 0, 0, 0;
the 3-year bond provides payments of 0.05*b*, 0.05*b*, 1.05*b*, 0; and
the 4-year bond provides payments of 0.07*c*, 0.07*c*, 0.07*c*, 1.07*c*.

The total payments one, two, three, and four years from now must match the liabilities. Therefore, we have

$$1.01a + 0.05b + 0.07c = 5766$$
$$0.05b + 0.07c = X$$
$$1.05b + 0.07c = 15421$$
$$1.07c = 7811$$

Note that to find *X*, we do not need the first equation.

Solving the fourth equation for *c* yields $c = \frac{7811}{1.07} = 7300$.

Substituting this value of *c* into the third equation and solving for *b* yields

$b = \frac{15421 - 0.07(7300)}{1.05} = 14200$.

Finally, substituting these values of *b* and *c* into the second equation yields

$X = 0.05(14200) + 0.07(7300) = 1221$.

345. Solution: E

$$r = \frac{49}{980}(2) = 0.10$$
$$i^{(2)} = 0.10 + 0.018 = 0.118$$
$$\frac{i^{(2)}}{2} = 0.059$$
$$915.70 = 49a_{\overline{2n}|0.059} + 1000v^{2n}$$
$$2n = 12$$
$$n = 6$$

346. Solution: B

The price of a one-year bond for 1000 is $\frac{30}{1.021}+\frac{1030}{1.021^2}=1017.45$.

Therefore, to match the payment at time 2 we need to invest $\frac{1500}{1030}1017.45=1481.72$ in the one-year bond.

The one-year bond gives a payment of $\frac{1500}{1030}30=43.69$ at time 0.5. Therefore, the amount that needs to be invested in the six month zero coupon bond is $=\frac{2000-43.69}{1.0175}=1922.66$.

The total cost of the dedicated portfolio is: 1481.72 + 1922.66 = 3404.38.

347. Solution: A

The current value for couch #2 is $260=1500e^{-0.1(4/12)}-X=1450.82-X \Rightarrow X=1190.82$.

The current value for couch #1 is $1500e^{-0.1(6/12)}-1190.82e^{0.1(2/12)}=1426.84-1210.83=216.$

348. Solution: A

$$PV_X=2500\left[\frac{1-\left(\frac{1.05}{1.04}\right)^{20}}{0.04-0.05}\right]=52,732.61$$

Annuity Y has the same increasing percentage as interest rate, so:

$PV_Y=30k$

$PV_X=PV_Y => k=1757.75$

349. Solution: A

Solution: $\left(1-\frac{d^{(1/2)}}{0.5}\right)^{-0.5(0.4)}=\exp\left(\int_{2.0}^{2.4}\frac{2}{10-t}dt\right)$

$\left(1-2d^{(1/2)}\right)^{-0.2}=\exp\left[-2\ln(10-t)\,|_{2.0}^{2.4}\right]$

$\left(1-2d^{(1/2)}\right)^{-0.2}=\left(\frac{8}{7.6}\right)^{2}$

$d^{(1/2)}=0.20063$

350. Solution: D

The price of Bond A is $60(1.04^{-1}+1.04^{-2}+1.04^{-3})+1000(1.04^{-3})=1055.50,$ while the Macaulay duration of Bond A is $\dfrac{60[1.04^{-1}+2(1.04^{-2})+3(1.04^{-3})]+3(1000)(1.04^{-3})}{1055.50}=2.838.$

Note that the one-year zero-coupon bond has duration 1.

Let w denote the proportion of wealth to invest in Bond A; then, $1-w$ is the proportion of wealth invested in Bond B. Then $2=2.838w+1(1-w),$ or w = 0.5440.

351. Solution: A

Let K be the amount that Bank B paid. Equating the amount borrowed (the 16 payments discounted at 7%) to the actual payments received (using the 6% yield rate) gives the equation

$$1000a_{\overline{16}|7\%}=1000a_{\overline{8}|6\%}+\frac{K}{1.06^8}.$$

Then,

$$\begin{aligned}K&=1000(a_{\overline{16}|7\%}-a_{\overline{8}|6\%})1.06^8\\&=1000(9.44665-6.20979)1.06^8\\&=3236.86(1.59385)=5159.06.\end{aligned}$$

352. Solution: A

$$640(1+i)^n=1000$$

$$(1+i)^n=1.5625$$

$$v^n=0.64$$

$$640=F\frac{i}{2}a_{\overline{n}|i}+Fv^n$$

$$640=F\left[\frac{i}{2}\frac{1-v^n}{i}+v^n\right]$$

$$640=F\left[\frac{1}{2}\left(1-0.64\right)+0.64\right]$$

$$F=780.49$$

353. Solution: E

The price is P = 950. The modified duration is $-\dfrac{P'}{P}=-\dfrac{-4750}{950}=5$.

Macaulay duration is $(1.09)(5)=5.45$

354. Solution: B

Because Bond A sold for its fact amount, the yield rate is the coupon rate of 6% per year.
The present values of the three bonds are:
Bond A: 1000 (given)
Bond B: $1000(1.06)^{-5} = 747.26$
Bond C: $1000(1.06)^{-10} = 558.39$
The durations are:
Bond A: 7.8017
Bond B: 5
Bond C: 10
The portfolio duration is the average of these three durations, weighted by the bond prices.

Duration = $\dfrac{1000(7.8017) + 747.26(5) + 558.39(10)}{1000 + 747.26 + 558.39} = 7.426$

355. Solution: A

The present value of the two payments is:

$$1,000,000 + \frac{1,000,000}{1.04^5} = 1,821,927.07$$

The present value of the perpetuity is:

$$(1.04)\left[\frac{X}{0.04} + \frac{1000}{0.04^2}\right] = 1,821,927.07$$

$$\left[\frac{X}{0.04} + \frac{1000}{0.04^2}\right] = 1,751,852.99$$

$$\frac{X}{0.04} = 1,126,852.99$$

$$X = 45,074.12$$

356. Solution: C

$$X = 1000\exp\left(\int_2^6 \frac{0.5}{5+0.5t}\,dt\right)$$

$$= 1000\exp\left[\ln(5+0.5t)\,|_2^6\right]$$

$$= 1000\exp\left(\ln 8 - \ln 6\right) = 1000\left(\frac{8}{6}\right) = 1333.33$$

$$1333.33 = Y\left[1 - \frac{0.08}{4}\right]^{-4(2)}$$

$$Y = 1134.35$$

357. Solution: D

The quarterly interest rate is $1.08^{1/4} - 1 = 0.01943$

$$1000\ddot{a}_{\overline{24}|0.01943} = X\left(1.08^{-1} + 1.08^{-3} + 1.08^{-5}\right)$$

$$19{,}406.51 = 2.40034X$$

$$X = 8085$$

358. Solution: E

$$\left(\frac{1}{0.98}\right)^{365/20} - 1 = 44.6\%$$

359. Solution: C

First calculate the three-year interest rate.

$$1.06^3 - 1 = 0.191016$$

This is an arithmetic increasing perpetuity.

$$X\left[\frac{1}{0.191016} + \frac{1}{0.191016^2}\right] = 655.56$$

$$X = 20.08$$

360. Solution: D

The present value of the first year's payments is:

$$1.08^{\frac{1}{12}} - 1 = 0.00643403$$

$$500a_{\overline{12}|0.006434} = 5756.43$$

This perpetuity can be thought of as a geometrically increasing perpetuity-due with first payment 5756.43. The present value is:

$$5756.43\left[\frac{1}{0.08 - 0.05}\right]1.08$$

$$= 207{,}231.44$$

361. Solution: D

$$Xs_{\overline{60}|i} = \frac{5000}{d} = \frac{5000}{i}(1+i) = \frac{5000(1.01)^{12}}{(1.01)^{12} - 1}$$

$$44{,}423.95 = 81.67X$$

$$X = 543.94$$

362. Solution: B

$$10,000(10)+\left[600s_{\overline{10}|0.04}+600\frac{s_{\overline{10}|0.04}-10}{0.04}\right]=137,295.27$$

$$55,400(1+i)^{10}=137,295.27$$

$$i=0.095.$$

363. Solution: E

$$1000(5)+\left[50s_{\overline{5}|0.04}+50\left[\frac{s_{\overline{5}|0.04}-5}{0.04}\right]\right]=5791.22$$

364. Solution: A

The total amount paid is *n*. The initial loan amount is $a_{\overline{n}|i}$

The total interest paid equals $n-a_{\overline{n}|i}$.

365. Solution: C

$$100(1+0.05)^2\exp\left(\int_2^5\frac{1}{1+t}\right)=100(1.1025)\exp\left[\ln(1+t)\Big|_2^5\right]$$

$$=110.25\left(\frac{6}{3}\right)=220.50$$

$$\frac{100}{(1-d)^5}=220.50$$

$$(1-d)^5=\frac{100}{220.50}=0.453515$$

$$1-d=0.853726$$

$$d=0.1463$$

366. Solution: A

$$\left[100\ddot{s}_{\overline{5}|0.13}+50\ddot{s}_{\overline{3}|0.13}\right](1.13)^5=1703.81$$

$$\frac{x}{1.12^{1/2}-1}=1703.81$$

$$X=99.33$$

367. Solution: B

$(1+j)^{12} = 1.08$

$j = 0.006434$

$225\ddot{s}_{\overline{240}|0.006434} = 128,848.51$

$128,848.51 = X\ddot{a}_{\overline{30}|0.07}$

$128,848.51 = X(13.27767)$

$X = 9704.15$

368. Solution: C

$10,000 = Pa_{\overline{20}|0.015}$

$P = 582.46$

$OB_4 = 582.46a_{\overline{16}|0.015}$

$OB_4 = 8230.86$

Remaining payments total:

$16(582.46) = 9319.32$

$9319.32 - 8230.86 = 1088.46.$

369. Solution: A

$1.0912 = (1+j)^{12}$

$j = 0.0073$

$100,000 = Pa_{\overline{180}|0.0073}$

$P = 1000.02$

$OB_{59} = 1000.02a_{\overline{121}|0.0073}$

$OB_{59} = 80,174.59$

$1.0576 = (1+j)^{12}$

$j = 0.004678$

Interest portion of first revised payment:

$80,174.59(0.004678) = 375.04.$

370. Solution: A

Define i' as the quarterly effective interest rate for the loan

Solve for i'

$$291.23a_{\overline{20}|\,i'} = 5000$$

$$i' = 1.5\%$$

Then, solve for *j* using the formula

$$\left(1+\frac{j}{12}\right)^{12} = (1+0.015)^4$$

$$j = 0.0597.$$

371. Solution: B

All prices imply an interest rate of 6% indicating that the yield rate does not change with duration, that is. the yield curve is flat.

372. Solution: E

Macaulay duration at an interest rate of 5% is

$$\frac{100(Ia)_{\overline{5}|} + 5000v^5}{100a_{\overline{5}|} + 1000v^5}$$

$$= \frac{100(12.56639) + 3917.63}{100(4.32947) + 783.53} = 4.2535$$

373. Solution: C

$$OB_{12} = 1000a_{\overline{24}|0.0075} = 21{,}889.15$$

$$21{,}889.15 = 2000a_{\overline{n}|0.0075}$$

$$n = 11.46$$

Drop payment will be made at the 12th month.

374. Solution: E

$$30{,}000 = Pa_{\overline{120}|0.0075}$$

$$P = 380.03$$

$$33{,}000 = 380.03a_{\overline{n}|0.0075}$$

$$n = 141$$

375. Solution: B

$$\bar{v} = \frac{\bar{d}}{1+r}$$

$$5.76 = \frac{6}{1+r}$$

$$r = 0.041667$$

$$\bar{v} = \frac{5}{1.041666} = 4.8$$

376. Solution: D

$$9.26 = \frac{10}{1+i}$$

$$i = 0.079914$$

$$\bar{d} = \frac{(Ia)_{\overline{10|}}}{a_{\overline{10|}}} = \frac{32.70395}{6.71270} = 4.8720.$$

377. Solution: E

$$Xa_{\overline{n|}0.01} = 2.01Xa_{\overline{200|}0.0201}$$

$$a_{\overline{n|}0.01} = 2.01a_{\overline{200|}0.0201}$$

$$a_{\overline{n|}0.01} = 98.13168$$

$$n = 400$$

378. Solution: C

Let r be the coupon rate.

$$2300 = 2000ra_{\overline{20|}0.07} + 2000v^{20}$$

$$2000r = 168.32$$

Bond is bought at a premium, so assume called as early as possible at year 18.

$$P = 168.32a_{\overline{18|}0.07} + 2000v^{18}$$

$$P = 2284.85$$

379. Solution: C

Since the annual effective discount rate is 3.2%, the present value of an amount is calculated by multiplying it by a discounting factor of $(1-0.032)^t = (0.968)^t$, where t is the number of years since the deposit.

At time t = 0, an initial deposit of 50,000 is made just after the balance is 0 (the account is new just before the deposit). The withdrawals are then X at each of times t = 2, 4, 6, 8, 10, 12 or equivalently at time $t = 2k$ for each whole number k from 1 to 6 inclusive.

Then to make the final balance 0, an additional withdrawal of 45,000 at time t = 12 would be needed.

Since the net present value of the cash flows (withdrawals minus deposits) must be zero, in a time period from a zero balance to another zero balance, we have

$$45{,}000(0.968)^{12} + X\sum_{k=1}^{6}(0.968)^{2k} - 50{,}000 = 0$$

$$45{,}000(0.968)^{12} + X\sum_{k=1}^{6}(0.968)^{2k} = 50{,}000.$$

380. Solution: E

$$3400 = 240a_{\overline{n}|0.045}$$

$$n = 23.05$$

Use 23 for the number of full payments of 240. X will be paid at time 24.

$$3400 = 240a_{\overline{23}|0.045} + Xv^{24}$$

$$3400 = 3395.47 + Xv^{24}$$

$$4.53 = Xv^{24}$$

$$X = 13.04$$

381. Solution: C

The minimum yield will occur at the earliest redemption date since the bond is brought at a premium. Therefore n = 18.

$$1321 = 111.10a_{\overline{18}|i} + 1111v^{18}$$

$$i = 0.07985$$

382. Solution: A

Conditions for full immunization of a single liability cash flow are:

1. PV (Assets) = PV (Liability)
2. Duration (Assets) = Duration (Liability)
3. The asset cash flows occur before and after the liability cash flow.

Checking Condition 1, only I and II are present value-matched, so can rule out III at this point.
Checking Condition 2, only I is duration-matched, so can rule out II at this point.
Checking Condition 3, the asset cash flows do occur before and after the liability for I.

So only I fully immunizes the liability.

383. Solution: B

Let i represent the effective market annual yield rate and $v = \frac{1}{1+i}$. The Macaulay duration is 3.70 years, which is equal to the present-value-weighted times of the liabilities. Therefore, we have

$$3.70 = \frac{20,000(0)+100,000v^5(5)}{20,000+100,000v^5} = \frac{25v^5}{1+5v^5}$$

$$3.70 + 18.5v^5 = 25v^5$$

$$3.70 = 6.5v^5$$

$$v = 0.89342$$

$$1+i = 1.11929$$

Modified duration equals Macaulay duration divided by (1 + i), so the modified duration is $\frac{3.70}{1.11929} = 3.30567$ years.

384. Solution: A

Using the basic formula:

$$BV_2 - BV_3 = 6.88$$

$$\left(87.5a_{\overline{58|}} + Cv^{58}\right) - \left(87.5a_{\overline{57|}} + Cv^{57}\right) = 6.88$$

$$39.0667 - 0.00625C = 6.88$$

$$C = 5150$$

385. Solution: E

$P = Xa_{\overline{12}|0.02} + 1000v^{12}$

$500 + P = 2Xa_{\overline{12}|0.02} + 1000v^{12}$

Subtract the first equation from the second.

$500 = Xa_{\overline{12}|0.02}$

$X = 47.28$

386. Solution: C

First, note that the monthly rate of interest is 0.072/12 = 0.006.
Also, the first payment occurs 60 months out, and the last payment occurs 240 months out. Thus, there are 181 payments in total.

Then, the present value of the annuity is

$$\frac{750}{(1.006)^{60}}\left[1+\frac{1.01}{1.006}+\ldots+\left(\frac{1.01}{1.006}\right)^{180}\right]$$

$$=\frac{750}{(1.006)^{60}}\left[\sum_{k=0}^{180}\left(\frac{1.01}{1.006}\right)^{180}\right]=\frac{750}{(1.006)^{60}}\left[\frac{1-\left(\frac{1.01}{1.006}\right)^{181}}{1-\left(\frac{1.01}{1.006}\right)}\right]$$

$$=523.82(1.05085/0.0039761)=138,440.$$

387. Solution: C

The price of the geometric perpetuity-due, as a function of the annual effective interest rate i, is given by

$$P(i)=1+\frac{0.99}{1+i}+\left(\frac{0.99}{1+i}\right)^2+\left(\frac{0.99}{1+i}\right)^3+\ldots=\frac{1}{1-\frac{0.99}{1+i}}=\frac{1+i}{0.01+i}=1+\frac{0.99}{0.01+i}.$$

Therefore, the modified duration is given by

$$-\frac{P'(i)}{P(i)}=-\frac{-\frac{0.99}{(0.01+i)^2}}{\frac{1+i}{0.01+i}}=\frac{0.99}{(0.01+i)(1+i)}.$$

Since Macaulay duration is modified duration times $(1+i)$, the Macaulay duration is $\frac{0.99}{0.01+i}=12$. Solving for i gives $i=0.0725$.

Therefore, the modified duration is $\frac{12}{1+i}=\frac{12}{1+0.0725}=11.18881$ years.

388. Solution: E

Since the annual nominal interest rate is 6%, compounded monthly, the monthly effective interest rate is $\frac{6\%}{12} = 0.5\%$ and the monthly discounting factor is $\frac{1}{1.005}$.

The 10,000 borrowed today is the present value of a 30-month annuity immediate with monthly payments of P each, followed by a 40-month annuity immediate with monthly payments of 1.1P but with payments deferred by 30 months, followed by a 50-month annuity immediate with monthly payments of 1.2P but with payments deferred by $30+40=70$ months. Therefore,

$$10,000 = Pa_{\overline{30}|0.005} + (1.005)^{-30}\left(1.1Pa_{\overline{40}|0.005}\right) + (1.005)^{-70}\left(1.2Pa_{\overline{50}|0.005}\right).$$

389. Solution: C

Since the Macaulay duration of a set of cash flows is a weighted sum of the payment times, deferring by six years must add exactly 6 to this duration. The Macaulay duration is $(1 + j)$ times the modified duration, so that $(1+j)(9-4)=6$, which implies $j = 0.2$.

If $P(i)$ denotes the present value of the original set of cashflows at rate i, we know that the modified duration $4 = \frac{-P'(j)}{P(j)} = \frac{-P'(j)}{10} \Rightarrow P'(j) = -40$.

The modified duration of the altered payments is $-\frac{0+P'(j)}{3+P(j)} = \frac{40}{13} = 3.077.$

The Macaulay duration is 1.2(3.077) = 3.69.

390. Solution: B

For all 3 investments, convexity is $P''(i)/P(i)$. Let C be the redemption value of the bond and the initial payments of the perpetuities (it turns out the value is irrelevant).

For i)

$P(i) = C(1+i)^{-50}, P'(i) = -50C(1+i)^{-51}, P''(i) = 2550C(1+i)^{-52}$

$\text{Convexity} = 2550(1.05)^{-52}/1.05^{-50} = 2550/1.05^2 = 2313.$

For ii)

$P(i) = Ci^{-1}, P'(i) = -Ci^{-2}, P''(i) = 2Ci^{-3}$

$\text{Convexity} = 2(0.05)^{-3}/0.05^{-1} = 2/0.05^2 = 800.$

For iii)

$P(i) = C(i-0.03)^{-1}, P'(i) = -C(i-0.03)^{-2}, P''(i) = 2C(i-0.03)^{-3}$

$\text{Convexity} = 2(0.05-0.03)^{-3}/(0.05-0.03)^{-1} = 2/0.02^2 = 5000.$

Thus, convexity when ranked from lowest to highest is:
800 < 2,313 < 5,000, or $y < x < z$.

391. Solution: E

$$Xv^2 - Yv^4 + Xv^6 = 0$$

$$Xv^4 - Yv^2 + X = 0$$

$$v^2 = \frac{Y \pm \sqrt{Y^2 - 4X^2}}{2X}$$

$$(1+i)^2 = \frac{2X}{Y \pm \sqrt{Y^2 - 4X^2}}$$

$$(1+i) = \sqrt{\frac{2X}{Y \pm \sqrt{Y^2 - 4X^2}}}$$

$$i = \sqrt{\frac{2X}{Y \pm \sqrt{Y^2 - 4X^2}}} - 1$$

392. Solution: A

$$1260 = 200(5) + 200i\left(Is\right)_{\overline{5|}\,0.06} = 1000 + 200i\left[\frac{\ddot{s}_{\overline{5|}\,0.06} - 5}{0.06}\right]$$

$$= 1000 + 200i\left(16.2553\right)$$

$$i = 0.08$$

393. Solution: C

$$1{,}000{,}000 = 20{,}000\ddot{a}_{\overline{5|}0.03} + (1.03)^{-5}50{,}000\ddot{a}_{\overline{\infty|}i}$$

$$1{,}000{,}000 = 94{,}341.97 + (1.03)^{-5}50{,}000\frac{1+i}{i}$$

$$1{,}049{,}905.88 = \frac{50{,}000(1+i)}{i}$$

$$i = 0.05$$

394. Solution: D

$$1+i_5 = e^{\int_4^5 \frac{1}{t+8}dt}$$

$$= e^{\ln(t+8)|_4^5}$$

$$= \frac{13}{12}$$

$$= 1.08333$$

$$i_5 = 8.3\%$$

395. Solution: D

$$\left[\frac{1}{0.05}+\frac{1}{0.05^2}\right](1.05)=441.$$

396. Solution: E

If *j* is the semi-annual effective interest rate, then

1 + (1/*j*) = 23

j = 1/22

Then the annual effective interest rate is $(1+1/22)^2-1=0.093$.

397. Solution: E

First determine the present value of the four payments within a year. Then determine the present value of ten of these.

$$\left(1+\frac{0.012}{4}\right)^4-1=0.12551$$

$$P=\left[100(Ia)_{\overline{4}|0.03}\right]\ddot{a}_{\overline{10}|0.12551}$$

398. Solution: D

$$(Ia)_{\overline{\infty}|}=\frac{1}{i}+\frac{1}{i^2}=\frac{1}{0.09}+\frac{1}{0.09^2}=134.5679$$

$$\begin{aligned}\text{X}&=(Ia)_{\overline{\infty}|}-2v^{10}(Ia)_{\overline{\infty}|}+v^{15}(Ia)_{\overline{\infty}|}\\&=134.5679(1-2\times 0.42241+0.274538)\\&=57.83\end{aligned}$$

399. Solution: A

Accumulate values to time ten.

$$X(1.085)^{10}=500s_{\overline{10}|0.08}+10,000$$

$$X=\frac{s_{\overline{10}|0.08}500+10,000}{1.085^{10}}$$

$$=\frac{7,243.28+10,000}{1.085^{10}}$$

$$=7626.45$$

400. Solution: D

Repayment = $\dfrac{1000}{a_{\overline{120}|.08/12}} = 12.13276$

Accumulated loan repayments at 6% = $12.13276 s_{\overline{120}|.005} = 1988.31$

$1000(1+j)^{10} = 1988.31 \Rightarrow j = 0.0711$

401. Solution: A

$65.42 = P(1-v^{n-n+1}) = P(1-1/1.07)$

$P = 1000$

$I_1 = 13,650(0.07) = 955.50$

$P_1 = 1000 - 955.50 = 44.50$

402. Solution: B

$(1+i')(1+r) = (1+i)$

$1+i'+r+i'r = 1+i$

$i = i'+r+i'r$

403. Solution: B

$0.09/4 = 0.0225 \quad 1.0225 \wedge 4 = 1.09308 \quad 0.093083 * 10,000 = 930.83$

$5461 = 930.83 s_{\overline{5}|i}$

$i = 8.00\%$

404. Solution: D

$MV = \dfrac{100}{1.03} + \dfrac{100}{1.035^2} + \dfrac{1100}{1.04^3} = 1168.33$

$1168.33 = 100 a_{\overline{3}|i} + 1000 v^3$

$i = 3.94\%$

405. Solution: A
Let P be the initial payment. Then,

$$12,000 = \sum_{t=1}^{60} P(1.01)^{t-1} / (1+i)^t = 60P / 1.01$$

$$P = 202.$$

The interest in the first month is 0.01(12,000) = 120.

The principal being repaid in the first month is 202 – 120 = 82.

406. Solution: B

$$\frac{1.09}{1.05} - 1 = 0.038$$

407. Solution: E
At the end of 5 years and 60 payments, the borrower still owes 150,000.
It will take n monthly payments of 1000 to retire the loan where

$$1000a_{\overline{n}|0.005} = 150,000$$

$$n = 278$$

Thus, it will take 338 total payments.

408. Solution: C
Suppose level payment is 500.

$$\begin{aligned} PV &= \frac{500}{1.0425} + \frac{500}{1.045^2} + \frac{500}{1.0475^3} + \frac{500}{1.054} \\ &= 479.62 + 457.86 + 435.02 + 411.35 \\ &= 1783.85 \end{aligned}$$

$$500a_{\overline{4}|i} = 1783.85$$

$$i = 4.74\%$$

409. Solution: B

$$\bar{d} = \frac{(Ia)_{\overline{10}|0.06}}{a_{\overline{10}|0.06}} = \frac{36.96241}{7.36009} = 5.02201$$

410. Solution: A
The percentage by which the balance accumulates each year is the effective interest rate.

411. Solution: E
We have
$P(i) = 1/i,\ P'(i) = -1/i^2,\ d_{\text{mod}} = 1/i = 15 \Rightarrow i = 1/15$
$d_{mac} = d_{\text{mod}}(1+i) = 15(1+1/15) = 16$

412. Solution: A
The Macaulay duration is
$$\frac{e^{-\delta} + 2e^{-2\delta}}{e^{-\delta} + e^{-2\delta}} = \frac{1+2e^{-\delta}}{1+e^{-\delta}}.$$

413. Solution: E
The effective rate per month is $i = \frac{0.03}{12} = 0.0025$. Hence the original loan amount is $L = Pa_{\overline{60}|i} = 359.37a_{\overline{60}|i} = 20,000$. Using the retrospective approach, the outstanding loan balance is given by
$$OLB_{36} = L(1+i)^{36} - \{Ps_{\overline{36}|i} - P(1+i)^{11} - P(1+i)^{10}\}$$
$$= 20,000 \times (1.0025)^{36} - \{359.37s_{\overline{36}|i} - 359.37 \times (1.0025)^{11} - 359.37 \times (1.0025)^{10}\} = 9099.17$$

414. Solution: C
The total interest payment of 400 in the 1-year period is 5% of the <u>new</u> balance of 8000, so by definition, 5% is the <u>effective discount rate</u>.

415. Solution: A
The effective monthly interest rate is $i = \frac{0.072}{12} = 0.006$. Let
L_A = the amount of Loan A and L_B = the amount of Loan B. We then have
$L_A = m \cdot a_{\overline{48}|\,i} + 1.5 \cdot m \cdot a_{\overline{48}|\,i} / (1+i)^{48}$
$L_B = 1.2 \cdot m \cdot \ddot{a}_{\overline{48}|\,i} + 0.9 \cdot m \cdot \ddot{a}_{\overline{48}|\,i} / (1+i)^{48}$
Hence $$\frac{L_A}{L_B} = \frac{ma_{\overline{48}|\,i} + 1.5ma_{\overline{48}|\,i} / (1+i)^{48}}{1.2m\ddot{a}_{\overline{48}|\,i} + 0.9m\ddot{a}_{\overline{48}|\,i} / (1+i)^{48}} = \frac{1+1.5/1.006^{48}}{(1.2+0.9/1.006^{48})1.006} = 1.12668$$

416. Solution: C
Let i = yield rate. Then, for bond A:
$$800 = 1000(i/2)a_{\overline{m}|i} + 1000v^m = 500(1-v^m) + 1000v^m \Rightarrow v^m = 0.6.$$
The price of bond B is:
$$1000(i/2)a_{\overline{3m}|i} + 1000v^{3m} = 500(1-v^{3m}) + 1000v^{3m} = 500(1+v^{3m}) = 500(1+0.6^3) = 608.$$

417. Solution: D
No computations are needed. Since the desired yield rate of 6% is higher than the coupon rate of 5%, the investor is at the greatest disadvantage when the bond is called at maturity.

Note that the two bonds are alike in all other ways, with the same maturity date, face value, desired yield rate, and coupon rate. So, the maximum price the investor should be willing to pay for the callable bond matches the price of the non-callable bond, namely 4361.

418. Solution: A
The amount charged to the card was 1500 immediately. The annual fees were 20 after 1 year and 20 after 2 years.

The monthly payments were X after $\frac{k}{12}$ years, for each whole number k from 1 to 24 inclusive. Then to make the final balance 0, the final payment would have been 200 after 2 years.

Therefore,

$$1500 + 20e^{-0.18(1)} + 20e^{-0.18(2)} - X\sum_{k=1}^{24} e^{-0.18\left(\frac{k}{12}\right)} - 200e^{-0.18(2)} = 0$$
$$1500 + 20e^{-0.18} + 20e^{-0.36} - X\sum_{k=1}^{24} e^{-0.015k} = 200e^{-0.36}$$
.

419. Solution: D
Let j be the quarterly interest rate. Then,
$$3{,}000 = 5{,}000v^{40}$$
$$j = .01285$$
$$1{,}000 = 950ra_{\overline{60}|0.01285} + 950v^{60} = 39{,}565.26r + 441.59$$
$$r = (1{,}000 - 441.59)/39{,}565.26 = 0.014114.$$
This is the quarterly rate. The annual rate is 0.014114(4) = 0.056456 = 5.65%.

420. Solution: D

$$X = 350(1.07)\left[\frac{1-\left(\frac{1.03}{1.07}\right)^5}{0.07-0.03}\right] + 350(1.03)^4 v^4 \left(\frac{1}{0.07}\right)$$

$$X = 1623.96 + 4293.23 = 5917.19.$$

421. Solution: E

The definition of “purchased at premium” is P > C; purchase price greater than redemption value.

422. Solution: A

$$225,000 = 1960 a_{\overline{180}|i}$$

$$i = 0.0054167$$

$$225,000 = X\ddot{a}_{\overline{360}|0.0054167}$$

$$X = 1414.50$$

423. Solution: A

The Macaulay duration of bond B is $MacD^B = 10$.

The Macaulay duration of bond A is $MacD^A = 2MacD^B = 20$

The price of bond A is $P = \dfrac{1}{i}$, where i is the annual yield. So, the modified duration of bond A is

$$ModD^A = -\frac{P'(i)}{P(i)} = \frac{1}{i}.$$

$MacD^A = ModD^A \times (1+i) = \dfrac{1}{i}(1+i) = 20 \Rightarrow i = 1/19$, therefore $ModD^A = \dfrac{1}{i} = 19$.

424. Solution: D

$$PV = 1500\left[\frac{1}{0.087-(-0.02)}\right]$$

$$PV = 14,018.69$$

$$14,018.69 = P\left[\frac{1}{0.078-(-0.02)}\right]$$

$$P = 1373.83$$

.

425. Solution: C

$$PV = 0 = 1000(1+i)^{-2} + 750(1+i)^{-4} - X - Y(1+i)^{-3}$$

$$PV' = 0 = -2000(1+i)^{-3} - 3000(1+i)^{-5} + 3Y(1+i)^{-4}$$

$$2000(1.06)^{-3} + 3000(1.06)^{-5} = 3Y(1.06)^{-4}$$

$$1679.24 + 2241.77 = 2.37628Y$$

$$Y = 1650.06$$

426. Solution: B

Matching the asset and liability present values and durations, we have the system of equations

$$Xv^{0.5} + Yv = 10,000v^{0.75}$$

$$0.5Xv^{0.5} + Yv = 7,500v^{0.75}$$

$$0.5Xv^{0.5} = 2,500v^{0.75}$$

$$X = 5,000v^{0.25} = 5,000(1.02)^{-0.25} = 4,975$$

427. Solution: D

Each coupon payment is $7000(0.06) = 420$.

The accumulated value of the reinvested coupons after 18 years is: $420s_{\overline{18}|0.057} = 12,617.50.$

Therefore,

$$P(1.0518)^{18} = 12,617.50 + 7,500 \Rightarrow P = 8105.45$$

Finally,

$$P = 8105.45 = 420a_{\overline{18}|i} + 7500v^{18} \Rightarrow i = 4.91\%$$

428. Solution: A

The *PV* of perpetuity X is $\dfrac{3000}{j_1} + 3000 = 125,939.41,\quad \text{where } (1.0494) = (1+j_1)^2$

The *PV* of perpetuity Y is $\dfrac{1.0494k}{j_2} = 10.36543134k,\ \textit{where } 1.0494^2 = 1 + j_2$

Setting *PV*'s equal gives *k* = 12,149.94.

429. Solution: A

$$\bar{d} = \frac{0.09(Ia)_{\overline{10}|0.10} + 10v^{10}}{0.09a_{\overline{10}|0.10} + v^{10}}$$

$$\bar{d} = \frac{2.61323 + 3.85543}{0.55301 + 0.38554} = 6.8922$$

$$\bar{v} = \frac{6.8922}{1.10} = 6.2656$$

430. Solution: B

Let i = yield rate.
Let n be the term in years of the two bonds.
Let F = the face value of the second bond.

Then the coupon rate for the second bond = $i/2$.
The equation of value for the first bond is $890 = 1000v^n \Rightarrow v^n = 0.89$.
For the second bond we have

$$890 = \frac{i}{2}Fa_{\overline{n}|i} + Fv^n = \frac{i}{2}F\frac{1-v^n}{i} + Fv^n = \frac{F}{2}(1 - v^n + 2v^n) = \frac{F}{2}(1+v^n)$$

$$F = \frac{1780}{1+v^n} = \frac{1780}{1.89} = 941.80.$$

431. Solution. E

$$Fr = 1000(0.03) = 30$$

$$j = (1.05)^{1/2} - 1 = 0.024695$$

$$B_{11} = 1000v_j^9 + 30a_{\overline{9}|j} = 1042.35$$

$$B_{12} = 1000v_j^8 + 30a_{\overline{8}|j} = 1038.09$$

$$1042.35 - 1038.09 = 4.26$$

432. Solution: E

(A) It does need to be satisfied
(B) Requires more rebalancing
(C) Convexity of assets must be greater than for liabilities
(D) Convexity is greater than 0
(E) True

433. Solution: A

OB, after year $5 = v^5 100 a_{\overline{10|}} = 378.17$

OB, after year $15 = 100 a_{\overline{5|}} = 378.11$

|378.17 – 378.11| = 0.06, round to 0

434. Solution: E

First, use bond A to determine the interest rate. Because the equation for the interest rate cannot be solved algebraically, use a financial calculator to get *i*/2, which yields *i*/2 = 0.035, or $i^{(2)} = 0.07$.

For bond B,

$$P = 1000(0.08/2)a_{\overline{30|}0.035} + 1000(0.10/2)1.035^{-30}a_{\overline{30|}0.035} + 1000(1.035)^{-60}$$
$$= 735.68 + 327.63 + 126.93 = 1190.24,$$

435. Solution: A

$$\text{Call after 20 years}: 30a_{\overline{40|}0.025} + 1000v^{40} = 1125.51$$
$$\text{Call after 15 years}: 30a_{\overline{30|}0.025} + 1030v^{30} = 1118.95$$
$$\text{Call after 10 years}: 30a_{\overline{20|}0.025} + 1060v^{20} = 1114.56$$

The correct price is then the smallest or 1115.

436. Solution: B

First, calculate the initial payment *P* as follows:

$$\left(\frac{P}{i} + \frac{Q}{i^2}\right)(1.10) = 1,000,000$$
$$\left(\frac{P}{0.1} + \frac{1000}{0.1^2}\right) = 909,090.91$$
$$P = 80,909.09.$$

Thus, the payment on January 1, 2020 will be *P* + 20*Q* = 80,909 + 20(1000) = 100,909.

The balance on 1 January 2020, immediately after the payment at that time, will be

$$\left(\frac{100,909.09 + 1000}{0.10} + \frac{1000}{0.10^2}\right) = 1,119,090.90.$$

437. Solution: D

The accumulation of discount at time 8 is 491.51, which is equal to 50 $s_{\overline{8}|i}$. Using a BA-II calculator, i can be computed as 5.8099441%. The accumulation of discount at time 25 is 50 $s_{\overline{25}|i}$, or 2670.91, the bond's redemption value is therefore 20,000+2671=22,671.

438. Solution: E

$$0.975X = 250 + (X - 250)v^{(60-15)/365}$$

$$0.975X = 250 + (X - 250)(1.242)^{-45/365}$$

$$X = 4827.18$$

439. Solution: D

$$20,000 \big/ a_{\overline{40}|0.08} = 1677.20$$

$$1677.20 a_{\overline{20}|0.08} = 16,467.03$$

$$16,407.03 \big/ a_{\overline{20}|0.06} = 1435.67$$

$$20\left(1677.20 + 1435.67\right) = 62,257$$

440. Solution: C

The present value of the perpetuity-due, at annual effective interest rate i, is given by

$$P(i) = \frac{1+i}{i} = \frac{1.0625}{0.0625} = 17$$

$$P'(i) = \frac{-1}{i^2}$$

.

$$\bar{v} = \frac{-P'(i)}{P(i)} = \frac{\frac{1}{i^2}}{\frac{1+i}{i}} = \frac{1}{i(1+i)} = \frac{1}{0.0625(1+0.0625)} = 15.058824.$$

The first-order modified approximation of the present value, at an interest rate i near an initial interest rate i_0, is therefore given by

$$P_{\text{approx}}(i) = P\left(i_0\right)\left[1 - \left(i - i_0\right) D_{\text{mod}}\left(i_0\right)\right] = 17\left[1 - \left(-0.005\right)(15.058824)\right] = 18.28.$$

Actual new price is $\frac{1.0575}{.0575} = 18.39130.$

$$\frac{X - Y}{Y} = \frac{18.28 - 18.39130}{18.39130} = -0.00605 = -0.61\%.$$

441. Solution: E

Let v denote the discount factor applicable to a period of one month and let X be the constant periodic amount received by both charities. For Charity A, $198,000 = \frac{X}{1-v}$.

Since v^2 is the discount factor applicable to a two-month period, for Charity B, $100,000 = \frac{X}{1-v^2}$.

Dividing the first equation by the second gives $1+v = 1.98$ so that $v = 0.98$. The yearly discount factor is $0.98^{12} = 0.78472$ so that the annual effective interest rate is

$$\frac{1}{0.78472} - 1 = 0.274 = 27.4\%.$$

442. Solution: C

$$1+i = \exp\left(\int_4^5 \frac{1}{20+t}\,dt\right) = \exp\left[\ln(20+t)\Big|_4^5\right] = \exp(\ln 25 - \ln 24) = \frac{25}{24}$$

$$i = \frac{1}{24} = 0.041667 = 4.17\%$$

443. Solution: C

Let X be the monthly deposit of year 1. Then determine the present value of the first year's monthly payments and then use geometric formula to accumulate to year 10.

$$i = \left[1 + \frac{0.024}{12}\right]^{12} - 1 = 0.024266.$$

$$X\ddot{a}_{\overline{12}|0.002}\left[\frac{1 - \left(\frac{1.10}{1.024266}\right)^{10}}{0.024266 - 0.10}\right](1.024266)(1.024266)^{10} = 21,234.05$$

$$X(11.869136)[13.74278] = 16,311.46$$

$$X = 100$$

The payments in year 5 will be $100(1.1)^4 = 146.41$.

444. Solution: D
The yield rate is equal to the coupon rate for Bond Z, so the price of Bond Z is 1000. This implies the price of Bond Y is 1,169.30.

$1169.30 = 20a_{\overline{4n}|j} + 1000v_j^{4n}$, where $1.0325 = (1+j)^2 \Rightarrow 4n = 76 \Rightarrow n = 19$

$1169.30 - 401.50 = Ra_{\overline{19}|i} + 1000v_i^{19}$, where $1.0325^2 = 1+i \Rightarrow R = 44.25$

445. Solution: E
Use Bond 1 to find information about $\dfrac{1+i_0}{1+i}$:

$$P(i) \approx P(i_0)\left(\frac{1+i_0}{1+i}\right)^{D_{mac}} \Rightarrow 21{,}635.83 = 20{,}400\left(\frac{1+i_0}{1+i}\right)^{11.735*(1+i)} \Rightarrow$$

$$\left(\frac{1+i_0}{1+i}\right)^{11.735*(1+i)} = 1.060579902$$

Next,

$$\left(\frac{1+i_0}{1+i}\right)^{13.101*(1+i)} = 1.060579902^{13.101/11.735} = 1.067865963$$

The first-order Macaulay approximation for Bond 2 is:

$$P(i) = 20{,}400\left(\frac{1+i_0}{1+i}\right)^{13.101*(1+i)} = 21{,}784.47$$

446. Solution: D
There are two blocks of 42 payments (paid every two months) increasing in an arithmetic progression. The value of each of these blocks two months before the first payment is:

$$100a_{\overline{42}|j} + 5\left(\frac{a_{\overline{42}|j} - 42v^{42}}{j}\right) = 3554.1141 + 5\left(\frac{35.54114068 - 30.03429308}{0.008016}\right) = 6989.024,$$

where $j = (1+0.048/12)^2 - 1 = 0.008016$

The present value is then:

$6989.024(1.008016) + 6989.024(1+.048/12) = 14{,}062.03.$

447. Solution: A
First, the size of the level payment does not have an impact on the duration. So, for the perpetuity-immediate:

$$D_{mac}(i) = 32.25 = \frac{\frac{1}{i}+\frac{1}{i^2}}{\frac{1}{i}} = \frac{1+i}{i} => d = \frac{1}{32.25} => i = 3.2\%$$

Next, the Macaulay duration of a perpetuity-due is:

$$D_{mac}(i) = \frac{\frac{1}{i}+\frac{1}{i^2}}{\frac{1}{d}} = \frac{\frac{1+i}{i^2}}{\frac{1+i}{i}} = \frac{1}{i} = \frac{1}{0.032} = 31.25 => D_{\text{mod}}(0.032) = \frac{31.25}{1.032} = 30.281$$

448. Solution: D
Using the financial calculator to solve for the yield rate with
$N = 15,\ PV = 2092.88,\ PMT = -30.25$ [because $2500(.0484)/4 = 30.25$], $FV = -2316.20$, i = 2.06% is the nominal quarterly yield rate.
$2316.3(0.0206) - 30.25 = 17.46578$ is the amount for accumulation of discount in the 20th coupon and $17.46578(1.0206)^{28} = 30.91$ is the amount for accumulation of discount in the last coupon.

449. Solution: C
Just after the 20th coupon is paid, there are still 40 coupon periods left, and the book value of the bond is B_{20} = PV(remaining cash flows) =
PV(10 coupons of 35, 30 coupons of 45, and 1 redemption value of 1,000) =
$35a_{\overline{10}|0.04} + 45a_{\overline{30}|0.04}(1.04)^{-10} + 1000(1.04)^{-40} = 283.88 + 515.68 + 202.29 = 1017.85.$

450. Solution: B
The PV and duration of the assets must match the liabilities, 24,000 + 1,000 = 25,000 and [24,000(7.5) + 1,000(20)]/25,000 = 8. The convexity of the assets must be greater than $[24{,}000(7.5^2) + 1{,}000(20_2)]/25{,}000 = 70$.

Let f represent the fraction of 25,000 that is invested in the n-year bond. The duration of the assets is $fn + (1 - f)2n$ and the convexity is $fn^2 + (1 - f)4n^2$.

For duration, $8 = fn + (1 - f)2n$ which implies $f = 2 - 8/n$. Since f must be between 0 and 1, n must be between 4 and 8 (inclusive).

Substituting for f, for convexity:

$$(2-8/n)n^2+(1-2+8/n)4n^2>70$$

$$-2n^2+24n>70$$

$$n^2-12n+35<0$$

$$(n-7)(n-5)<0.$$

This forces n to be between 5 and 7 which is the answer.

451. Solution: E
The quarterly rate of interest is 0.05/4 = 0.0125.
There are 20 deposits of 1,000, and 60deposits that are geometrically increasing above 1,000.

Then, $AV_{20} = 1{,}000\ddot{s}_{\overline{20}|0.0125}(1.0125)^{60}+1{,}000(1.02)(1.0125)^{60}\left[1+\frac{1.02}{1.0125}+\cdots+\left(\frac{1.02}{1.0125}\right)^{59}\right]$

$$=1{,}000\ddot{s}_{\overline{20}|.0125}(1.0125)^{60}+1{,}000(1.02)(1.0125)^{60}\left[\frac{1-\left(\frac{1.02}{1.0125}\right)^{60}}{1-\left(\frac{1.02}{1.0125}\right)}\right]$$

$$=48{,}138.59+161{,}639.07=209{,}778.$$

452. Solution B

Because the outlays all occur before the inflows, there is a unique yield rate. At a 50% interest rate, the equation is $-30-40/1.5+60/1.5^2+90/1.5^n=0 \Rightarrow 1.5^n=3$ for $n=\ln(3)/\ln(1.5)=2.71<3$.. For the final payment to be later, the yield rate must less than 50%.

453. Solution: D

The accumulation of discount of at time 8 is 491.51, which is equal to $50s_{\overline{8|}i}$. Using a BA-II Plus calculator, i can be computed as 5.8099441%. The accumulation of discount at time 25 is $50s_{\overline{25|}i}$, or 2670.91, the bond's redemption value is therefore 20,000+2671=22,671.

There is also an algebraic solution for the interest rate:

$$491.51 = s_{\overline{8|}} = \frac{(1+i)^8-1}{i}$$

$$1263.74 = s_{\overline{16|}} = \frac{(1+i)^{16}-1}{i}$$

$$2.57114 = \frac{(1+i)^{16}-1}{(1+i)^8-1} = \frac{[(1+i)^8+1][(1+i)^8-1]}{(1+i)^8-1} = (1+i)^8+1$$

$$(1+i)^8 = 1.57114$$

$$i = 0.0581.$$

454. Solution: E

The amounts for accumulation of the discount in the annual coupon payments (amounts of increase in book value) form a geometric progression with common ratio $(1+i)$, where i is the annual effective yield rate.

The amount for accumulation of discount in the 9th coupon payment is $4730-4478=252$. Since the redemption value is the book value just after the last (in this case 10th) coupon payment but before the redemption payment, the amount for accumulation of the discount in the 10th coupon payment is $5000-4730=270$.

Therefore, $1+i=\frac{270}{252}$, which leads to the conclusion that the annual effective yield rate is

$$i=\frac{270}{252}-1=0.071429..$$

455. Solution: E
First note that the yield rate is equal to the coupon rate for Bond Z, so the price of Bond Z is 1000. This implies the price of Bond Y is 1,073.78. The semiannual yield rate is $1.015^2 - 1 = 0.030225$. Then, $1073.78 = 34a_{\overline{2n}|0.030225} + 1000(1.030225)^{-2n} \Rightarrow 2n = 30 \Rightarrow n = 15$. The price of bond X is 1073.78 – 138.88 = 934.90. The annual yield rate is $1.015^4 - 1 = 0.06136$. Then,

$$934.90 = Ra_{\overline{15}|0.06136} + 1000(1.06136)^{-15} = 9.62652R + 409.3165 \Rightarrow R = 54.60.$$

456. Solution: E
Assume the face amount is 1. The price of the bond is $0.015a_{\overline{40}|0.01} + 1.01^{-40} = 1.1642$.
At the given coupon and return rates, the premium is 1.1642 – 1 = 0.1642.
We are looking for the time at which the book value is 1 + 0.1642/2 = 1.0821.
Then,

$$\begin{aligned}1.0821 &= 0.015a_{\overline{40-k}|0.01} + 1.01^{-(40-k)} \\ &= 0.015\frac{1-1.01^{-(40-k)}}{0.01} + 1.01^{-(40-k)} \\ &= 1.5 - 0.5(1.01)^{-(40-k)}\end{aligned}$$

$$0.8328 = 1.01^{-(40-k)}$$

$$40 - k = -\frac{\ln 0.8328}{\ln 1.01} = 18.03$$

$$k = 21.97.$$

For the book value to exceed 1.0821, round up to k = 22.

457. Solution: B
Let x be the amount of a coupon payment. The book value right after the n^{th} coupon is

$$95.5087 = xa_{\overline{n}|0.02} + 100v^n$$

The price of the bond is the present value of the time n book value plus the present value of the first n coupons.

$$91.8243 = 95.5087v^n + xa_{\overline{n}|0.02} = 95.5087v^n + 95.5087 - 100v^n$$

$$3.6844 = 4.4913v^n$$

$$n = -\ln(3.6844/4.4913)/\ln(1/1.02) = 10.$$

458. Solution: B

$$P_A(i) = 1000(1+i)^{-2} + 3000(1+i)^{-3} + X(1+i)^{-5}$$

$$P_A(0.1) = 3089.39 + 0.62092X$$

$$-P'_A(i) = 2000(1+i)^{-3} + 9000(1+i)^{-4} + 5X(1+i)^{-6}$$

$$-P'_A(0.1) = 7649.75 + 2.82237X$$

$$4 = \frac{7649.75 + 2.82237X}{3089.39 + 0.62092X}$$

$$X = 13,793.98$$

$$P_B(i) = 1000(1+i)^{-3} + 3000(1+i)^{-4} + 13,793.98(1+i)^{-7}$$

$$P_B(0.1) = 9878.85$$

$$-P'_B(i) = 3000(1+i)^{-4} + 12,000(1+i)^{-5} + 7(13,793.98)X(1+i)^{-8}$$

$$-P'_B(0.1) = 54,545.05$$

$$Y = \frac{54,545.05}{9878.85} = 5.5214$$

459. Solution: A

$$L = \frac{10,000}{0.07}(1.07) = 152,857.14$$

$$\bar{d} = \frac{(Ia)_{\overline{n}|}}{\ddot{a}_{\overline{n}|}} = \frac{\dfrac{1}{0.07} + \dfrac{1}{(0.07)^2}}{\left(\dfrac{1}{0.07}\right)1.07} = 14.2857$$

$$\bar{v} = \frac{14.285714}{1.07} = 13.351135$$

$$M = 152,857.14\left[1 - (-0.02)(13.351135\right] = 193,673.47$$

$$M - L = 193,673.47 - 152,857.14 = 40,816.33$$

460. Solution: B

The present value of the geometric payment perpetuity-immediate, at annual effective interest rate *i* and with payment growth rate $r = -0.005$, is given by

$$P(i) = \frac{1}{i-r} = \frac{1}{i+0.005}.$$

The modified duration is therefore given by

$$\bar{v} = -\frac{P'(i)}{P(i)} = -\frac{-\dfrac{1}{(i+0.005)^2}}{\dfrac{1}{i+0.005}} = \frac{1}{i+0.005}.$$

The first-order modified approximation of the present value, at an interest rate *i* near an initial interest rate i_0, is therefore given by

$$E = P(i_0)\left[1-(i-i_0)\bar{v}\right] = \frac{1}{i_0+0.005}\left(1-\frac{i-i_0}{i_0+0.005}\right).$$

Therefore, using 0.065 for i_0 and 0.055 for *i*, we find that the percentage error in this approximation is

$$\frac{E-P}{P} = \frac{\left(\dfrac{1}{0.065+0.005}\right)\left(1-\dfrac{0.055-0.065}{0.065+0.005}\right)-\left(\dfrac{1}{0.055+0.005}\right)}{\left(\dfrac{1}{0.055+0.005}\right)} = -0.0204 = -2.04\%.$$

461. Solution: D

The present value of the geometric payment perpetuity-due, at annual effective interest rate *i* and with payment growth rate $r=-0.02$, is given by

$$P(i)=\frac{1+i}{i-r}=\frac{1+i}{i+0.02}.$$

The Macaulay duration is therefore given by

$$\bar{d}=(1+i)\bar{v}=\frac{(1+i)\left(-P'(i)\right)}{P(i)}=-\frac{(1+i)\left[\frac{0.02-1}{(i+0.02)^2}\right]}{\frac{1+i}{i+0.02}}=\frac{\frac{1-0.02}{(i+0.02)^2}}{\frac{1}{i+0.02}}=\frac{0.98}{i+0.02}.$$

The first-order Macaulay approximation of the present value, at an interest rate *i* near an initial interest rate i_0, is therefore given by

$$E=P\left(i_0\right)\left(\frac{1+i_0}{1+i}\right)^{D_{\text{mac}}\left(i_0\right)}=\left(\frac{1+i_0}{i_0+0.02}\right)\left(\frac{1+i_0}{1+i}\right)^{\frac{1-0.02}{i_0+0.02}}.$$

Using 0.08 for i_0 and 0.07 for *i*, we find that the percentage error in this approximation is

$$\frac{E-P}{P}=\frac{\left[\left(\frac{1.08}{0.08+0.02}\right)\left(\frac{1.08}{1.07}\right)^{\frac{0.98}{0.08+0.02}}\right]-\frac{1.07}{0.07+0.02}}{\frac{1.07}{0.07+0.02}}=-0.004883=-0.49\%.$$

462. Solution: D

The outstanding loan balance here can be calculated using either the prospective approach or the retrospective approach, although the retrospective approach is easier, since it does not require the value of *n*.

$$5,000,000(1.0325)^{10}-303,244.53s_{\overline{10}|0.0325}=3,367,821.18.$$