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Geospatial Metrics for Insurance Risk Concentration and Diversification

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Abstract

(Re)insurance practitioners view geospatial variability of insurable losses and claims as a significant risk factor in the definition of key business processes and their outcomes. This second-order, volatility type of risk factor impacts the construction of insurance rates and reinsurance treaty premiums, the computation of reserve capital and the management of concentrations of physical and financial risk. With increased industry emphasis on analytics, modeling and measuring of all types of physical and financial variability and volatility, both temporal and geospatial, new efforts are needed to enrich the scope of metrics that capture the nature of second-order risk.

In its first generation, second-order metrics are pairwise by nature. Significant effort by academics and practitioners is under way to develop a new generation of such metrics, which capture and express the complexities of concentration and interconnectedness of multiple risk factors that are physical, geospatial and financial in nature. Clarity of and intellectual discipline in the definition of second-order geospatial risk metrics helps (re)insurance practitioners to adopt these statistical and computational methodologies effectively and promptly. Further, clarity, consistency and coherence of second-order geospatial risk metrics allows practitioners to relate them efficiently to the main business workflows of (re)insurance firms, to apply them in effective measurement when mitigating and hedging situations, and to promulgate them easily to executive-level decision makers.

[1] The views and opinions expressed in this article are those of the author and do not necessarily reflect the position of any other organization, employer or company. Assumptions made in this paper are not reflective of the position of any entity other than the author, nor do they reflect on any real-world business performance.

Achieving a sustainable scale of business operations is a key objective for insurance and reinsurance firms today. This drive toward market scale takes many forms. It is expressed in the motivation to acquire larger market shares of gross underwritten premiums by various means, the most obvious being through acquisitions of business lines or whole books of insurance business. Tangible economies of scale are also an internal objective and a driving force in pursuing optimal cost and distribution of reserve capital among the firm's business units. Whatever the business realization of the drive toward market scale, the achievement of this goal has two very different but intersecting implications for the insurance firm. The first is the emerging demand for measuring and accounting for the effects of diversification in a (re)insurance book of business due to geospatial insurance coverage or financial risk factor interdependencies. The second is measuring and accounting for the concentration and clustering of insurance risks, which are subject to mutually, highly contingent outcomes. For top-level practitioners at a (re)insurance firm who want to achieve scale in insurance operations, a critical task is to master the modeling and measurement of interdependence among risk factors in all forms—including the fundamental ones of exposure and expected loss diversification.

The objective of this paper is to review existing geospatial risk metrics, contribute to the development of new ones and attempt to enhance the market applicability of such techniques and methodologies for measuring the interdependence for insurance risk factors. Another objective is to outline their direct use and applications in practical risk management for an insurer portfolio, more particularly in managing an underwriting concentration of insurance exposures and henceforth defining and following up with capital reserving tasks. The paper derives a set of metrics and indices from modeled and simulated insurance losses for a detailed physical and geographical distribution of insured exposures that comprise a book of business for a notional regional firm.

Section 1 reviews the economic theory and motivation for developing diversification and concentration risk metrics from the perspective of the insurance firm. It also outlines the structure of the insured exposure of the case study (notional insurer) used to compute and construct the numerical risk metrics and indices. Section 2 provides the details of the insurance loss modeling methodology and simulation, which produces the outputs for the development and construction of all numerical metrics and indices. These are presented in section 3, which develops and discusses three new geospatial risk metrics. Section 4 shows how the process of back allocating losses and risk metrics from a global portfolio level down to single risk components affects the interdependence of these same metrics within the overall risk profile of the insurance firm's book of business. Section 5 continues to focus on applied risk management by introducing three new diversification and concentration portfolio indices. Section 6 concludes with an analysis of the applied utility of these metrics for insurance risk management professionals and outlines some further research directions.

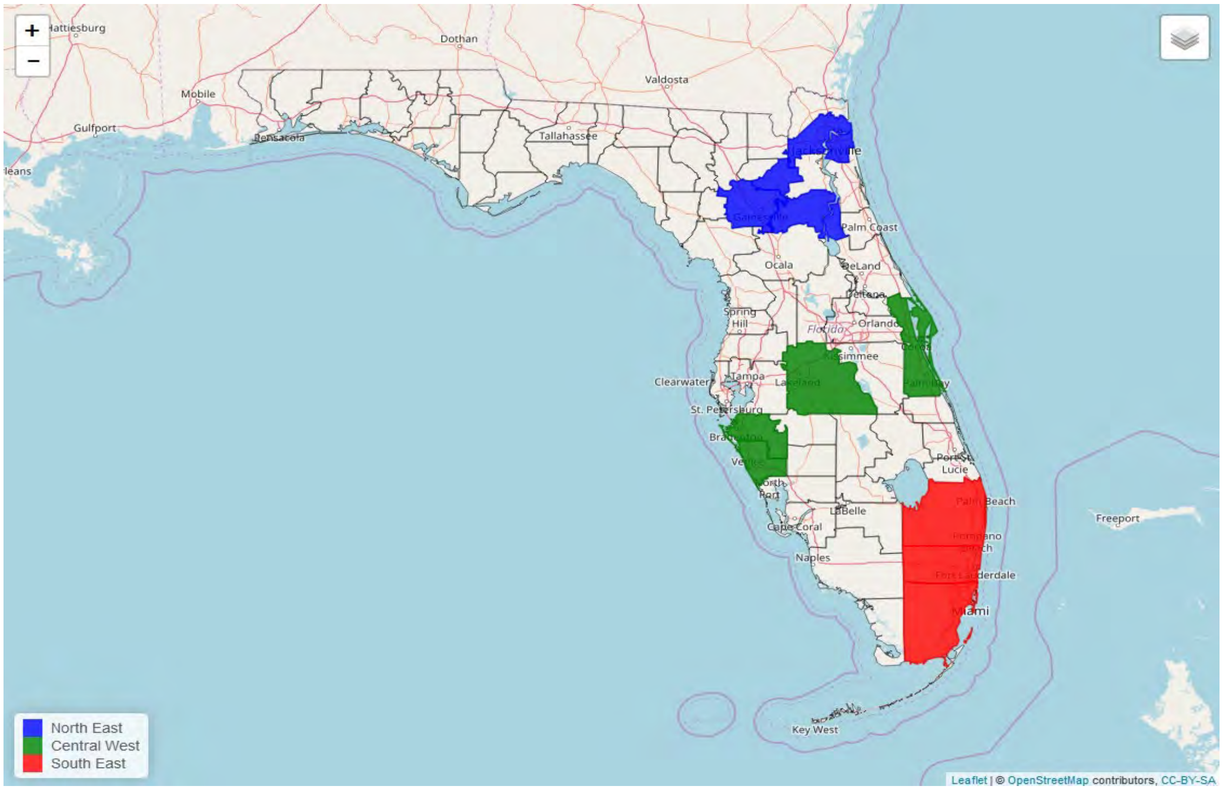
1. Economic Theory and Motivation

Connectedness, also described as *interconnectedness*, has become a central theme in modern risk management, but it has remained technically ambiguous and less well defined than traditional statistical, pairwise correlation metrics. Typically, modern financial and insurance portfolio correlation metrics are pairwise and are applied in linear Gaussian methodological and modeling frameworks. The impact of insurance risk interconnectedness on critical market and business tasks such as pricing, underwriting, concentration management and reserving demands further improvements in analytical and measurement practices. This article focuses on measuring this interconnectedness and its effects on a critical portfolio risk factor—the geospatial distribution of insurance claims and losses among the company business units of an insurance firm. Loss analytics, rate making and premium pricing practices, which are among the *company units* within an insurance firm, are interconnected through the impact of claims from extreme events with large geospatial footprints that cause what practitioners term “clustering” or “concentrations” in the accumulation of losses. Such undesirable loss accumulations may take place as well, during smaller catastrophic events that occur in clustered patterns both spatially and temporally (in a smaller geographic area with less time between events), a phenomenon known as *temporal clustering*. Since premium prices, insurance rates and reserves greatly depend on a modeled expectation of insurance loss, practitioners are also concerned about the volatility, or various expressions of uncertainty, in such loss expectations. Section 3 of this paper examines how identifiable and measurable connectedness impacts the variability in loss expectations and the definition of key business metrics. It shows that in the presence of strong and measurable connectedness among risk factors, both volatility of loss and variability of risk metrics are magnified by extreme events and extreme disaster scenarios.

To provide an analytical framework for deriving and testing risk metrics, let’s study a theoretical insurance firm based in Florida. It is composed of three business units covering insured risks in 12 geoadministrative areas, defined as state counties. These three business units are named South East (SE), Central Unit (CU) and North East (NE). The risks in two of the business units—South East and North East—are located in immediately neighboring administrative areas (counties), while those in the third (Central) are in proximity but not strictly adjacent to each other, as illustrated in Figure 1. In addition, SE and NE business units comprise risks from four geographically bordering counties, while CU comprises risks from only three counties. Finally, these three business units constitute the whole book of business for this notional and regional insurer.

The general purpose of this analysis is the modeling, measurement and understanding of dependencies among these business units and then the presentation of results through the case study developed in sections 3 and 4. Let’s begin by discussing three new metrics, which are proposed to facilitate the tasks of measuring second-order risk and interdependence among business units (but more generally among risk factors) in an insurance firm. The contribution to risk management of these new metrics is mainly twofold: (1) the potential to enhance the accuracy of capital reserving, and (2) the potential to facilitate optimal selection of second-order, risk-mitigating insurance and financial contracts and instruments.

Figure 1. Three Business Units Constituting the Insurer's Book of Business



We will examine traditional pairwise connectedness measured between any two of the firm's business units, as well as between any two geoadministrative counties within the same unit, and in parallel within the full portfolio. We will also compute and analyze "marginal connectedness," which is defined in this study as the connectedness of a single company unit or geographic area to the remainder of the insured portfolio. Subsequent sections will show how both metrics of pairwise connectedness and metrics of marginal connectedness become valuable tools in managing concentration and risk-ranking analysis, pricing and underwriting, and reserving and risk management.

2. Physical Peril Simulation for Insurance Loss

The numerical study of this article uses simulated and modeled insurance losses produced by AIR Worldwide tropical cyclone model for North America, which includes coverage for Florida, where the insured exposures of our notional firm are located. The model can be viewed as a sequence of conditional statistical algorithms, which this section describes at a very high level. The primary objective of this work is not to review and provide detailed analysis of this type of natural catastrophe model for the insured peril of tropical cyclones. Rather, it is to use the model's loss outputs to illustrate numerically, as well as theoretically, methodologies for second-order, geospatial risk management, which in turn can be used by insurance industry practitioners.

The first set of modeling algorithms derives statistical probability distributions from historical annual frequency data of recorded storms. The critical variables in defining the physical model of a tropical cyclone are central barometric pressure, radius of maximum winds and forward speed. Central pressure is the primary physical determinant of the storm and, therefore, the primary modeling function to define the key variables for the computation of its intensity. Theoretical statistical distributions for all three variables are used as baseline generators and tested for goodness of fit to historical meteorological data. In the third stage of the model sequence of algorithms, a large sample of simulated and fully probabilistic storm tracks are generated. These tracks are generated from conditional probabilistic distributions derived from a large historical data set of recorded storm events. The sampled and simulated physical parameters are propagated through the storm track, and an analytical equation estimates the storm intensity on a predefined geospatial grid of a very high three-dimensional metric and temporal granularity. In the last step of this sequence, modeled storm intensity is linked through a nonlinear damageability response function to produce the final insurable loss values for a predefined insured exposure with a known monetary value, located by latitude and longitude coordinates on the "global" geospatial model grid. Then insured loss is computed by traditional actuarial formulas that apply insurance policy limits and deductibles on the actual insurable loss coming from the natural peril model simulation.

$$Insured\ loss = \min[limit, \max(Insurable\ loss - Deductible, 0)]$$

3. Toward Second-Order Insurance Risk Management

At the technical and quantitative level, this work is directed toward developing principles, techniques and tools for comprehensive second-order insurance risk management. In the context of a single insurance firm, this problem is defined as the measurement and management of single insurance risk and business unit dependencies and connectedness. For a typical insurance firm, such interdependencies in its book of business are manifold, including those of a geospatial, physical and financial nature. In parallel and at the macroeconomic level, the risk measurement problem becomes one of defining and quantifying the systemic nature of the interconnectedness of insurance and financial firms. This article focuses on the geospatial and physical interconnectedness of insurance risks at the microstructure of a single insurance firm. Thus, the data, computations, examples and case study reflect this scenario.

Let's define a geospatial, second-order risk metric to be a function ρ, which assigns a real number from the spatially aggregated distribution of insured losses $[X_{i,n}]$ by any predefined geographic and administrative unit, which corresponds to a business unit of our hypothetical insurance firm. Later in the study, we will also use noncoherent risk measures such as value-at-risk (VaR), also known as probable maximum loss (PML):

$$\rho[X_n] = VaR_\alpha(X_n) = \inf\{x \mid P(X_n > x) 1 - \alpha\}$$

In the process of deriving geospatial risk metrics, we will make extensive use of coherent measures such as tail-value at risk (TVaR), which is defined as:

$$\rho'[X_n] = TVaR_\alpha(X_n) = \frac{1}{1-\alpha}\int_\alpha^1 VaR(X_n)dt$$

We can expect noncoherent risk measures, such as VaR to display both superadditive and subadditive properties in accumulation, depending on the numerical and empirical case and the physical and statistical premises of the simulation. In short, they will not behave in a theoretically persistent manner.

$$Superadditivity = \sum_n \rho[X_i] < \rho\left[\sum X_{i,n}\right]$$

$$Subadditivity = \rho\left[\sum X_{i,n}\right] < \sum_n \rho[X_i]$$

However, a coherent risk measure, such as TVaR, can be expected to behave strictly in a subadditive manner in accumulations from portfolio units to the global risk profile of the book of business. We will also make use of the positive homogeneity property of risk measures:

$$\rho[kX_i] = k\rho[X_i], k \geqq 0$$

The homogeneity property is expected to hold for both coherent and noncoherent classifications of second-order geospatial risk metrics. In our notional insurance book, comprising three business units, we notate the respective loss distributions by unit as:

for $i = 1\ to\ n$, South East as $\left[X_{SE,1}, \dots, X_{SE,n}\right]$
for $j = 1\ to\ n$, Central Unit as $\left[X_{CW,1}, \dots, X_{CW,n}\right]$
for $k = 1\ to\ n$, North East as $\left[X_{NE,1}, \dots, X_{NE,n}\right]$

Where *n* is 10,000 stochastic, simulated loss scenarios for each of these spatial loss distributions.

The first geospatial risk metric is most appropriately defined as a *covariance ratio*. This metric is computed as the ratio of the covariance of insured losses of any two of the business unit pairs to the sum of the variances of these same loss distributions. For example, the covariance ratio metric for business units South East and Central Unit is shown as $CR_{SE,CU}$ and is expressed in its traditional statistical form:

$$CR_{SE,CU} = \frac{COV[X_i : X_j]}{VAR[X_i] + VAR[X_j]}$$

The covariance ratio metric is computed for all three pairs of business units in Table 1.

Table 1. Covariance Ratio for Each Pair of Business Units

	South East	Central Unit	North East
South East	—	0.24	0.05
Central Unit	—	—	0.19
North East	—	—	—

With this first risk metric, numerical results converge to first principles from expectations of the importance of cumulative geospatial distances, being a main determinant in its explanatory strength. The distances $d_{i,k}$ between any two pairs of individual insured risks, such as $r_{SE,i}$ *and* $r_{CU,k}$, are accumulated to measure the total conceptual and cumulative geospatial distance *D* between any two business units, in this case South East and Central Unit:

$$D_{SE,CU} = \sum_{i \neq k}^{n} d_{i,k}$$

The two business units of South East and North East, which have the largest cumulative geospatial distance between their respective composite risks, as expected, have the lowest covariance ratio.

The second risk metric—called *covariance percent share,* or *covariance share* (CS)—is a partial transformation of the covariance ratio. This metric is defined as the percent share of the covariance of any two business units' losses to the full sum of the covariance matrix of all pairs of business units' losses. Again, in the example case of South East and Central Unit, the new metric ($CS_{SE,CU}$) takes the following form:

$$CS_{SE,CU} = \frac{COV[X_i : X_j]}{\sum \sum COV[X_{i,j,k} : X_{k,j,i}]}$$

The covariance percent share metric is computed for all three notional business unit pairs in table 2.

Table 2. Covariance Percent Share for Each Pair of Business Units

	South East	Central Unit	North East
South East	—	0.81	0.15
Central Unit	—	—	0.05
North East	—	—	—

The third metric quantifies the marginal impact of second-order insurance risk by measuring the geospatial interdependence of each business unit to the accumulated insurance loss of all other business units. Thus, in our case study, the remainder of the book of business is constructed of two units, as the whole book comprises three. This metric is called the *marginal covariance* (MC) *ratio*. To compute this metric, we must create three marginal portfolio loss distributions, which are defined as the aggregated sum of each combination of any two of the three business units' simulated losses. For example, the aggregated loss distribution without the North East business unit losses $[X_{NE,k}]$ is defined as the accumulated loss of the other two business units—Central Unit and South East. This combined partial loss $Q_{1,n}$ is produced by a joint generating function q:

$$Q_1 = q\left[X_{1,CU}, X_{SE,1}\right], \dots, Q_n = q\left[X_i, X_j\right]$$

The other two biregional loss distributions are expressed in similar joint, partial form with aggregate loss generating functions $q' and\ q''$, respectively .

$$\text{Central Unit and North East} = \left[q'(X_{1,CU}, X_{1,NE}), \dots, q'(X_j, X_k)\right]$$

$$\text{South East and North East } = \left[q''(X_{1,SE}, X_{1,NE}), \dots, q''(X_i, X_k)\right]$$

$$\text{for } i, j, k \text{ from 1 to } n = 10{,}000$$

With the two components of single unit losses and partial portfolio losses, the marginal covariance itself is computed between any single business unit and the aggregated loss of the remainder of the portfolio. In our example, the marginal covariance MC_{NE} of losses for North East, $X_{NE,k}$ will be expressed to the aggregate loss $q(X_{SE,i}, X_{CU,j})$ from South East and Central Unit. Then the marginal covariance takes traditional statistical form:

$$MC_{NE} = COV[X_{NE,k}: q(X_{SE,i}, X_{CU,j})]$$

In turn, the marginal covariance ratio for any business unit (including North East, MCR_{NE}, in our example) is formally defined as the ratio of the actual marginal covariance to the sum of the single unit variance and the partial variance of the remainder of the book of business:

$$MCR_{NE} = \frac{COV[X_{NE,k}: q(X_{SE,i}, X_{CU,j})]}{VAR\left[X_{NE,k}\right] + VAR\left[q(X_{SE,i}, X_{CU,j})\right]}$$

This new marginal covariance ratio metric is computed for the portfolio's three business units in table 3.

Table 3. Marginal Covariance Ratio for Each Business Unit

	Central Unit & South East	North East & South East	Central Unit & North East
South East	—	—	0.23
Central Unit	—	0.28	—
North East	0.05	—	—

Each of these three metrics presents a different but mutually complementary picture of active practices for measuring geospatial variability and interdependence in second order of insurance risk. We started by developing a purely pairwise metric of covariance ratio and then transformed it to a still pairwise but now advantageously unitary, covariance percent share metric. Last, we arrived at a marginal impact type of metric—marginal covariance ratio—that is no longer purely pairwise and is not unitary; rather it describes the relationship of every single unit to the accumulated remainder of the insurance book of business. As expected, business units, which comprise geographically neighboring counties, produce metrics that indicate higher degrees of dependence in second order of geospatial insurance risk. Then in a reverse relationship, we observe that business units with the largest cumulative spatial distances produce geospatial metrics of the lowest interdependence. These new metrics allow for the explicit quantification and ranking of business units by magnitude of "second-order riskiness." This ranking practice can also be used effectively within a business unit to understand the relationships between and the contributions of individual risks to the overall profile of the unit; hence, the metric becomes a valuable ranking tool for concentration and underwriting management. Understanding the exposure and expected loss outcomes of the portfolio geospatial risk distributions provides a much improved environment in which practitioners can use these metrics to design and structure mitigating insurance and financial products, as well as for underwriting and risk management tasks and strategies.

4. Covariance Back-Allocation and Geospatial Interdependence of Metrics

This section explicitly develops the links between statistical measures of geospatial interdependence and the practical, business-minded understanding and measurement of portfolio diversification. We can derive further spatial risk measures from insurance risk factors by using portfolio and business unit TVaR, which is a coherent market risk metric and has broad acceptance and understanding among practitioners. The individual business unit risk metrics of TVaR are derived by a procedure of back-allocation, executed downward from the global portfolio level of the total, corporate "global" metric. Back-allocating risk metrics, which practitioners often call a top-down approach, from an insurance or a financial portfolio total metric reflects that interdependence among risk factors at the portfolio level is present and captured in the modeled and global loss distribution of the portfolio risk factors. These effects of risk factor interdependence are contained in the modeled portfolio metric due to the very nature of the joint multivariable insurance loss accumulation. Section 2 described the physical peril model intensity simulation procedure, which models interdependent insurance losses in the geospatial domain on a grid-based, granular system of 30-meter resolution. Two types of physical peril intensity interdependencies are explicitly modeled: the first is geospatial, which captures dependencies among losses within a single simulated scenario; and the second is temporal and interevent, which captures dependencies across simulated, temporally clustered catastrophe scenarios. Analysis of the modeling principles and techniques of these effects is a large and separate research subject that is beyond the scope of this article.

Expected and significant benefits of working with accurate and comprehensive second-order metrics for risk management and underwriting guidelines are that both pure technical premiums and risk measurements, such as VaR and TVaR, are optimally back-allocated to individual risks and policies, thus providing a sustainable and competitive edge in pricing and reserving. Capturing interdependencies with back-allocation practices reflects the effects of diversification or concentration in an insurance book of business, both of which are critical in making business decisions. The rest of this study will use the metric of TVaR as the theoretically coherent choice for a portfolio and business unit risk metric. With respect to coherence, the most significant and fully required premise for this study is the support of the subadditive principle of accumulations of a risk metric ρ. In our notional case study, this principle is expected to hold in accumulations of the risk metric ρ from the business unit to the portfolio global level:

$$\rho[Portfolio] \leq \sum_{i=1}^{k} \rho[Business\ unit]$$

In the processes of corporate risk management, one practical translation of measuring interdependence is the identification and measurement of portfolio diversification. With a good understanding and effective application of such principles, practitioners can then take advantage of measuring and understanding diversification while defining and optimizing cost savings from optimal capital reserves allocation (Zvezdov and Rath 2017).

This algorithmic flow for measuring portfolio diversification takes a few steps of allocation and transformation. It begins by measuring a corporate, global TVaR from the aggregated and full loss distribution Y_p of the entire insurance book of business, composed of its three business units, with a joint aggregate loss generating function g:

$$Y_1 = g\left[X_{CW,1}, X_{SE,1}, X_{NE,1}\right], \dots, Y_n = g\left[X_{CW,n}, X_{SE,n}, X_{NE,n}\right]$$

$$\text{for } i, j, k \text{ from 1 to } n = 10{,}000$$

$$VaR_\alpha(Y_n) = \inf\{y \mid P(Y_n > y)1 - \alpha\}$$

$$TVaR_\alpha(Y_n) = \frac{1}{1-\alpha}\int_\alpha^1 VaR(Y_n)dt$$

After computing the numerical $TVaR(s)$ from the individual loss distributions of the business units and then from the combined and joint loss distribution Y_p of the entire portfolio, we can observe a subadditive relationship, as is expected by first theoretical principles of a coherent risk measure. We can now restate this subadditive relationship in a form of TVaR metric accumulation:

$$TVaR[Portfolio] \leq \sum_{i=1}^{k} TVaR[Business\ unit]$$

As a next step in the procedure to measure diversification, this total and global portfolio $TVaR_\alpha(Y_n)$ metric is allocated down to each single contributing risk—in this case, to each business unit.

Practitioners use various numerical techniques for portfolio TVaR back-allocation. In this study, we will examine the covariance and the marginal covariance-variance (MCov/Var) back-allocation principles, which we develop, express, and modify specifically for the needs of our case study. These two capital and risk metric back-allocation techniques are first constructed by expressing the decomposition of the full portfolio covariance matrix $\sum_{i=1}^{n}\sum_{k=1}^{n} COVAR(X_i, X_k)$ into its business unit covariance components:

$$\sum_{i=1}^{n}\sum_{k=1}^{n} COV(X_i, X_k) = COV\left(\sum_{i=1}^{n} X_i, \sum_{k=1}^{n} X_k\right)$$

$$= \sum_{k=1}^{n} COV(X_1, X_k) + \ldots + \sum_{k=1}^{n} COV(X_n, X_k)$$

The marginal covariance of each individual business unit with the total portfolio is then expressed more robustly:

$$\sum_{k=1}^{n} COV(X_i, X_{j+k}) = COV(X_i, \sum_{k=1}^{n} X_{j+k})$$

An interdependent TVaR for each business unit is constructed, with the contribution of a marginal covariance principle back-allocation weight w_i, computed in a ratio form:

$$w_i = \frac{COV(X_i, \sum_{k=1}^{n} X_{j+k})}{COV\left(X_i, \sum_{k=1}^{n} X_{j+k}\right) + COV\left(X_j, \sum_{k=1}^{n} X_{i+k}\right) + COV\left(X_k, \sum_{k=1}^{n} X_{j+i}\right)}$$

The business unit TVaR back-allocation procedure itself is complete, with the following expression showing the case for the South East unit risk metric being dependent on the total portfolio $TVaR_\alpha(Y_n)$. This interdependent business unit metric is defined as a covariance back-allocated measure $TVaR_{CoVAR}[BU]$:

$$TVaR_{COVAR}[BU] = w_{i,BU} TVaR_\alpha(Y_n)$$

This covariance back-allocation relationship inevitably enforces a comonotonic relationship between the sum of the allocated business unit metrics $TVaR_{CoVAR}[BU]$ and the total global portfolio $TVaR_\alpha(Y_n)$:

$$TVaR_\alpha(Y_n) = \sum\nolimits_{i=1}^{k} TVaR[BU]_{COVAR} \leq \sum\nolimits_{i=1}^{k} TVaR[BU]_{Ind}$$

This relationship between the global metric, the back-allocated sum and the sum of independent, stand-alone business unit metrics is a foundation for many daily tasks in the insurance portfolio risk management process. It also serves as a motivation and justification to search for optimal and cost-efficient distribution of capital reserves within the business.

Some practitioners use a back-allocation procedure for both risk metrics and capital reserves based on the marginal business unit covariance and the full variance of the total portfolio loss distribution Y_n:

$$TVaR[X_i]_{MCov/Var} = TVaR_\alpha(Y_n) \frac{COV(X_i, \sum_{k=1}^{n} X_{j+k})}{VAR[Y_n]}$$

This second back-allocation technique does not support a strong theoretical and unitary relationship, as the sum of the marginal covariance of each unit—$COV\left(X_i, \sum_{k=1}^{n} X_{j+k}\right) + COV\left(X_j, \sum_{k=1}^{n} X_{i+k}\right) + COV\left(X_k, \sum_{k=1}^{n} X_{j+i}\right)$—will not equal the variance of the global portfolio loss distribution, $VAR[Y_n]$. This is evident by pure statistical mechanics, which show that the sum of the marginal covariance(s) does not theoretically or necessarily equal the theoretical variance $VAR[Y_n]$ of the combined loss distribution:

$$VAR[Y_n] = VAR[X_i] + VAR\left[X_j\right] + VAR[X_k] + 2\sum_{i=1}^{n}\sum_{k=1}^{n} COV(X_i, X_k)$$

$$VAR[Y_n] \neq COV\left(X_i, \sum_{k=1}^{n} X_{j+k}\right) + COV\left(X_j, \sum_{k=1}^{n} X_{i+k}\right) + COV\left(X_k, \sum_{k=1}^{n} X_{j+i}\right)$$

With three types of metric computed—one stand-alone and independent, a second back-allocated by the pure covariance principle, and a third back-allocated by the marginal covariance-variance principle—we will next explore their ranking and relative standing:

$$\sum TVaR[BU]_{MCov/Var} < \sum TVaR[BU]_{COVAR} < \sum TVaR[BU]_{Ind}$$

The right side of this inequality expresses support for subadditive principles in accumulation, while the left demonstrates the lack of a unitary relationship as described earlier. On one hand, theoretical principles of subadditive accumulations guarantee that the independent sum of risk metrics exceeds the dependent and back-allocated sums through covariance participations in the granularity of business unit risk metrics. On the other hand, the relationship between the pure covariance business unit metrics and those computed through back-allocation by the marginal covariance-variance principle cannot be strongly guaranteed by theoretical and statistical principles. The numerical ratios of the back-allocated business unit metrics to the independent stand-alone business unit metric are summarized in table 4, where the theoretical expressions of the ratios take the following statistical forms:

$$Ratio\ of\ pure\ COVAR\ to\ independent = \frac{TVaR[BU]_{COVAR}}{TVaR[BU]_{Ind}}$$

$$Ratio\ of\ marginal\ COVAR\ to\ independent = \frac{TVaR[BU]_{MCov/Var}}{TVaR[BU]_{Ind}}$$

Table 4. Ratios of Back-Allocated Business Unit Metrics to the Independent Business Unit Metric

	Independent Unit to	
	Pure COVAR	**MCov/Var**
North East	2.73	1.27
Central Unit	2.11	0.98
South East	0.63	0.29

The numerical behavior of subadditive accumulations of insurance portfolio $TVaR(s)$ by business unit supports an economic motivation and proposition that economies of scale and cost-savings benefits can be realized in capital reserving due to risk dispersion and geospatial diversification. The pure covariance back-allocation principle is most widely accepted among practitioners, and it is also the more coherent in mathematical risk theory.

5. Insurance Portfolio Diversification and Concentration Indices

Many of the requirements for cost-effective, optimal capital reserving are dependent on measuring and quantifying portfolio diversification, which is theoretically justified under the mathematical principle of subadditive accumulations. To provide numerical support and proof for this analysis, we will continue to use our three business units—South East, North East and Central Unit—which compose our single notional insurance firm. The first two business units comprise risks from bordering and clustered geoadministrative geographies— counties in Florida. The risks in the last one, Central Unit, are less spatially concentrated (see figure 1).

In the context of our case study, let's examine some indexed diversification measures DI derived from business unit and portfolio covariance and TVaR metrics. For all three business units and the entire portfolio, TVaR is measured at $\alpha = 0.05$. The index construction relies on the mathematical properties of subadditive accumulation of TVaR, which we reviewed in the previous section. Computed from the loss distributions of SE, NE and CU (X_i, X_j and X_k, respectively) and the distribution of the entire insurance book of business (Y_n), these risk metrics numerically provide support for first theoretical principles, expressed in a generalized form:

$$TVaR[Y_n] = TVaR\left[\sum_{i=1}^{n} X_{i,j,k}\right] \leqq \sum_{i,j,k=1}^{n} TVaR\left[X_{i,j,k}\right]$$

The first index, $DI_{VaR/TVaR}$, is composed of two types of components: (1) business unit and portfolio VaR, and (2) TVaR. This first index includes VaR metrics, at both the business and portfolio levels, which theoretically do not satisfy the requirements for mathematical coherence, particularly risk metric subadditivity. In the context of natural catastrophe risk management, VaR metrics, also known as probable maximum loss, are not coherent and subadditive, as are their TVaR counterparts. For the same reason of lack of coherence, VaR metrics are not subject to portfolio back-allocation and are generally computed independently from their underlying, single factor loss distribution for the exposure or unit at risk. Thus, for the construction of the first diversification index, we use risk metrics computed only from the stand-alone and independent loss distributions of the business units (X_i, X_j and X_k, respectively), with i, j and k from 1 to 10,000 simulation scenarios and not derived through the top-down back-allocation procedure, developed in section 4. To detail the expression, with an example case for the North East business unit, the index is constructed as the ratio of the difference of its independent VaR_{Ind} and $TVaR_{Ind}$ at $\alpha = 0.05$ to the sum of the same differences of independent metrics in all three units:

$$DI_{VaR/TVaR} = \frac{VaR_{Ind}[X_i] - TVaR_{Ind}[X_i]}{\sum_{i,j,k=1}^{n}\{VaR_{Ind}[X_i] - TVaR_{Ind}[X_i]\}}$$

The second diversification index is constructed from interdependent business unit $TVaR(s)$. These metrics are back-allocated, by the top-down procedure, from the portfolio-level global metric by the covariance principle outlined in section 4. Logically the index is directly dependent on subadditive relations, so it is appropriately noted as $DI_{COVAR/TVaR}$. We saw in the previous section that due to mathematical coherence of back-allocation, the business unit metrics have a comonotonic relationship to the portfolio level metric. Again for the North East business, the index is formally expressed by:

$$SE\ DI_{COVAR/TVaR} = \frac{TVaR_{COVAR}[X_i]}{TVaR\left[\sum_{i=1}^{n} X_{i,j,k}\right]}$$

To revisit the back-allocation process in the context of index construction, the global portfolio metric $TVaR[Y_n]$ is measured from the stochastically simulated, multivariate and joint distribution of all insurance losses in all businesses of the entire portfolio. Geospatial interdependencies among the individual business units, as well as individual risks, are captured by the covariance back-allocation principle applied to the global portfolio metric. As outlined in section 4, this principle constructs a back-allocation ratio, also known as "back-allocation weight," from each business unit's marginal covariance $COV\left(X_i, \sum_{k=1}^{n} X_{j+k}\right)$ share of the sum total of the portfolio marginal units' covariance matrix $\sum_{i=1}^{n}\sum_{k=1}^{n} COV(\,X_i, X_k)$. The $DI_{COVAR/TVaR}$ index reflects the presence of these correlation effects, while the $DI_{VaR/TVaR}$ index does not. Both diversification indices $DI_{VaR/TVaR}$ and $DI_{COVAR/TVaR}$ are computed in table 5.

Table 5. VaR/TVaR- and COVAR/TVaR-Based Diversification Indices for Each Business Unit

	Diversification Indices	
	VaR/TVaR	**COVAR/TVaR**
North East	0.06	0.10
Central Unit	0.28	0.42
South East	0.67	0.48

Another two diversification indices, based on a covariance allocated metric and an independently computed risk metric, are proposed by Tasche (2008). These indices measure the relationship between a marginal covariance allocated risk metric that fits into the definition of our business unit $TVaR[BU]_{COVAR}$, which is back-allocated through the covariance principle, and a fully independent risk metric such as those which we compute and define as $TVaR[BU]_{Ind}$. The latter, independent metric fits into our definition of a stand-alone business unit $TVaR$, computed from the stand-alone and independent loss distributions of its accumulated and aggregated insurable risks. This diversification metric is defined formally, in the following North East business example, using the adopted notation of Tasche (2008) in the first expression and that of this paper in the second:

$$DI(X_k|Y_n)_{TVaR\ ratio} = \frac{TVaR[X_k|Y_n]}{TVaR_{Ind}[X_k]}$$

$$DI(X_k|Y_n)_{TVaR\ ratio} = \frac{TVaR_{COVAR}[X_k]}{TVaR_{Ind}[X_k]}$$

For the whole insurance book of business, with a joint modeled loss distribution $[Y_n]$ with $n = 10{,}000$ simulation scenarios, the same metric is expressed in similar mathematical logic:

$$DI(Y_n) = \frac{TVaR[Y_n]}{\sum_{i,j,k=1}^{n} TVaR_{Ind}\left[X_{i,j,k}\right]}$$

This set of five indices for each business unit and the whole book of business is computed in table 6.

Table 6. Business Unit Diversification Ratio Indices Based on Covariance and Independent TVaR(s)

	COVAR-TVaR/Independent Ratio
North East	2.73
Central Unit	2.11
South East	0.63
Portfolio	0.92

To summarize our numerical analysis, we have produced, generalized and quantified three diversification and concentration indices: $DI_{VaR/TVaR}$, $DI_{COVAR/TVaR}$ and $DI_{TVaR\ ratio}$. All three point to the same relative ranking of business units, by the quantified patterns and metrics of risk clustering and risk concentration, summarized in table 7.

Table 7. Relative Ranking of Risk Concentration and Clustering by Business Unit

	Ranking*		
	VaR/TVaR	COVAR/TVaR	COVAR/TVaR Ratio
North East	3	3	3
Central Unit	2	2	2
South East	1	1	1

*1 = the highest concentration and clustering, while 3 = the lowest.

6. Conclusions

The interpretation of diversification indices and geospatial metrics provides meaningful analytics and a support tool for practitioners to use in systemically examining both actual and observed claims and simulated loss relationships of subadditive accumulations, geospatial interdependence and diversification. Such indices and metrics inform practitioners of risk clustering in the spatial and temporal domains, which is an opposing effect to aims and promises of portfolio diversification. Risk clustering and concentration is a highly undesirable effect of insurance underwriting by all risk managers and business unit managers. Furthermore, the effects of concentration and clustering of risk in physical, geospatial and temporal patterns are measurable, quantifiable and manageable using various exposure redistributions techniques, risk dispersion and the transfer of reinsurance and capital market contracts.

It is clear that the traditional pairwise metrics of Gaussian model domains are insufficiently equipped to capture and describe the complexities of risk factor interconnectedness, which new generations of computationally powerful models make it possible to derive in enterprise IT environments today. The analytical and numerical efforts in this paper attempt to contribute toward the development and propagation of a set of geospatial and second-order risk metrics that capture in a more coherent manner the effects of diversification, concentration and connectedness among complex risk factors in an insurance book of business. Coherence, consistence and clarity are indispensable requirements for such metrics, so they can be easily understood and have an impact on decisions made by executive stakeholders.

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