SOCIETY OF ACTUARIES

Advanced Long-Term Actuarial Mathematics Exam

Exam ALTAM

Date: October 22, 2024

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 6 questions numbered 1 through 6 with a total of 60 points. The points for each question are indicated at the beginning of the question.

2. **Question 1 is to be answered in the Excel workbook. For this question only the work in the Excel workbook will be graded**.

3. Questions 2-6 are to be answered in pen in the Yellow Answer Booklet provided. For these questions graders will only look at the work in the Yellow Answer Booklet. Excel may be used for calculations, for referencing tables, or for statistical functions, but any work in the Excel booklet will not be graded.

4. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document

Excel Answer Instructions

1. For Question 1, you should answer directly in the Excel Question worksheet. The question will indicate where to record your answers.
2. You should generally use formulas in Excel rather than entering solutions as hard coded numbers. This will aid graders in assigning appropriate credit for your work.
3. Graders for Excel questions will not have access to any comments or calculations provided in the Yellow Answer Booklet.
4. For Question 1, you may add notes to the Excel Question worksheet if you feel that might help graders. However, these should be entered directly into the Excel Question worksheet. Graders may not be able to read notes entered as comments.
5. When you finish, save your Excel workbook with a filename in the format xxxxx_ALTAM where xxxxx is your candidate number. Your name must not appear in the filename.

Pen and Paper Answer Instructions

1. Write your candidate number and the number of the question you are answering at the top of each sheet. Your name must not appear.

2. Start each question on a fresh sheet. You do not need to start each sub-part of a question on a new sheet.

3. Write in pen on the lined side of the answer sheet.

4. The answer should be confined to the question as set.

5. When you are asked to calculate, show all your work including any applicable formulas in the Yellow Answer Booklet.

6. If you use Excel for calculations for pen and paper answers, you should include as much information in the Yellow Answer Booklet as if you had used a calculator, including formulas and intermediate calculations where relevant. Written answers without sufficient support may not receive full or partial credit.

7. When you finish, hand in <u>all</u> your written answer sheets to the Prometric Center staff. Be sure to hand in all your answer sheets because they cannot be accepted later.

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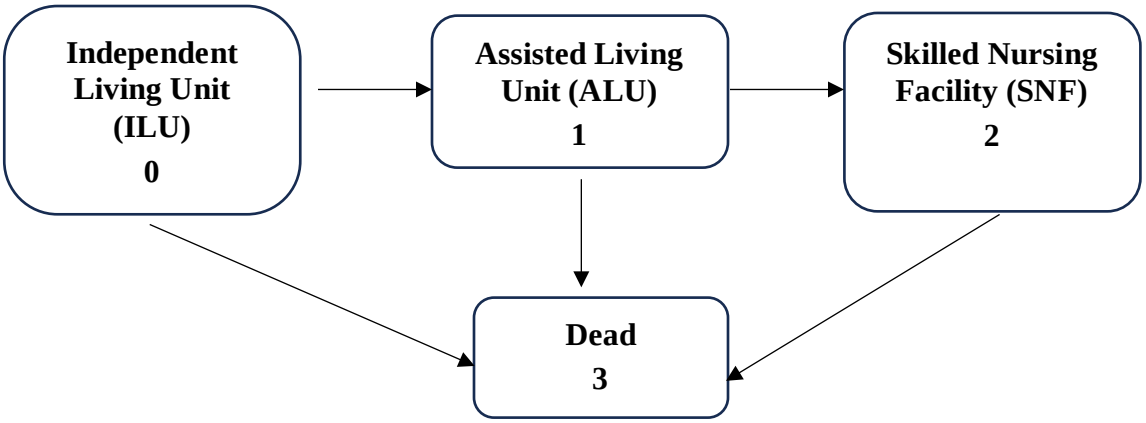
BEGINNING OF EXAMINATION*
****ADVANCED LONG-TERM ACTUARIAL MATHEMATICS****

1.

(*11 points*)

Provide the response for this question in the Excel spreadsheet.

You are using the four state model illustrated below to value costs and fees for an individual in a CCRC.



The transition intensities are:

$\mu_x^{01} = 0.04 \qquad \mu_x^{12} = 0.04$

$\mu_x^{03} = A + Bc^x \quad \mu_x^{13} = 2\mu_x^{03} \quad \mu_x^{23} = 5\mu_x^{03}$

where $A = 0.0002, \quad B = 2.00 \times 10^{-6}, \quad c = 1.13$

Interest rate for valuing cash flows: $i = 0.04$

Age at entry: $x = 80$

1. Continued

(a) (*4 points*)

(i) You have been asked to calculate transition probabilities for lives age 80 entering the Independent Living Unit (ILU).

Complete the table in columns T to AC of the Excel worksheet, using Euler's forward method.

Use a step size of $h = 0.25$.

You may add columns for your own calculations to the right of the table. Do not insert columns or rows into the table.

(ii) Calculate the probability that a life currently age 80, living in the ILU, will still be alive at age 90. You should find the answer is 0.43 to the nearest 0.01.

For parts (b), (c), and (d), use the probability table from (a).

You may add columns for your own calculations to the right of the table. Do not insert columns or rows into the table.

(b) (*3 points*)

(i) Calculate $\ddot{a}_{80}^{00(4)}$. You should find the answer is 6.5 to the nearest 0.1

(ii) Calculate $\ddot{a}_{80}^{01(4)}$. You should find the answer is 1.0 to the nearest 0.1

(iii) Calculate $\ddot{a}_{80}^{02(4)}$. You should find the answer is 0.1 to the nearest 0.1

You are given the following information about costs and fees for the CCRC. Assume all costs are payable at the start of each quarter.

Cost of ILU care, per quarter-year	10,000
Cost of ALU care, per quarter-year	22,500
Cost of SNF care, per quarter-year	30,000
Entry fee for Type A contract	100,000

1. Continued

(c) (2 *points*) After paying the entry fee, residents pay level fees at the start of each quarter, independent of the level of care, throughout their stay in the CCRC. Calculate the annual fee rate for an 80-year-old new entrant to the ILU, using the equivalence principle.

(d) (*2 points*) The CCRC is considering adding a 50% refund of the entry fee, payable at the end of the quarter of death. Calculate the increase in the annual fee rate for the 80-year-old new entrant if this change is implemented.

2.

(*8 points*) A life age 60 purchases a 5-year critical illness policy which pays 100,000 at the end of the year of death or of diagnosis of a critical illness, whichever occurs first. There is no benefit on lapse.

The insurer uses the following multiple decrement table, where decrement (1) represents deaths, decrement (2) represents critical illness (CI) diagnoses, and decrement (3) represents lapses.

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
60	10,000	50	105	400
61	9,445	52	115	500
62	8,778	53	125	600
63	8,000	54	140	500
64	7,306	55	152	0

(a) (*2 points*)

(i) Calculate the probability that the policy generates a claim.

(ii) Given that the policy generates a claim, calculate the probability that the triggering event was a CI diagnosis.

(b) (*3 points*) Assume that the respective forces of decrement for deaths and critical illness diagnoses are constant within each year of age, and assume that all lapses occur at the year end.

(i) Show that the independent rate of decrement for lapses at age 60 is 0.041 to the nearest 0.001. You should calculate the value to the nearest 0.0001.

(ii) Calculate the independent rate of decrement for deaths at age 60.

(iii) Calculate the independent rate of decrement for CI diagnoses at age 60.

2. Continued

(c) (*3 points*) The insurer decides to restrict the definition of critical illnesses covered by the policy for new policyholders, such that the new independent rates of decrement for CI diagnoses will be 50% of the current rates. At the same time, the insurer reduces the independent lapse rate assumption to 60% of current rates. Assume that the independent mortality rates are not affected. All other assumptions are as in (b).

(i) Calculate the revised value of $d_{60}^{(3)}$.

(ii) Explain in words why the revised value of $d_{60}^{(1)}$ will be greater than the original value.

3.

(*10 points*) You are the actuary for a small defined benefit pension plan. You are given the following information.

(i) The plan is a career average earnings plan with an accrual rate of 2%.

(ii) The retirement benefit for each plan member is a whole life annuity payable monthly in advance from age 65.

(iii) On the death of a plan member after retirement a 75% reversionary benefit is payable to their surviving spouse for the remainder of the spouse's lifetime.

(iv) The current membership information, as of December 31, 2023, is given in the following table.

Active Members				
Employee	Exact Age	Years of Service	Accrued Benefits	2023 Salary
A	25	2	1,100	30,000
B	45	10	7,400	40,000
C	45	15	10,500	40,000
D	55	25	20,000	50,000
E	55	30	25,000	50,000
Retired Member				
F	70		25,000	

The valuation assumptions are as follows:

(i) There are no exits before age 65.

(ii) Each member, including the retired member, is assumed to have a spouse of the same age as the member.

(iii) Post-retirement mortality for members and their spouses follows the Standard Ultimate Mortality Model.

(iv) Members and their spouses are assumed to have independent future lifetimes.

(v) All salaries increased by 2% on January 1, 2024.

(vi) Monthly annuity values are determined using Woolhouse's 2-term approximation.

3. Continued

(vii) The Traditional Unit Credit funding method is used for valuing the accrued benifits and for determining the Normal Contribution.

(viii) $i = 0.05$

(a) (*1 point*) Show that $\ddot{a}^{(12)}_{65|65} = 1.9$ to the nearest 0.1. You should calculate the value to the nearest 0.0001.

(b) (*3 points*) Show that the Actuarial Liability, as of January 1, 2024, is 827,000 to the nearest 1000. You should calculate the value to the nearest 1.

(c) (*2 points*) Calculate the Normal Contribution for 2024 as a percentage of the total active members' salaries in 2024.

(d) (*3 points*) Following your valuation, the Plan Sponsor decides to improve benefits for active members as follows. The annual retirement benefit will be equal to

- 2% of 2023 salary per year of service, for service prior to January 1, 2024 plus
- 2% of all earnings after January 1, 2024.

(i) Calculate the increase in the Actuarial Liability, as of the valuation date January 1, 2024.

(ii) State with reasons whether the Normal Contribution would increase, decrease, or stay the same.

(e) (*1 point*) Assume that the marginal post-retirement mortality of members and their spouses is unchanged, but that there is a common shock risk while both are alive. State with reasons whether the Actuarial Liability for the pension plan would increase, decrease or stay the same.

4.

(*11 points*) An insurance company sells fully discrete 2-year term life insurance policies to lives age 60. The death benefit is 1,000,000. The gross premium is 4,300.

You are given the following profit testing assumptions:

(i) Mortality follows the Standard Ultimate Mortality Model.

(ii) The precontract expense is 1000.

(iii) Commissions are 5% of the gross premium, paid at the beginning of each year.

(iv) Annual maintenance expenses are 100, paid at the beginning of each year including the first.

(v) The lapse rate is 8% of the policies in-force at the end of the first year. There is no benefit payable on lapse.

(vi) The earned interest rate is 6% per year.

(vii) The hurdle rate is 10% per year.

(viii) The insurer holds a reserve of 100 at the start of each year, including the first.

(a) (*1 point*) Describe two uses of profit testing in insurance practice.

(b) (*4 points*)

(i) (*3 points*) Calculate the profit vector, $\left(\text{Pr}_0, \text{Pr}_1, \text{Pr}_2\right)$.

(ii) (*1 point*) Calculate the profit signature, $\left(\Pi_0, \Pi_1, \Pi_2\right)$.

4. Continued

(c) (*3 points*)

(i) Show that the net present value of the contract is 70 to the nearest 10. You should calculate the value to the nearest 0.1.

(ii) Without further calculation, state with reasons whether the internal rate of return is greater than or less than the hurdle rate.

(iii) Calculate the profit margin.

(d) (*3 points*) Your manager wants to change the gross premium to achieve a profit margin of 5%. Calculate the revised gross premium that will meet this objective.

5.

(*11 points*) For a Type B universal life policy with annual premiums, you are given the following information:

(i) The Additional Death Benefit is 100,000.

(ii) The minimum annual premium is 2,000.

(iii) The death benefit is paid immediately on death.

(iv) The expense charge is 100 plus 2% of the premium.

(v) The CoI is calculated based on 120% of the Standard Ultimate Mortality Model.

(vi) CoIs are not discounted (that is, $i_q = 0$).

(vii) The annual effective credited interest rate of $i_c = 0.06$ is credited continuously to account values.

(viii) There is no corridor factor.

A life age 50 purchases this policy. She pays annual premiums of 2,000 throughout the term of the policy.

(a) (*3 points*)

(i) Show that the Account Value at the end of the second year is 3730 to the nearest 10. You should calculate the value to the nearest 0.1.

(ii) Roughly sketch a graph of the account values, AV_t, as a function of *t*, for $0 \le t < 2$. You should identify key values on your sketch.

(b) (*2 points*) Show that the youngest age at which the deductions (cost of insurance and expenses) are greater than the premium for this policyholder is age 74.

5. Continued

(c) (*3 points*)

(i) Assume that the policyholder holds the policy until her death at age 91.6.

You are given that $AV_{41} = 109,467$.

Calculate the total amount of death benefit paid.

(ii) Roughly sketch a graph of the account values, AV_t, as a function of *t*, for $41 \le t < 41.6$.

(d) (*3 points*) The insurer offers a special joint life and last survivor version of the policy, under which a death benefit of 50,000 is paid out on the first death of a couple. On the second death, the death benefit is 50,000 plus the account value. The CoI is calculated assuming that the lives have independent future lifetimes and that the mortality of each life is 120% of the Standard Ultimate Mortality Model. All other policy details and assumptions are as given above.

A couple, Linda and Xiaobai, who are both age 50, purchases this policy. Linda also buys the single life policy.

(i) Show that in the first year, the CoI for this policy is the same as the CoI for the single life policy.

(ii) Describe how the account values of the single life policy and joint life policy would compare at the end of the *t*-th year if both Linda and Xiaobai survive to time $t-1$.

(iii) Describe how the account values of the single life policy and joint life policy would compare at the end of the *t*-th year if only Linda survives to time $t-1$.

6.

(*9 points*) A 10-year equity-linked policy is issued to a life age 60. The policyholder pays a single premium of 1000. The policy offers a guaranteed minimum maturity benefit (GMMB) of 100% of the single premium. The insurer deducts a management charge at a rate of 2% per year, payable continuously, from the policyholder's funds. There are no other expense deductions.

The insurer constructs a hedge portfolio at the issue date based on the Black-Scholes option valuation. You are given the following assumptions.

(i) The risk free rate is $r = 0.03$ continuously compounded.

(ii) The volatility is $\sigma = 0.20$

(iii) Mortality follows the Standard Ultimate Mortality Model.

(iv) There are no exits other than death.

(v) $\bar{a}_{60:\overline{10}|\delta=0.02} = 8.8569$

(a) (*4 points*)

(i) Show that the GMMB hedge portfolio value at the issue date is 150 to the nearest 10. You should calculate the value to the nearest 0.01.

(ii) Let *c* denote the cost of the hedge portfolio expressed as a continuous risk premium. Show that $c = 0.017$ to the nearest 0.001. You should calculate the value to the nearest 0.0001.

Over the first 3 years, the price of the underlying stocks in the policyholder's fund increases by 50%.

(b) (*3 points*)

(i) Calculate the policyholder's fund value at $t = 3$.

(ii) Calculate the value of the total GMMB hedge portfolio required at $t = 3$.

(iii) You are given that $\bar{a}_{63:\overline{7}|\delta=0.02} = 6.4021$. State with reasons whether the charge determined in (a)(ii) is sufficient to pay the future costs of the GMMB as of $t = 3$.

6. Continued

(c) (*2 points*) The insurer is considering introducing a reset option. The minimum contract term is 10 years. Resets would be allowed at any policy anniversary.

(i) Assume that the policyholder exercised the reset option at $t = 3$. Explain in words the effect of the reset on the policy.

(ii) Describe one advantage and one disadvantage of the reset option from the insurer's perspective.

****END OF EXAMINATION****